COMMUNICATIONS

UNCERTAINTIES IN THE WHOLE RANGE OF CALIBRATION OF A THERMOCOUPLE

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This contribution describes the procedure of evaluating the calibration of a thermocouple by means of its comparison with the thermocouple standard. In the process of thermocouple calibration by means of comparison, the resulting uncertainty is specified by applying the generalized procedure for evaluating the calibration of measuring devices with continuous scale. The advantage of this method of evaluation is the determination of uncertainties in the whole range of calibration. The conclusion of this paper presents illustrated differences between cases when covariances are taken into account or not.

**w o r d s: calibration, thermocouple, estimation unknown parameters, uncertainties, covariance

1 INTRODUCTION

The best way for increasing the accuracy of measurement in modern metrology is often the application of a modern mathematical-statistical method which until now has not been sufficiently utilized for evaluating the calibration of instruments. This attitude is reasonable because current technical solutions are so perfect that their development stagnates. For a measuring instrument with a continuous scale a generalized procedure for evaluating the calibration uncertainties and covariances has been developed by Palenčár, Wimmer [1, 2] and Kubáček [6]. In this paper authors are presenting these procedures for evaluating the uncertainties of calibration of a type S thermocouple (hereafter TC only) by means of comparison.

2 CALIBRATION PROCEDURE

Calibration is carried out by comparison of the unit under test against a standard TC of type S calibrated in defined fixed points according to ITS-90 (Fig. 1). The thermoelectric voltage (emf) is measured by a digital voltmeter connected to a PC through GPIB port for simulta-

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temperature is 23°C, the number of calibration points is higher than the number of unknown parameters, and generally it is given as a function of temperature \( t \). Thus the model is overdetermined. The left side of model (1) or (2), the observation vector \( \mathbf{W} \) is the measurement model of the unit under test.

\[
W = \mathbf{T} \mathbf{a}
\]  
where \( \mathbf{T} \) is a matrix which contains the values, arithmetical means of series of measurements in each calibration point measured by the standard TC:

\[
\mathbf{T} = \begin{pmatrix}
1 & t_{100} & t_{100}^2 & t_{100}^3 & t_{100}^4 \\
1 & t_{200} & t_{200}^2 & t_{200}^3 & t_{200}^4 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & t_{1100} & t_{1100}^2 & t_{1100}^3 & t_{1100}^4
\end{pmatrix}
\]  

The vector of deviations from the reference function (5). The reference function is given by IEC 584.2 standard (6)

\[
E_{\text{ref}} = \sum_{k=1}^{8} b_k t_i^k, \quad i = 100, 200, \ldots, 1100,
\]

\[
\Delta \mathbf{E} = \begin{pmatrix}
E_{100} - E_{\text{ref,100}} \\
E_{200} - E_{\text{ref,200}} \\
\vdots \\
E_{1100} - E_{\text{ref,1100}}
\end{pmatrix}
\]

in product of \( C_K \lambda \) are reflected features concerning the unit under test TC. The vector of correction \( \lambda \) is given by

\[
\lambda_{1 \times 20}^T = \begin{pmatrix}
\delta E_{1H} & \delta E_{RV} & \delta E_K & \delta E_D & \delta E_{CK} \\
\delta E_N & \delta t_{RO} & \delta t_F & \delta t_{RF}
\end{pmatrix}
\]

where \( \delta E_{1H} \) – correction linked to the reading of the voltmeter \( \delta E_{RV} \) – correction linked to the resolution the voltmeter \( \delta E_K \) – correction obtained from the calibration of the voltmeter \( \delta E_D \) – correction linked to the drift of the voltmeter \( \delta E_{CK} \) – correction linked to the compensation cable \( \delta E_N \) – correction due to the inhomogeneity of the thermocouple wires \( \delta t_{RD} \) – correction due to the deviation of the ice bath temperature \( \delta t_F \) – correction linked to the nonuniformity of the temperature profile \( \delta t_{RF} \) – error of reference function.

