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RECURSIVE IDENTIFICATION OF HAMMERSTEIN SYSTEMS WITH POLYNOMIAL NONLINEARITIES

Jozef Vörös *

The paper deals with recursive identification of nonlinear dynamic systems using Hammerstein models with two-segment polynomial nonlinearities. A special form of Hammerstein model is considered, which is linear-in-parameters. The proposed algorithm is a direct application of the known recursive least squares method supplemented with the estimation of internal variables.

K e y w o r d s: nonlinear systems, recursive identification, Hammerstein model, polynomial nonlinearity

1 INTRODUCTION

For the subclasses of nonlinear dynamic systems that can be considered as block-oriented systems there exist several identification methods using topologically identical models [1]. One of the simplest nonlinear models of this category is the so-called Hammerstein model consisting of one nonlinear static block and one linear dynamic block and many processes have this structure (*eg* , [2, 3]). The linear block is mostly described by its transfer function and the static nonlinearity is often characterized by a simple polynomial approximation, although other approaches have also been applied (*eg* , Haar approximations [4], piecewise linear maps [5], *etc*).

Recursive identification methods are important not only for the property that they can be computed in real time, but they may be combined with on-line control strategies to produce adaptive control algorithms. Some of them were applied to the nonlinear systems of Hammerstein type where a certain redundancy was considered in the chosen form of model description because of the combination of linear and nonlinear block parameters [6–8]. Despite the fact that this increases the total number of parameters and it is not easy to separate them, this approach is the only one where the chosen model is linear-in-parameters.

In this paper a special form of the Hammerstein model, based on a decomposition technique [9], is considered where the nonlinear static block is characterized by a two-segment polynomial approximation. This model is linear-in-parameters and is used for the recursive identification of nonlinear dynamic systems. The proposed algorithm is a direct application of the known recursive least squares method [10] supplemented with the estimation of internal variables. It enables the estimation of both the parameters of linear block transfer functions and the coefficients of polynomials approximating nonlinear characteristics using the system inputs, outputs and estimated internal variables.

2 HAMMERSTEIN MODELS WITH POLYNOMIAL NONLINEARITIES

The Hammerstein model is given by the cascade connection of a static nonlinearity block followed by a linear dynamic system shown in Fig. 1. The linear dynamic system can be described as

$$y(t) = q^{-d}B(q^{-1})x(t) - [A(q^{-1}) - 1]y(t) + v(t) \qquad (1)$$

where q^{-d} represents the pure time delay of the system, $x(t)$ and $y(t)$ are the inputs and outputs, respectively, $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_mq^{-m}\,, \qquad (2)$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_nq^{-n} \qquad (3)$$

and $v(t)$ is the output noise. The nonlinear block is characterized by the mapping $C(\cdot)$:

$$x(t) = C[u(t)] \qquad (4)$$

where $u(t)$ and $x(t)$ are the inputs and outputs, respectively.

Although a polynomial approximation of the mapping $C(\cdot)$ may be quite appropriate, there are nonlinearities with significantly different characteristics for the positive and negative inputs, with discontinuities or discontinuous derivatives in the origin, where a single polynomial approximation may fail. In these cases the so-called two-segment polynomial approximation can be used [11] where the nonlinear block is characterized as follows:

$$x(t) = \begin{cases} f[u(t)] & \text{if } u(t) \ge 0\,, \\ g[u(t)] & \text{if } u(t) < 0\,. \end{cases} \qquad (5)$$

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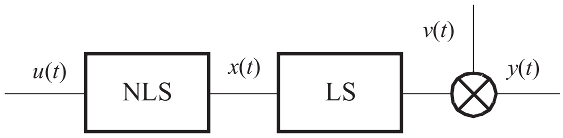

Fig. 1. Hammerstein model

After defining the switching function

$$h(t) = h[u(t)] = \begin{cases} 0 & \text{if } u(t) \geq 0\,, \\ 1 & \text{if } u(t) < 0 \end{cases} \tag{6}$$

the relation between the input $u(t)$ and the output $x(t)$ of the assumed nonlinearity can be written as follows:

$$x(t) = C[u(t)] = f[u(t)] + \big\{g[u(t)] - f[u(t)]\big\}h(t)\,. \tag{7}$$

If the nonlinear maps $f(\cdot)$ and $g(\cdot)$ can be approximated by the polynomials

$$f[u(t)] = \sum_{k=0}^{r} f_k u^k(t)\,, \tag{8}$$

$$g[u(t)] = \sum_{k=0}^{r} g_k u^k(t) \tag{9}$$

then (7) can be written as follows:

