Journal of ELECTRICAL ENGINEERING, VOL. 58, NO. 1, 2007, 51–54

COMMUNICATIONS

UNCERTAINTIES IN THE WHOLE RANGE OF CALIBRATION OF A THERMOCOUPLE

Peter Benkó * — Rudolf Palenčár **

This contribution describes the procedure of evaluating the calibration of a thermocouple by means of its comparison with the thermocouple standard. In the process of thermocouple calibration by means of comparison, the resulting uncertainty is specified by applying the generalized procedure for evaluating the calibration of measuring devices with continuous scale. The advantage of this method of evaluation is the determination of uncertainties in the whole range of calibration. The conclusion of this paper presents illustrated differences between cases when covariances are taken into account or not.

K e y w o r d s: calibration, thermocouple, estimation unknown parameters, uncertainties, covariance

1 INTRODUCTION

The best way for increasing the accuracy of measurement in modern metrology is often the application of a modern mathematical-statistical method which until now has not been sufficiently utilized for evaluating the calibration of instruments. This attitude is reasonable because current technical solutions are so perfect that their development stagnates. For a measuring instrument with a continuous scale a generalized procedure for evaluating the calibration uncertainties and covariances has been developed by Palenčár, Wimmer [1, 2] and Kubáček [6]. In this paper authors are presenting these procedures for evaluating the uncertainties of calibration of a type S thermocouple (hereafter TC only) by means of comparison.

2 CALIBRATION PROCEDURE

Calibration is carried out by comparison of the unit under test against a standard TC of type S calibrated in defined fixed points according to ITS-90 (Fig. 1). The thermoelectric voltage (emf) is measured by a digital voltmeter connected to a PC through GPIB port for simulta-

* Slovak Legal Metrology, Geologická 1, 822 11 Bratislava, Slovakia. E-mail: benko@slm.sk

** STU Faculty of Mechanical Engineering nám. Slobody 17, 812 31 Bratislava, Slovakia. E-mail: rudolf.palencar@stuba.sk

ISSN 1335-3632

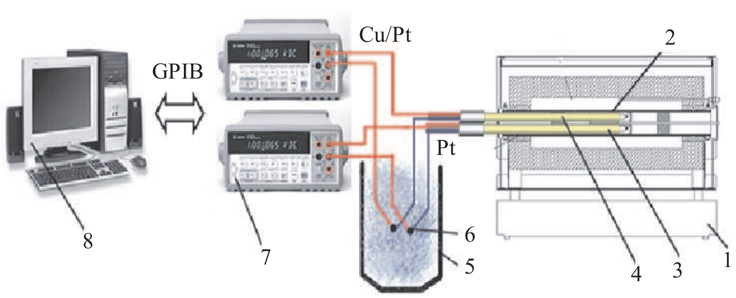

Fig. 1. Scheme of calibration 1 – Calibration furnace, 2 – Isothermal block, 3 – Standard TC, 4 – Unit under test, 5 – Dewar flask, 6 – Reference junction of TC, 7 – Voltmeters, 8 – Computer with GPIB port.

Table 1. Measured and computed values i – calibration points (nominal values), $t(\overline{E}_{S_i})$ (°C) – values measured by standard TC, $\overline{E}_{K_i})$ (°C) – values measured by TC under test.

Cal. Points i	$t(\overline{E}_{S_i})$ (°C)	$\overline{E}_{K_i})$ (°C)
100	99.8188	643.85
200	199.7252	1437.82
300	299.7120	2319.79
400	399.7737	3255.32
500	499.8191	4228.52
600	599.6689	5230.63
700	699.7653	6265.91
800	799.8341	7334.86
900	899.7653	8437.38
1000	999.5335	9569.7
1100	1099.4	10736.91

neous recording of values. As a source of heat a horizontal pipe calibration furnace is used. Here the TC measuring junction is placed and the reference junction is maintained at 0 °C in a Dewar flask. Calibration is carried out in the range from 0 °C to 1100 °C. In each calibration point, measurement is repeated ten times. The ambient temperature is 23 °C ± 1 °C. Calibration is represented as a curve fitted to the measured values of the deviation $E - E_{\mathrm{ref}}$ and generally it is given as a function of temperature t. This curve represents the deviation function.

