DESIGN OF CONTROLLERS FOR TIME DELAY SYSTEMS
PART II: INTEGRATING AND UNSTABLE SYSTEMS

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The paper deals with the design of controllers for integrating and unstable time delay systems. The proposed method is based on the time delay approximation and the polynomial approach. A simple control structure with two feedback controllers is considered. Resulting continuous-time controllers obtained via polynomial equations and the LQ control technique ensure asymptotic tracking of step references as well as step load disturbances attenuation. Simulation results are presented to illustrate the proposed method.

Key words: time delay system, time delay approximation, polynomial method, LQ control

1 INTRODUCTION

The presence of a time delay is a common property of many technological processes. In addition, a part of time delay systems can be unstable or have integrating properties. Typical examples of such processes are pumps, liquid storing tanks or some types of chemical reactors.

Plants with a time delay often cannot be controlled by usual controllers designed without consideration of the dead-time. There are various ways to control such systems. A number of methods utilize PI or PID controllers in the classical feedback closed-loop structure, e.g [1], [2], [3]. Other methods employ ideas of the IMC [4] or robust control [5]. Control results of a good quality can be achieved by modified Smith predictor methods [6], [7], [8], [9].

This paper follows the results obtained for stable time delay systems [10] and extends these on integrating and unstable time delay systems. Principles of the method and design procedures in the 1DOF and 2DOF control system structures have been presented by authors of this paper in [11], [12]. Here, the control system structure with two feedback controllers is considered [13]. The procedure of obtaining controllers is based on the time delay first order Padé approximation and on the polynomial approach [14]. For tuning of the controller parameters, the pole assignment method exploiting the LQ control technique is used [15]. The resulting proper and stable controllers obtained via polynomial Diophantine equations and spectral factorization techniques ensure the asymptotic tracking of step references as well as step disturbances attenuation. The structures of developed controllers together with analytically derived formulas for computation of their parameters are presented for the four typical plant types of integrating and unstable time delay systems: The integrating time delay system (ITDS), the unstable first order time delay system (UFOTDS), the stable first order plus integrating time delay system (SFOPITDS) and the unstable plus integrating time delay system (UFOPITDS). Presented simulation results document usefulness of the proposed method providing stable control responses of a good quality also for a higher ratio between the time delay and unstable time constants of the controlled system.

2 APPROXIMATE TRANSFER FUNCTIONS

The transfer functions in the sequence ITDS, UFOTDS, SFOPITDS and UFOPITDS have forms

\[ G_1(s) = \frac{K e^{-\tau_d s}}{s}, \]

\[ G_2(s) = \frac{K}{\tau s - 1} e^{-\tau_d s}, \]

\[ G_{3,4}(s) = \frac{K}{s(\tau s \pm 1)} e^{-\tau_d s}. \]

Using the first order Padé approximation, the time delay term in (1)–(3) is approximated by

\[ e^{-\tau_d s} \approx \frac{2 - \tau_d s}{2 + \tau_d s}. \]

Then, approximate transfer functions take forms

\[ G_{A1} = \frac{K(2 - \tau_d s)}{s(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^2 + a_1 s} \]

where \( b_0 = \frac{2K}{\tau_d}, \quad b_1 = K \) and \( a_1 = \frac{2}{\tau_d} \) for the ITDS,

\[ G_{A2}(s) = \frac{K(2 - \tau_d s)}{(\tau s - 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^2 + a_1 s + a_0} \]

with \( b_0 = \frac{2K}{\tau_d}, \quad b_1 = \frac{K}{\tau_d}, \quad a_0 = -\frac{1}{\tau_d}, \quad a_1 = \frac{2K}{\tau_d} \) and \( \tau_d \neq 2\tau \) for the UFOTDS, and

\[ G_{A3,4} = \frac{K(2 - \tau_d s)}{s(\tau s \pm 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^3 + a_2 s^2 + a_1 s} \]

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where $b_0 = \frac{2\tau}{\tau_d}$, $b_1 = \frac{\tau}{\tau_d}$, $a_1 = \pm \frac{1}{\tau_d}$, $a_2 = \frac{2\tau\pm\tau_d}{\tau_d}$ and $\tau_d \neq 2\tau$ for the SFOPITDS and UFOPITDS, respectively.

