

# MODELLING OBSTRUCTIONS IN STRAIGHT AND CURVED RECTANGULAR TUNNELS BY FINITE ELEMENT APPROACH

Kamran Arshad\* — Ferdinand Katsriku\*\* — Aboubaker Lasebae\*\*

Accurate analysis of electromagnetic wave propagation in tunnels has important applications in the areas of mobile and wireless communications. In this paper a fully vectorial finite element based propagation model is developed for blocked straight and curved tunnels. Effect of different vehicles, location of vehicles and number of vehicles inside tunnels is analysed. It is shown that the basic propagation loss inside tunnels is mainly determined by the number of vehicles and their size. Location of vehicles have influence on radio wave propagation only in curved tunnels.

**Key words:** finite element method, parabolic equation method, curved tunnels, blocked tunnels

## 1 INTRODUCTION

Number of mobile device users are growing very fast especially in the past decade so radio waves in the frequency range from high frequency (HF) to microwave band are now densely utilized everywhere. It is a common observation that signals cannot be heard in tunnels. Most obvious reason is either cut-off propagation of waves or severe attenuation of signals. Many researchers have therefore been focusing their attention on radio propagation in tunnels [1].

Many theoretical studies [2] and practical approaches [3] have been conducted on radio wave propagation characteristics in hollow tunnels. Actual tunnels have vehicles most of the time hence for future development of mobile communication systems, study of propagation characteristics inside tunnels with vehicles is a must. A number of measurements have been carried out to study the influence of vehicles inside tunnels for example, see [4],[7]. To the authors knowledge, only few analytical investigations have been done on propagation characteristics in non hollow tunnels [8].

Main purpose of this paper is to analyze propagation loss in tunnels with obstructions like vehicles. Vehicles cause some extra loss inside tunnels, so in the presence of vehicles total propagation loss is the sum of loss due to tunnels and vehicle loss. Propagation loss due to vehicles depends on a number of factors: number, location, size of vehicle and structure of tunnel. It is shown that the basic propagation loss due to vehicles in tunnel is determined mainly by the number and size of the vehicles.

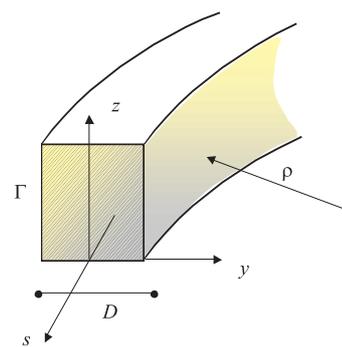
In our previous work, finite element based V-PEM (FEM-VPEM) was used as a suitable way for analytical modelling of wave propagation in hollow tunnels [9]. Finite element formulation of V-PEM for straight and curved rectangular tunnels is described in this paper. V-PEM seems to be an adequate mathematical model of wave propagation in tunnels due to selective wall absorption filtering out higher Brillouin angles and forming a

paraxial wave packet even if tunnel axis is a curved one [10].

The remainder of the paper is organized as follows. In section 2, vectorial parabolic equation for the curved tunnel along with boundary conditions is given. Finite element formulation of the model is given in section 3. Finally, results and discussion is given in section 4. Section 5 concludes this paper.

## 2 V-PEM MODELLING FOR CURVED TUNNEL

Consider a curved tunnel shown in Fig. 1, where  $y$  and  $z$  are the transverse dimensions and  $\rho$  denotes the curvature radius so for straight tunnels  $\rho = \infty$ . Suppose  $D$  is the cross sectional dimension of the tunnel so in UHF/VHF band it can be assumed that  $\lambda/D \simeq \nu$ ,  $\beta \simeq \nu$ ,  $D/\rho \simeq \nu^2$  with  $\nu \ll 1$  where  $\lambda$  is the wavelength and  $\beta$  is the Brillouin angle [10].