3 METHODOLOGY

We consider the case, when the number of calibration points r is higher than the number of unknown parameters p, $r > p$, thus the model is overdetermined. The calibration model should be established using the following relations (1)

$$
\begin{aligned}
W_{100} &= a_0 + a_1 t_{100} + a_2 t_{100}^2 + a_3 t_{100}^3 + a_4 t_{100}^4 \\
W_{200} &= a_0 + a_1 t_{200} + a_2 t_{200}^2 + a_3 t_{200}^3 + a_4 t_{200}^4 \\
&\vdots \\
W_{1100} &= a_0 + a_1 t_{1100} + a_2 t_{1100}^2 + a_3 t_{1100}^3 + a_4 t_{1100}^4
\end{aligned}
\tag{1}
$$

in matrix notation

$$\mathbf{W} = \mathbf{T}\boldsymbol{a} \tag{2}$$

where $\mathbf{T}$ is a matrix which contains the values, arithmetical means of series of measurements in each calibration point measured by the standard TC:

$$
\mathbf{T} = \begin{pmatrix}
1 & t_{100} & t_{100}^2 & t_{100}^3 & t_{100}^4 \\
1 & t_{200} & t_{200}^2 & t_{200}^3 & t_{200}^4 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
1 & t_{1100} & t_{1100}^2 & t_{1100}^3 & t_{1100}^4
\end{pmatrix} . \tag{3}
$$

The left side of model (1) or (2), the observation vector $\mathbf{W}$ is the measurement model of the unit under test

$$\mathbf{W} = \Delta\mathbf{E} + \mathbf{C}_K \boldsymbol{\Lambda} \tag{4}$$

where $\Delta\mathbf{E}$ is the vector of deviations from the reference function (5). The reference function is given by IEC 584.2 standard (6)

$$E_{\mathrm{ref}_i} = \sum_{k=1}^{8} b_k t_i^k \,, \; i = 100, 200, \ldots, 1100 \,, \tag{5}$$

$$
\Delta\mathbf{E} = \begin{pmatrix}
\overline{E}_{100} - E_{\mathrm{ref}_{100}} \\
\overline{E}_{200} - E_{\mathrm{ref}_{200}} \\
\vdots \\
\overline{E}_{1100} - E_{\mathrm{ref}_{1100}}
\end{pmatrix} \tag{6}
$$

in product of $\mathbf{C}_K \boldsymbol{\Lambda}$ are reflected features concerning the unit under test TC.

The vector of correction $\boldsymbol{\lambda}$ is given by

$$
\boldsymbol{\Lambda}_{1\times 20}^{\top} = \begin{pmatrix}
\delta E_{IH} & \delta E_{RV} & \delta E_K & \delta E_D & \delta E_{CK} \\
 & \delta E_N & \delta t_{RO} & \delta t_F & \delta t_{RF}
\end{pmatrix} \tag{7}
$$

where

δE_{IH} – correction linked to the reading of the voltmeter
δE_{RV} – correction linked to the resolution the voltmeter
δE_K – correction obtained from the calibration of the voltmeter
δE_D – correction linked to the drift of the voltmeter
δE_{CK} – correction linked to the compensation cable
δE_N – correction due to the inhomogeneity of the thermocouple wires
δt_{RD} – correction due to the deviation of the ice bath temperature
δt_F – correction linked to the nonuniformity of the temperature profile
δt_{RF} – error of reference function

and matrix $\mathbf{C}_K$ is the known matrix; usually its elements are sensitivity coefficients.