All approximate transfer functions (5), (6) and (7) are strictly proper transfer functions

$$G_A(s) = \frac{b(s)}{a(s)}$$

where $b$ and $a$ are coprime polynomials in $s$ that fulfill the inequality $\deg b < \deg a$. The polynomial $a(s)$ in their denominators can be expressed as a product of the stable and unstable part

$$a(s) = a^+(s)a^-(s)$$

so that for ITDS, UFOTDS and SFOPITDS the equality holds

$$\deg a^+ = \deg a - 1.$$  \hfill (10)

### 3 CONTROL SYSTEM DESCRIPTION

The control system is depicted in Fig. 1. In the scheme, $w$ is the reference, $v$ is the load disturbance, $e$ is the tracking error, $u_0$ is the controller output, $y$ is the controlled output, $u$ is the control input and $G_A$ represents one of the approximate transfer functions (5)–(7) in the general form (8). Both $w$ and $v$ are considered to be step functions with transforms

$$W(s) = \frac{u_0}{s}, \quad V(s) = \frac{v_0}{s^2}. $$  \hfill (11)

The transfer functions of controllers are assumed as

$$Q(s) = \frac{\tilde{q}(s)}{p(s)}, \quad R(s) = \frac{r(s)}{p(s)} $$  \hfill (12)

where $\tilde{q}$, $r$ and $\tilde{p}$ are polynomials in $s$.

### 4 APPLICATION OF THE POLYNOMIAL METHOD

The controller design described in this section follows the polynomial approach. The general requirements on the control system are formulated as its internal properness and strong stability (in addition to the control system stability, also the controller stability is required), asymptotic tracking of the reference and load disturbance attenuation. The procedure to derive admissible controllers can be performed as follows:

Transforms of the basic signals in the closed-loop system take following forms (for simplification, the argument $s$ is in some equations omitted)

$$Y(s) = \frac{b}{d}[rW(s) + \tilde{p}V(s)].$$  \hfill (13)

$$E(s) = \frac{1}{d}[(a\tilde{p} + b\tilde{q})W(s) - b\tilde{p}V(s)].$$  \hfill (14)

$$U(s) = \frac{a}{d}[rW(s) + \tilde{p}V(s)].$$  \hfill (15)

Here,$$
\quad d(s) = a(s)\tilde{p}(s) + b(s)[r(s) + \tilde{q}(s)]$$  \hfill (16)

is the characteristic polynomial with roots as poles of the closed-loop.

Establishing the polynomial $t$ as

$$t(s) = r(s) + \tilde{q}(s)$$  \hfill (17)

and substituting (17) into (16), the condition of the control system stability is ensured when polynomials $\tilde{p}$ and $t$ are given by a solution of the polynomial Diophantine equation

$$a(s)\tilde{p}(s) + b(s)t(s) = d(s)$$  \hfill (18)

with a stable polynomial $d$ on the right side.

With regard to the transforms (11), the asymptotic tracking and load disturbance attenuation are provided by divisibility of both terms $a\tilde{p} + b\tilde{q}$ and $\tilde{p}$ in (14) by $s$. This condition is fulfilled for polynomials $\tilde{p}$ and $\tilde{q}$ having forms

$$\tilde{p}(s) = sp(s), \quad \tilde{q}(s) = sq(s). $$  \hfill (19)

Subsequently, the transfer functions (12) take forms

$$Q(s) = \frac{q(s)}{p(s)}, \quad R(s) = \frac{r(s)}{sp(s)} $$  \hfill (20)

and, a stable polynomial $p(s)$ in their denominators ensures the stability of controllers.