$$x(t) = \sum_{k=0}^{r} f_k u^k(t) + \sum_{k=0}^{r} p_k u^k(t) h(t) \tag{10}$$

where $p_k = g_k - f_k$. After substituting (10) into (1) the Hammerstein model is described by the equation

$$y(t) = \sum_{i=1}^{n} b_i \Big\{ \sum_{k=0}^{r} f_k u^k(t-d-i) + \sum_{k=0}^{r} p_k u^k(t-d-i)h(t-d-i) \Big\} - \sum_{j=1}^{m} a_j y(t-j) + v(t) \tag{11}$$

which is linear in the parameters a_j and the combinations of parameters $b_i f_k$ and $b_i p_k$. The estimates of unknown parameters can be generated by the standard recursive least squares (RLS) algorithm [10] using the measured system inputs and outputs. However, the number of parameters is $2(r+1)(n+1)+m$ and it is not a trivial problem to separate the parameters b_i, f_k and p_k.

To separate the parameters in the above model, the decomposition of Hammerstein operator can be performed using the key term separation principle [9]. Assuming $b_0 = 1$ (one parameter can be always fixed in this model), the internal variable can be separated in the linear block description as follows:

$$y(t) = x(t-d) + \sum_{i=1}^{n} b_i x(t-d-i) - \sum_{j=1}^{m} a_j y(t-j) + v(t)\,. \tag{12}$$

Then the half-substitution of (10) into (12) only for the separated x(t-d) will lead to the Hammerstein model output equation in the form

$$y(t) = \sum_{k=0}^{r} f_k u^k(t-d) + \sum_{k=0}^{r} p_k u^k(t-d)h(t-d) + \sum_{i=1}^{n} b_i x(t-d-i) - \sum_{j=1}^{m} a_j y(t-j) + v(t)\,. \tag{13}$$

Now the decomposed Hammerstein model is given by Eqs. (10) and (13). Both equations are linear-in-parameters and the output equation contains the minimum number of parameters, *ie* , $2(r+1)+n+m$, because all the model parameters are separated.

3 RECURSIVE IDENTIFICATION

The problem with the decomposed form of Hammerstein model given by (10) and (13) is that the internal variable $x(t)$ is not accessible for measurement. Therefore an iterative identification method was proposed with the internal variable estimation [9]. The values of internal variable $x(t)$ are recomputed in each iteration using the previous estimates of nonlinear block parameters. This off-line (batch) method can be easily converted into an on-line version.

Defining the following parameter and data vectors

$$\theta = \big[f_0, f_1, \dots, f_r, p_0, p_1, \dots, p_r, b_1, \dots, b_n, a_1 \dots, b_m\big]^\top, \tag{14}$$

$$\varphi(t) = \big[1, u(t-d), \dots, u^r(t-d), h(t-d), u(t-d)h(t-d), \dots, u^r(t-d)h(t-d), x(t-d-1), \dots, x(t-d-n), -y(t-1), \dots, -y(t-m)\big]^\top\,. \tag{15}$$

the Hammerstein model equation can be written in the concise form as:

$$y(t) = \varphi^\top(t)\theta \tag{16}$$

where the internal variable $x(t)$ depends on the vector θ and therefore (implicitly) $\varphi(t) = \varphi(t,\theta)$.

The estimates of the parameter vector can be evaluated using the modified RLS algorithm, minimizing the least-squares criterion

$$\hat{\theta} = \arg\min_{\theta} \sum_{k=1}^{t} \big[y(k) - \hat{\varphi}^\top(k)\theta\big] \tag{17}$$