Our aim is to get estimation for unknown parameters of the deviation function. This aim could be reached by using the least-square method. Uncertainties are taken into account as well. We apply the following expression iteratively because of the stochastic nature of quantity t.

$$\hat{\mathbf{a}} = \left(\mathbf{T}^\top \mathbf{U}_W^{-1} \mathbf{T}\right)^{-1} \mathbf{T}^\top \mathbf{U}_W^{-1} \mathbf{W}. \tag{8}$$

Initial values of unknown parameters $\hat{\mathbf{a}}$ of the deviation function are determined by zero estimation. Then the covariance matrix of input quantities $\mathbf{U}_W$ is

$$\mathbf{U}_W = \mathbf{U}_{\Delta E} + \mathbf{C}_K \mathbf{U}_\Lambda \mathbf{C}_K^\top \tag{9}$$

where
$\mathbf{U}_{\Delta E}$ – covariance matrix of vector $\Delta\mathbf{E}$ is a diagonal matrix; principal-diagonal elements present the square of uncertainties estimated by type A method,
$\mathbf{C}_K \mathbf{U}_\Lambda \mathbf{C}_K^\top$ – product of these matrix gives a diagonal covariance matrix; principal-diagonal elements present the square of uncertainties estimated by type B method,
$\mathbf{U}_\Lambda$ – uncertainties of correction measurement by unit under test TC are included in this covariance matrix.

Covariance matrix $\mathbf{U}_{\hat{a}}$ is represented by the matrix of uncertainties of the estimates

$$\mathbf{U}_{\hat{a}} = \left(\mathbf{T}^\top \mathbf{U}_W^{-1} \mathbf{T}\right)^{-1}. \tag{10}$$

Deviation associated with the reference function is solved by

$$\Delta\widehat{\mathbf{E}} = \mathbf{T}\hat{\mathbf{a}} \tag{11}$$

and uncertainty of the deviation can be achieved by applying the law of propagations of uncertainties

$$\boldsymbol{u}_{\Delta\mathbf{E}}^2 = \mathbf{T}\mathbf{U}_{\hat{a}}\mathbf{T}^\top. \tag{12}$$

The zero estimation of vector $\hat{\mathbf{a}}$ is biased, see Fig. 2. It is caused by the stochastic nature of quantity t. Therefore the model is nonlinear and requires a solution procedure. It is linearized by applying a Taylor series and higher elements of estimated values are neglected. After linearization, the left side of model vector $\mathbf{W}$ will be

$$\mathbf{W} = \Delta\mathbf{E} + \mathbf{C}_K\boldsymbol{\Lambda} + \mathbf{D}(\boldsymbol{\delta t}_1 + \mathbf{C}_S\boldsymbol{\delta t}_2). \tag{13}$$

$\mathbf{D} = \mathrm{diag}\big(d_{100} \quad d_{200} \quad \ldots \quad d_{1100}\big)$ is the known matrix obtained by applying the Taylor series

$$\begin{aligned} d_{100} &= \hat{a}_1 + 2\hat{a}_2 t_{T_{100}} + 3\hat{a}_3 t_{T_{100}}^2 + 4\hat{a}_4 t_{T_{100}}^3 \\ d_{200} &= \hat{a}_1 + 2\hat{a}_2 t_{T_{200}} + 3\hat{a}_3 t_{T_{200}}^2 + 4\hat{a}_4 t_{T_{200}}^3 \\ &\vdots \\ d_{1100} &= \hat{a}_1 + 2\hat{a}_2 t_{T_{1100}} + 3\hat{a}_3 t_{T_{1100}}^2 + 4\hat{a}_4 t_{T_{1100}}^3 \end{aligned} \tag{14}$$

$\big(\boldsymbol{\delta t}_1 + \mathbf{C}_S\boldsymbol{\delta t}_2\big)$ – this part is valid for standard,
vector $\boldsymbol{\delta t}_1$ – error given by standard,
product $\mathbf{C}_S\boldsymbol{\delta t}_2$ – contains the influence quantities concerning a standard TC in the same way as in expression (4) for the unit under test.

After linearization, the covariance matrix $\mathbf{U}_W$ has the form

$$\mathbf{U}_W = \mathbf{U}_{\Delta\mathbf{E}} + \mathbf{C}_K\mathbf{U}_\Lambda\mathbf{C}_K^\top + \mathbf{D}\big(\mathbf{U}_{\delta t_1} + \mathbf{C}_S\mathbf{U}_{\delta t_2}\mathbf{C}_S^\top\big)\mathbf{D}^\top \tag{15}$$

where
$\mathbf{U}_{\delta t_1}$ – covariance matrix of vector $\boldsymbol{\delta t}_1$ is a diagonal matrix; principal-diagonal elements present the square of uncertainties estimated by type A method
$\mathbf{C}_S\mathbf{U}_{\delta t_2}\mathbf{C}_S^\top$ – product of this matrix is given by diagonal covariance matrix; principal-diagonal elements present square of uncertainties estimated by type B method,
$\mathbf{U}_{\delta t_2}$ – uncertainties of correction of measurements by standard are included in this covariance matrix.