The control system satisfies the internal properness condition when the transfer functions of all its components are proper. Consequently, the degrees of polynomials $q$ and $r$ must fulfill these inequalities

$$\deg q \leq \deg p, \quad \deg r \leq \deg p + 1.$$  \hfill (21)

Now, the polynomial $t$ can be rewritten to the form

$$t(s) = r(s) + sq(s). $$  \hfill (22)
Taking into account solvability of (18) and conditions (21), the degrees of polynomials in (17) and (18) can be easily derived as
\[
\deg t = \deg r = \deg a, \quad \deg q = \deg a - 1, \\
\deg p \geq \deg a = 1, \quad \deg d \geq \deg a. \tag{23}
\]
Denoting \(\deg a = n\), polynomials \(t\), \(r\) and \(q\) have forms
\[
t(s) = \sum i = 0^n t_is^i, \quad r(s) = \sum i = 0^n r_is^i, \\
g(s) = \sum i = 1^n q_is^i-1
\tag{24}
\]
and, relations among their coefficients are
\[
r_0 = t_0, \quad r_i + q_i = t_i \text{ for } i = 1, \ldots, n. \tag{25}
\]
Since by a solution of the polynomial equation (18) only coefficients \(t_i\) can be calculated, unknown coefficients \(r_i\) and \(q_i\) can be obtained by a choice of selectable coefficients \(\gamma_i \in (0, 1)\) such that
\[
r_i = \gamma_it_i, \quad q_i = (1 - \gamma_i)t_i \text{ for } i = 1, \ldots, n. \tag{26}
\]
The coefficients \(\gamma_i\) divide a weight between numerators of transfer functions \(Q\) and \(R\).

Remark. If \(\gamma_i = 1\) for all \(i\), the control system in Fig. 1 reduces to the 1DOF control configuration \((Q = 0)\). If \(\gamma_i = 0\) for all \(i\), and, both reference and load disturbance are step functions, the control system corresponds to the 2DOF control configuration. The controller parameters then result from solutions of the polynomial equation (18) and depend upon coefficients of the polynomial \(d\). The next problem here is to find a stable polynomial \(d\) that enables to obtain acceptable stabilizing and stable controllers.

### 5 POLE ASSIGNMENT

The polynomial \(d\) is considered as a product of two stable polynomials \(g\) and \(m\) in the form
\[
d(s) = g(s)m(s) \tag{27}
\]
where the polynomial \(g\) is a monic form of the polynomial \(g'\) obtained by the spectral factorization
\[
[sa(s)]^* + \varphi[sa(s)] + b^*(b(s) = g''(s)g'(s) \tag{28}
\]
where \(\varphi > 0\) is a weighting coefficient.

In the LQ control theory, the polynomial \(g'\) results from the minimization of the quadratic cost function
\[
J = \int_0^\infty \{e + 2(t) + \varphi u^2(t)\} dt \tag{29}
\]
where \(e(t)\) is the tracking error and \(u(t)\) is the control input derivative.

The second polynomial \(m\) ensuring properness of controllers is given for the ITDS, UFOTDS as
\[
m(s) = a^+(s) = s + \frac{2}{\tau_d} \tag{30}
\]
and, for both UOFITDS and SFOPITDS in the form
\[
m(s) = \left( s + \frac{2}{\tau_d} \right) \left( s + \frac{1}{\tau} \right). \tag{31}
\]
The coefficients of the polynomial \(d\) include only a single selectable parameter \(\varphi\) and all other coefficients are given by parameters of polynomials \(b\) and \(a\). Consequently, the closed loop poles location can be affected by a single selectable parameter. As known, the closed loop poles location determines both step reference and step load disturbance responses. However, with respect to the transform (13), it may be expected that weighting coefficients \(\gamma\) influence only step reference responses.