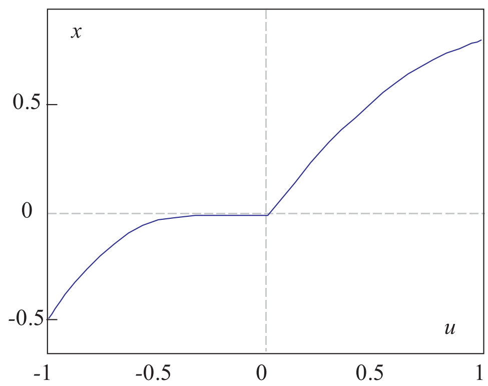

Fig. 2. Example 1 — nonlinearity

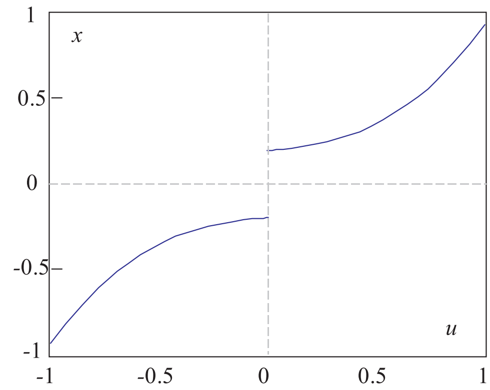

Fig. 4. Example 2 — nonlinearity

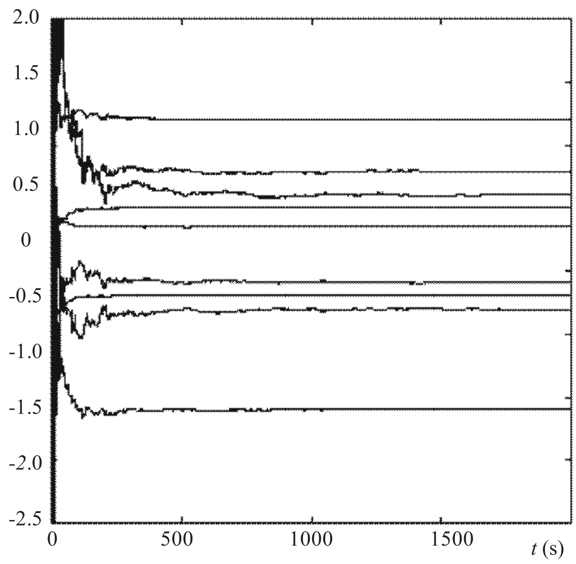

Fig. 3. Example 1 — parameter estimates

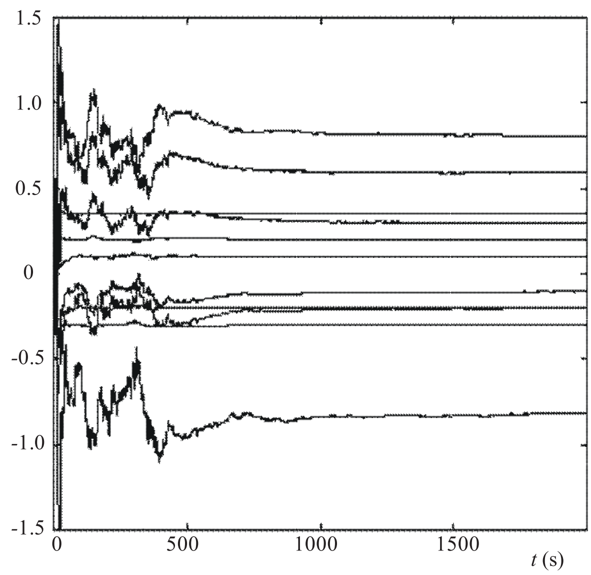

Fig. 5. Example 2 — parameter estimates

based on (16), where the data vector $\varphi(t)$ is replaced by $\hat{\varphi}(t)$ with the estimates of the internal variable. The formulae of recursive identification algorithm supplemented with the internal variable estimation are as follows:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-1)\hat{\varphi}(t)[y(t) - \hat{\varphi}^{\top}(t)\hat{\theta}(t-1)]}{\lambda + \hat{\varphi}^{\top}(t)P(t-1)\hat{\varphi}(t)}, \tag{18}$$

$$P(t) = \frac{1}{\lambda}\Big[P(t-1) - \frac{P(t-1)\hat{\varphi}(t)\hat{\varphi}^{\top}(t)P(t-1)}{\lambda + \hat{\varphi}^{\top}(t)P(t-1)\hat{\varphi}(t)}, \tag{19}$$

$$\hat{x}(t) = \sum_{k=0}^{r} \hat{f}_k(t-1)u^k(t) + \sum_{k=0}^{r} \hat{p}_k(t-1)u^k(t)h(t), \tag{20}$$

$$\begin{aligned}\hat{\varphi}(t) = \big[&1, u(t-d), \dots, u^r(t-d), h(t-d), u(t-d)h(t-d),\\ &\dots, u^r(t-d)h(t-d), \hat{x}(t-d-1), \dots,\\ &\hat{x}(t-d-n), -y(t-1), \dots, -y(t-m)\big]^{\top}\end{aligned} \tag{21}$$

$$P(0) = \mu I, \quad 0 < \mu < \infty \tag{22}$$

where the new values (estimates) of internal variable for the data vector (21) in each recursion are computed by (20) with the previous estimates of parameters f_k and p_k and $\lambda \leq 1$ is the so-called forgetting factor.

A bad initialization of a recursive algorithm leads generally to various problems such as convergence to a local minimum, to a wrong estimate or instability. However, there is no approach, which can be recommended as being universal to give an analytic solution to this problem. This fact is related to the many different features that the nonlinear relation can introduce.