Now in new iteration we consider the observation vector $\mathbf{W}$ (13) and covariance matrix $\mathbf{U}_W$ (15) and we use the formula for estimation of parameters (8).

Numerically, in most cases the design matrix $\mathbf{T}$ is badly scaled and its columns are nearly linearly dependent. For this it is reasonable to transform the quantities of t to interval $-1 \le t \le 1$ according to the following relationship

$$t_T = \frac{\big(t_i - t_{\max}\big) - \big(t_{\max} - t_i\big)}{t_{\max} - t_{\min}}\,;\ i = 100, 200, \ldots, 1100. \tag{16}$$

Backward transformation is carried out multiplying the rows of vector $\mathbf{t}_T$ determined by (16) by columns of matrices of estimated parameters $\hat{\mathbf{a}}$ and of covariance matrix $\mathbf{U}_{\hat{a}}$, consecutively. First from the left, then from the right side.

From the viewpoint of the user, relevant results are the temperature values and their uncertainties. The temperature value can be obtained by an interpolation table which can be edited from the deviation function and its uncertainty is determined by applying the theorem for an implicit function.

$$f(E, t, a) = E - g(t, \mathbf{a}) = 0 \tag{17}$$

$$\begin{aligned} g(t, \mathbf{a}) = a_0 &+ (a_1 + b_1)t + (a_2 + b_2)t^2 + (a_3 + b_3)t^3 + \\ &(a_4 + b_4)t^4 + b_5 t^5 + b_6 t^6 + b_7 t^7 + b_8 t^8. \end{aligned} \tag{18}$$

We get it by adding up the deviation function and reference function, where variable E is representing the current measured value of emf. Now consider function $t = (h, \boldsymbol{a})$ defined from the implicit function. This function is continuous and we have the partial derivative [3]

$$\frac{\partial h}{\partial a_k} = -\frac{\frac{\partial f}{\partial a_k}}{\frac{\partial f}{\partial t}} = -\frac{\frac{\partial g}{\partial a_k}}{\frac{\partial g}{\partial t}} = -\frac{t^k}{S(t)}\,,\ k = 0, \ldots, 4. \tag{19}$$

Derivative (19) is presenting elements of vector $\boldsymbol{h}$. Standard uncertainty is then obtained from relation

$$u^2(t) = \boldsymbol{h}^\top\mathbf{U}_{\hat{a}}\boldsymbol{h}. \tag{20}$$

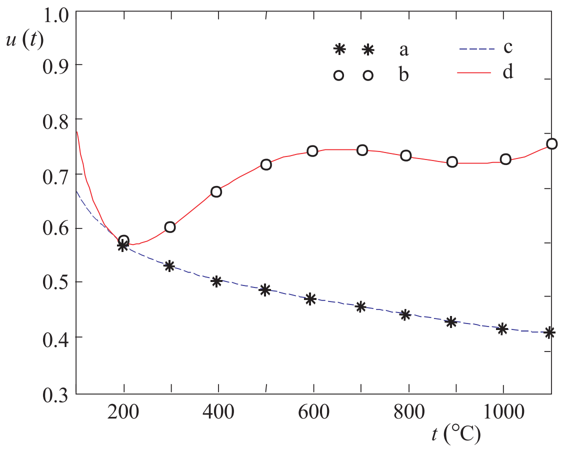

Fig. 2. Standard uncertainties of deviation function — difference between zero and third estimation of parameters: (a)- uncertainties in calibration points – zero estimation, (b)- uncertainties in calibration points – third estimation, (c)- uncertainties of deviation function – zero estimation, (d)- uncertainties of deviation function – third estimation