Then, the monic polynomial \(g\) and derived formulas for their parameters have forms
\[
g(s) = s^3 + g_2s^2 + g_1s + g_0 \tag{32}
\]
for both ITDS and UFOTDS, where
\[
\begin{align*}
g_0 &= \frac{2K}{\tau_d \sqrt{\varphi}}, \\
g_1 &= \frac{1}{\sqrt{\varphi}} \sqrt{4K + 2 + K^2}, \\
g_2 &= \frac{2}{\sqrt{\varphi}} g_1 + \frac{4}{\tau_d}
\end{align*} \tag{33}
\]
for the ITDS and
\[
\begin{align*}
g_0 &= \frac{2K}{\tau_d \sqrt{\varphi}}, \\
g_1 &= \frac{1}{\tau_d} \sqrt{4K + 1 + K^2}, \\
g_2 &= \frac{2}{\tau_d} \sqrt{\frac{1}{\varphi} g_1 + 4^2 + \tau^2}
\end{align*} \tag{34}
\]
for the UFOTDS, and,
\[
g(s) = s^4 + g_3s^3 + g_2s^2 + g_1s + g_0 \tag{35}
\]
for both SFOPITDS and UFOPITDS, where
\[
\begin{align*}
g_0 &= \frac{2K}{\tau_d \sqrt{\varphi}}, \\
g_1 &= \frac{1}{\varphi} \left( \frac{4K}{\tau_d} g_2 + \frac{K^2}{\tau^2} \right), \\
g_2 &= \frac{1}{\tau_d} \frac{g_2}{\varphi} \left( 4^2 + \frac{1}{\tau^2} \right) \tag{36}
\end{align*}
\]
for the UFOTDS, and,
The transfer functions of controllers are
\[ Q(s) = \frac{q_2 s + q_1}{s + p_0}, \quad R(s) = \frac{r_2 s^2 + r_1 s + r_0}{s(s + p_0)} \]

for both ITDS and UFOTDS, and,
\[ Q(s) = \frac{q_3 s^2 + q_2 s + q_1}{s^2 + p_1 s + p_0}, \]
\[ R(s) = \frac{r_3 s^3 + r_2 s^2 + r_1 s + r_0}{s^2 + p_1 s + p_0} \]

for both SFOPITDS and UFOPITDS.

## 6 CONTROLLER PARAMETERS

For the sake of limited space, formulas derived from (18) for all considered systems together with conditions of the controllers’ stability are introduced in the form of tables. Parameters \( r_i \) and \( q_i \) in (37) and (38) are calculated from \( t_i \) according to (26).

### Table 1. Controller parameters for the ITDS

| \( p_0 \) | \( p_0 = g_2 + \frac{r_2}{\tau_d}(2g_1 + \tau_d g_0) \), \( t_0 = \frac{1}{\tau_d} g_0 \) |
| \( t_1 \) | \( t_1 = \frac{1}{\tau_d}(g_1 + \tau_d g_0) \), \( t_2 = \frac{r_2}{\tau_d}(2g_1 + \tau_d g_0) \) |
| \( p_0 > 0 \) for all \( \tau_d \) |

### Table 2. Controller parameters for the UFOTDS

| \( p_0 \) | \( p_0 = \frac{\tau_d}{\tau_d + \tau}(g_1 + \tau d g_0) + \frac{1}{2} \) |
| \( t_0 \) | \( t_0 = \frac{1}{\tau_d} g_0 \), \( t_1 = \frac{1}{\tau_d}[g_1 + (\tau + \tau_d)g_0] \) |
| \( t_2 \) | \( t_2 = \left[ (2\tau + \tau_d)(2g_1 + \tau_d g_0) + 2\tau_d g_0 \right] \) |
| \( t_3 \) | \( t_3 = \frac{\tau_2}{\tau_0} \) |
| \( p_0 > 0 \) for all \( \tau_d \) |