For the decomposed form of Hammerstein model, one way to overcome the initialization difficulty is to consider the output as a linear parameter representation where the data vector contains only (measured) input and output data. There are some alternatives for the initialization of the above algorithm depending on the first estimates of internal variable $x(t)$. In all the cases the nonlinear block parameters, which determine the internal variable, are included into the estimation process immediately.

In the simplest case of a quasi-Hammerstein model the estimation of parameters b_i can be omitted in the first step, *ie* , $x(t-d-i)=0$. The second alternative is to start the recursion with the internal variable $x(t-d-i)=u(t-d-i)$ in the first step. Other possibilities are to run the recursion for some times (about 10–20 steps) only with the above alternatives and then to continue with the inclusion of the estimates of internal variable. This would correspond with the first step of the iterative method in [9].

Further problems may be caused by wild fluctuations occurring in the early steps of recursion. If they appear with the parameters of nonlinear block determining the internal variable estimates the recursive process can be prolonged or eventually become unstable. Using a proper forgetting factor in the RLS algorithm can positively influence these problems. Finally note that there is no general proof of convergence for the identification methods using the Hammerstein model [12], or block-oriented nonlinear models with internal variable estimation [13], although they are satisfactory for most practical applications.

4 SIMULATION STUDIES

The presented method for the recursive identification of nonlinear dynamic systems using the Hammerstein model with polynomial nonlinearities was implemented and tested by means of MATLAB. Several systems were simulated and the estimation of all the model parameters (those of linear and nonlinear blocks) and the internal variables were carried out on the basis of input/output records.

To illustrate the feasibility of the proposed identification method, the following examples show the parameter estimation process for simulated Hammerstein systems. In the first example the nonlinear block of the Hammerstein model was given by the two-segment characteristic shown in Fig. 2, where

$$f[u(t)] = 1.2u(t) - 0.3u^2(t) - 0.1u^3(t)$$
$$p[u(t)] = -1.1u(t) + 0.6u^2(t) + 0.8u^3(t)$$

and the linear block was described by

$$y(t) = x(t-1) + 0.5x(t-2) + 0.2y(t-1) - 0.35y(t-2)\,.$$

The identification was carried out with 2000 samples of uniformly distributed random inputs $|u(t)| \le 1$ and simulated outputs. The initial values of the parameters were chosen zero. The output noise was generated as a zero mean white noise and the signal to noise ratio (the square root of the ratio of output and noise variances) was $\mathrm{SNR} = 50$. Generally, the internal variable estimation requires the use of lower forgetting factor λ to reduce the influence of old data, while a value of λ close or equal to 1 is less sensitive to disturbance [10], [14]. Therefore two forgetting factors were used in this example, *ie* , $\lambda = 0.98$ for the first 200 samples and $\lambda = 1.0$ for the rest of data. The process of parameter estimation is shown in Fig. 3 (the top-down order of parameters is f_1, p_3, p_2, b_1, a_2, f_3, a_1, f_2, p_1) and the estimated values converge to the true values after about 750 steps.

A special case of two-segment nonlinearity with preloads (Fig. 4) characterized by

$$f[u(t)] = 0.2 - 0.1u(t) + 0.8u^2(t) - 0.2u^3(t)$$
$$p[u(t)] = -0.3 + 0.3u(t) - 0.8u^2(t) + 0.6u^3(t)$$

was assumed in the second example where the linear system was described by

$$y(t) = x(t-1) + 0.1x(t-2) + 0.2y(t-1) - 0.35y(t-2)\,.$$

The output noise was generated as a zero mean white noise with $\mathrm{SNR} = 100$. The identification was performed under the same condition as in the first example. The process of parameter estimation is shown in Fig. 5 (the order of parameters is f_2, p_3, a_2, p_1, f_0, b_1, f_1, $a_1 = f_3$, p_0, p_2) and the estimated values converge to the true values after about 1000 steps.

In the implementation of proposed recursive algorithm the MISO form of MATLAB function 'rarx' was used and the internal variable in the data vector was externally updated. The recursive algorithm proved good convergence. The parameter estimate fluctuations appearing in the early steps are not too severe.

5 CONCLUSION

Identification of nonlinear dynamic systems is a very difficult problem and no approach can be recommended as being universal. Although several identification methods using block-oriented models are available, further research is needed, especially in connection with adaptive control algorithms.

The proposed forms of the Hammerstein models with two-segment polynomial nonlinearities seem to be appropriate for the recursive identification of a broad subclass of nonlinear dynamic systems. The main potential of the method is in on-line process monitoring and analysis. As the model is linear-in-parameters, several approaches to the adaptive control using linear models can be adopted for the control of this subclass of nonlinear dynamic systems [14].

Finally, note that similar approach can be applied to the Wiener model decomposed into a form, which is linear-in-parameters [9].

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