3 METHODOLOGY

We consider the case, when the number of calibration points \( r \) is higher than the number of unknown parameters \( p \), thus the model is overdetermined. The calibration model should be established using the following relations (1)

\[
W_{100} = a_0 + a_1 t_{100} + a_2 t_{100}^2 + a_3 t_{100}^3 + a_4 t_{100}^4 \\
W_{200} = a_0 + a_1 t_{200} + a_2 t_{200}^2 + a_3 t_{200}^3 + a_4 t_{200}^4 \\
\vdots \\
W_{1100} = a_0 + a_1 t_{1100} + a_2 t_{1100}^2 + a_3 t_{1100}^3 + a_4 t_{1100}^4
\]  

in matrix notation

\[
\mathbf{W} = \mathbf{T} \mathbf{a}
\]

\[
\begin{align*}
\text{Table 1. Measured and computed values } & & \text{(nominal values), } \\
\text{Cal. Points } i & t(E_{iK}) \xi & t(E_{iK}) \xi (\text{C}) \\
100 & 99.8188 & 643.85 \\
200 & 199.7252 & 1437.82 \\
300 & 299.7120 & 2319.79 \\
400 & 399.7737 & 3255.32 \\
500 & 499.8191 & 4228.52 \\
600 & 599.6689 & 5230.63 \\
700 & 699.7653 & 6265.91 \\
800 & 799.8341 & 7334.86 \\
900 & 899.7653 & 8437.38 \\
1000 & 999.5335 & 9569.7 \\
1100 & 1099.4 & 10736.91
\end{align*}
\]
and matrix $C_K$ is the known matrix; usually its elements are sensitivity coefficients.

Our aim is to get estimation for unknown parameters of the deviation function. This aim could be reached by using the least-square method. Uncertainties are taken into account as well. We apply the following expression iteratively because of the stochastic nature of quantity $t$.

$$\hat{a} = (T^TU_W^{-1}T)^{-1}T^TU_W^{-1}U_W. \quad (8)$$

Initial values of unknown parameters $\hat{a}$ of the deviation function are determined by zero estimation. Then the covariance matrix of input quantities $U_W$ is

$$U_W = U_{AE} + C_KU_A\,C_K^T \quad (9)$$

where $U_{AE}$ — covariance matrix of vector $\Delta E$ is a diagonal matrix; principal-diagonal elements present the square of uncertainties estimated by type A method, $C_KU_A\,C_K^T$ — product of these matrices gives a diagonal covariance matrix; principal-diagonal elements present the square of uncertainties estimated by type B method, $U_A$ — uncertainties of correction of measurements by unit under test TC are included in this covariance matrix.

Covariance matrix $U_{\hat{a}}$ is represented by the matrix of uncertainties of the estimates

$$U_{\hat{a}} = (T^TU_W^{-1}T)^{-1} \quad (10)$$

Deviation associated with the reference function is solved by

$$\Delta \hat{E} = \mathbf{T} \hat{a} \quad (11)$$

and uncertainty of the deviation can be achieved by applying the law of propagations of uncertainties

$$U_{\Delta \hat{E}}^2 = \mathbf{T}U_{\hat{a}}\mathbf{T}^T. \quad (12)$$

The zero estimation of vector $\hat{a}$ is biased, see Fig. 2. It is caused by the stochastic nature of quantity $t$. Therefore the model is nonlinear and requires a solution procedure. It is linearized by applying a Taylor series and higher elements of estimated values are neglected. After linearization, the left side of model vector $W$ will be

$$W = \Delta \hat{E} + C_KA + D(\delta t_1 + C_S\delta t_2). \quad (13)$$

$$D = \text{diag}(d_{100}, d_{200}, \ldots, d_{1100})$$

is the known matrix obtained by applying the Taylor series

$$d_{100} = \hat{a}_1 + 2\hat{a}_2t_{T100} + 3\hat{a}_3t_{T100}^2 + 4\hat{a}_4t_{T100}^3 \quad (14)$$