### Table 3. Controller parameters for the SFOPITDS

| \( p_0 \) | \( p_0 = (2g_1 + \tau_d g_0) \), \( p_1 = g_3 + \frac{r_3}{\tau_d} \) |
| \( t_0 \) | \( t_0 = \frac{1}{\tau_d} g_0 \), \( t_1 = \frac{1}{\tau_d}[g_1 + (\tau + \tau_d)g_0] \) |
| \( t_2 \) | \( t_2 = \left[ (2\tau + \tau_d)(2g_1 + \tau_d g_0) + 2\tau_d g_0 \right] \) |
| \( t_3 \) | \( t_3 = \frac{\tau_2}{\tau_0} \) |
| \( p_1 > 0 \) for all \( \tau_d, p_0 > 0 \) for \( \tau_d < 2\tau \) |

### Table 4. Controller parameters for the UFOPITDS

| \( p_0 \) | \( p_0 = 4g_3 + (2\tau + \tau_d)(2g_1 + \tau_d g_0) + \frac{r_3}{\tau_d} \) |
| \( p_1 \) | \( p_1 = g_3 + \frac{r_3}{\tau_d} \) |
| \( t_0 \) | \( t_0 = \frac{1}{\tau_d} g_0 \), \( t_1 = \frac{1}{\tau_d}[g_1 + (\tau + \tau_d)g_0] \) |
| \( t_2 \) | \( t_2 = \left[ (2\tau + \tau_d)(2g_1 + \tau_d g_0) + 2\tau_d g_0 \right] \) |
| \( t_3 \) | \( t_3 = \frac{\tau_2}{\tau_0} \) |
| \( p_1 > 0 \) for all \( \tau_d, p_0 > 0 \) for \( \tau_d < 2\tau \) |

## 7 SIMULATION RESULTS

The simulations were performed by MATLAB-Simulink tools. For all simulations, the unit step reference \( v \) was introduced at the time \( t = 0 \) and the step load disturbance \( w \) after settling of the step reference responses.

### 7.1 ITDS

In the transfer function (1), let \( K = 1 \). The responses in Fig. 2 for \( \tau_d = 5 \) show the effect of \( \varphi \) upon the control
7.2 UFOTDS

For the simulation, the parameters in (2) have been chosen as $K = 4$, $\tau = 4$. The effect of $\varphi$ on the control responses is similar to the ITDS, as shown in Fig. 5. The responses in Fig. 6 demonstrate robustness of the proposed method against changes of $\tau_d$. The controller parameters were computed for a nominal model with $\tau_d = 2$ and subsequently used for perturbed models with the $+10\%$ and $+25\%$ estimation errors in the $\tau_d$ value ($\tau_d = 2.2$ and $\tau_d = 2.5$). The control response for $\tau_d = 4$ is shown in Fig. 7. Especially, the response without any overshoot documents usefulness of the proposed method. Figs. 5, 6, 7

7.3 SFOPITDS

For this model, the parameters in (3) have been chosen as $K = 1$, $\tau = 4$, $\tau_d = 4$. A suitable selection of parameters $\varphi$ and $\gamma$ provides control responses of a good quality, as illustrated in Fig. 8.

7.4 UFOPITDS

With regard to the presence of both integrating and unstable parts, the UFOPITDS belongs to hardly controllable systems. However, the control responses in Fig. 9 document usability of the proposed method also for such a system. Fig. 9
8 CONCLUSIONS

The problem of control design for integrating and unstable time delay systems has been solved and analyzed. The proposed method is based on the Padé time delay approximation. The controller design uses the polynomial synthesis and results of the LQ control theory. The presented procedure provides satisfactory control responses in the tracking of a step reference as well as in the step load disturbance attenuation. The procedure enables tuning of the controller parameters by a single selectable parameter. Using derived formulas, the controller parameters can be automatically computed. As a consequence, the method could also be used for adaptive control.

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