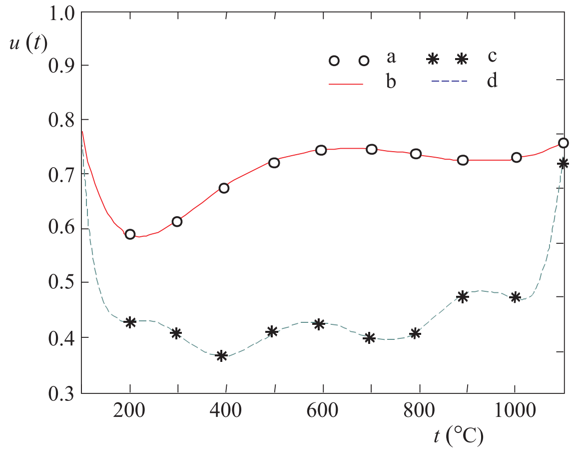

Fig. 3. Standard uncertainties derived from third estimation with and without considered covariance: (a)- standard uncertainties of temperature in calibration points (covariance), (b)- standard uncertainties of temperature (covariance), (c)- standard uncertainties of temperature in calibration points (without covariance), (d)- standard uncertainties of temperature (without covariance)

4 CONCLUSION

A procedure for evaluating the calibration of a TC was applied to demonstrate whether considering the covariances has an impact on the final result of standard uncertainty. For this reason, evaluation was carried out twice. The difference is shown in Fig. 3. As a conclusion we can claim that covariances had significant effect on the final result of a calibration.

Acknowledgements

This work, elaborated during the first author's PhD study at the Faculty of Mechanical Engineering in Bratislava, was partly supported by grant 1/3131/06 of the VEGA Grant Agency of the Slovak Republic. Peter Benkó wishes to thank Prof. Rudolf Palenčár for introducing him to the estimation theory of mathematical statistics.

References

[1] WIMMER, G.—WITKOVSKÝ, V.—PALENČÁR, R.: Handling and evaluation of measurements (Spracovanie a vyhodnocovanie meraní), VEDA, Bratislava, 2002. (in Slovak)

[2] PALENČÁR, R.—RUIZ, J.—JANIGA, I.—HORNÍKOVÁ, A.: Statistical methods in testing and calibration laboratories (Štatistické metódy v skúšobných a kalibračných laboratóriách), Grafické štúdio Ing. Peter Juriga, Bratislava, 2001. (in SLovak)

[3] LIRA, I.: Evaluating Measurement Uncertainty: Fundamentalsand Practical Guidance, IoP Bristol.

[4] ZVIZDIC, D.—SERFEZI, D.—BERMANEC, L. G.—BONIER, G.—RENAOT, E.: Estimation of Uncertainties in Comparison Calibration of Thermocouples, XVII IMEKO World Congress, June 22-27, 2003, Croatia.

[5] BENKÓ, P.: Neistoty pri kalibrácii termoelektrických snímačov teploty porovnávaním (Uncertaities in Comparison Calibration of Thermocouples), Proceedings of the 8th International Scientific Conference MECHANICAL ENGINEERING 2004, Bratislava. (in SLovak)

[6] KUBÁČEK, L.: Statistika a metrologie (Statistics and Metrology), Vydavatelství Univerzita Palackého v Olomouci, 2000. (in Czech)

Received 10 October 2006

Peter Benkó (Ing, PhD) was born in 1977 in Nové Zámky, Slovakia. He received his first degree (Ing) at the Slovak University of Technology, Faculty of Mechanical Engineering in 2001. He continued his PhD studies at the same faculty at the Department of Automation and Measurement, which he finished in 2006. At present he works in Slovak Legal Metrology as a senior metrologist.

Rudolf Palenčár (Prof, Ing, PhD) was born in Ruská Nová Ves, Slovakia, in 1949. He attended the Slovak Technical University Bratislava, Faculty of Mechanical Engineering where he received the first degree (Ing) in 1973 and the second degree (PhD) in 1979. In 1985 he was appointed Associate Professor of measurement technique at the Department of Automation and Measurement, STU, and since 2002 he is a Full Professor. His research activities include the problems of uncertainty of measurement.