$$d_{200} = \hat{a}_1 + 2\hat{a}_2t_{T200} + 3\hat{a}_3t_{T200}^2 + 4\hat{a}_4t_{T200}^3 \quad (15)$$

$$\vdots$$

$$d_{1100} = \hat{a}_1 + 2\hat{a}_2t_{T1100} + 3\hat{a}_3t_{T1100}^2 + 4\hat{a}_4t_{T1100}^3$$

$(\delta t_1 + C_S\delta t_2)$ — this part is valid for standard, vector $\delta t_1$ — error given by standard, product $C_S\delta t_2$ — contains the influence quantities concerning a standard TC in the same way as in expression (4) for the unit under test.

After linearization, the covariance matrix $U_W$ has the form

$$U_W = U_{\Delta \hat{E}} + C_KU_A\,C_K^T + D(U_{\delta t_1} + C_SU_{\delta t_2}C_S^T)D^T \quad (15)$$

where $U_{\delta t_1}$ — covariance matrix of vector $\delta t_1$ is a diagonal matrix; principal-diagonal elements present the square of uncertainties estimated by type A method, $C_SU_{\delta t_2}C_S^T$ — product of this matrix is given by diagonal covariance matrix; principal-diagonal elements present square of uncertainties estimated by type B method, $U_{\delta t_2}$ — uncertainties of correction of measurements by standard are included in this covariance matrix.

Now in new iteration we consider the observation vector $W$ (13) and covariance matrix $U_W$ (15) and we use the formula for estimation of parameters (8).

Numerically, in most cases the design matrix $\mathbf{T}$ is badly scaled and its columns are nearly linearly dependent. For this it is reasonable to transform the quantities of $t$ to interval $-1 \leq t \leq 1$ according to the following relationship

$$t_T = \frac{(t_i - t_{\text{max}}) - (t_{\text{max}} - t_i)}{t_{\text{max}} - t_{\text{min}}} \quad ; \quad i = 100, 200, \ldots, 1100. \quad (16)$$

Backward transformation is carried out multiplying the rows of vector $t_T$ defined by (16) by columns of matrices of estimated parameters $\hat{a}$ and of covariance matrix $U_{\hat{a}}$, consecutively. First from the left, then from the right side.

From the viewpoint of the user, relevant results are the temperature values and their uncertainties. The temperature value can be obtained by an interpolation table which can be edited from the deviation function and its uncertainty is determined by applying the theorem for an implicit function.

$$f(E, t, a) = E - g(t, a) = 0 \quad (17)$$

$$g(t, a) = a_0 + (a_1 + b_1)t + (a_2 + b_2)t^2 + (a_3 + b_3)t^3 + (a_4 + b_4)t^4 + b_5t^5 + b_6t^6 + b_7t^7 + b_8t^8. \quad (18)$$

We get it by adding up the deviation function and reference function, where variable $E$ is representing the current measured value of emf. Now consider function $t = (h, a)$ defined from the implicit function. This function is continuous and we have the partial derivative [3]

$$\frac{\partial h}{\partial a_k} = -\frac{\partial t}{\partial a_k} = -\frac{\partial t}{\partial a} = \frac{t_k}{S(t)}, \quad k = 0, \ldots, 4. \quad (19)$$

Derivative (19) is presenting elements of vector $h$. Standard uncertainty is then obtained from relation

$$u^2(t) = h^T U_{\hat{a}} h. \quad (20)$$
4 CONCLUSION

A procedure for evaluating the calibration of a TC was applied to demonstrate whether considering the covariances has an impact on the final result of standard uncertainty. For this reason, evaluation was carried out twice. The difference is shown in Fig. 3. As a conclusion we can claim that covariances had significant effect on the final result of a calibration.

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