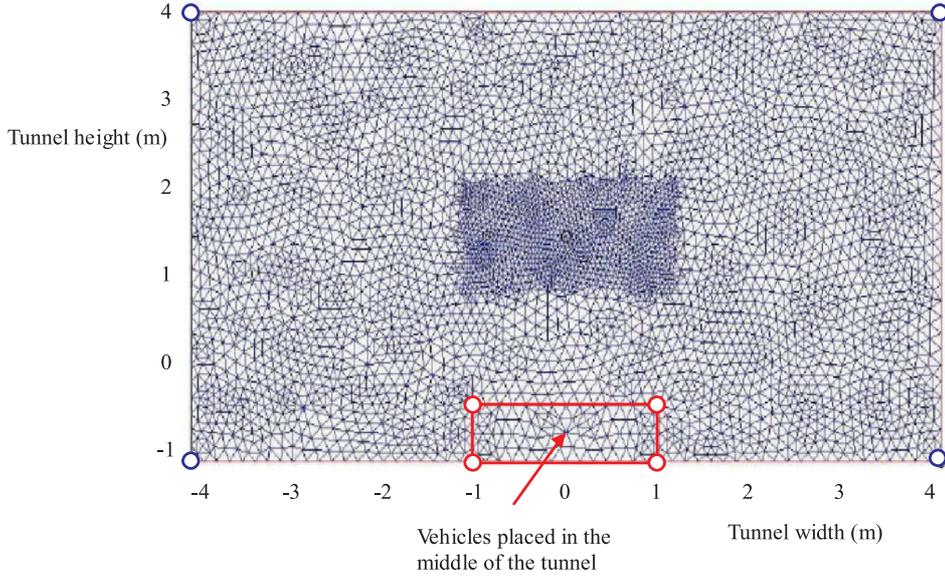


**Fig. 1.** Cross section of a curved tunnel with radius of curvature  $\rho$

The transverse electric field  $\mathbf{E}_\perp$  can be approximated as:

$$\mathbf{E}_\perp = \mathbf{W}e^{-jks} \quad (1)$$

Institute of Advanced Telecommunications, Swansea University, Swansea, UK; k.arshad@swan.ac.uk; \*\* School of Computing Science, Middlesex University, London, UK; f.katsriku@mdx.ac.uk, a.lasebae@mdx.ac.uk



**Fig. 2.** Mesh of cross section of tunnel

The attenuation function  $\mathbf{W}(s, y, z) = (W_y, W_z)$  must satisfy following equation [10]:

$$\frac{\partial^2 \mathbf{W}}{\partial y^2} + \frac{\partial^2 \mathbf{W}}{\partial z^2} - 2k^2 \frac{y \cos \theta(s) + z \sin \theta(s)}{\rho(s)} \mathbf{W} = 2kj \frac{\partial \mathbf{W}}{\partial s} \quad (2)$$

where  $\theta$  is the rotational angle [denoting a tunnel that is possibly curved in more than one dimension] and  $k$  is the wave number. At the boundary contour  $\Gamma$  of the tunnel cross section, Leontovich impedance boundary conditions can be implemented as an approximation of the wall electrical properties [11].

$$\mathbf{W} = \frac{j}{k} \tilde{\mathbf{A}} \tilde{\mathbf{B}} \tilde{\mathbf{A}} \frac{\partial \mathbf{W}}{\partial n} \quad (3) \quad \text{where,}$$

with,

$$\tilde{\mathbf{A}} = \begin{bmatrix} n_y & n_z \\ n_z & -n_y \end{bmatrix}, \quad \tilde{\mathbf{B}} = \begin{bmatrix} 1/Z & 0 \\ 0 & Z \end{bmatrix}$$

Here,  $n = (n_y, n_z)$  is the unit normal to  $\Gamma$  and  $Z$  is the normalized impedance of the tunnel walls. For a wall material with relative permittivity  $\epsilon_r$  and conductivity  $\sigma$ , wall impedance can be approximated as  $Z = 1/\sqrt{\epsilon_r - j60\lambda\sigma}$  [11]. Equation (2) is the vectorial version of scalar parabolic equation describing creeping and whispering gallery waves [10]. Equation (2) accounts for transversal diffusion of wave amplitude  $\mathbf{W}(s, y, z)$  whereas the matrix boundary condition (3) governs the effects of grazing angle reflection, selective mode absorption and depolarization in the tunnel walls. If the tunnel is blocked by vehicles, additional geometry is added for the length of tunnel and impose different boundary conditions on it.

### 3 FINITE ELEMENT FORMULATION OF VECTORIAL PARABOLIC EQUATION MODEL

Applying the finite element method (FEM) over the domain  $y_{min} \leq y \leq y_{max}$  and  $z_{min} \leq z \leq z_{max}$  to equation (2) will yield the following equations,

$$\begin{aligned} & -j2k[\mathbf{M}] \frac{\partial \mathbf{W}}{\partial s} \\ & + ([\mathbf{K}] - k^2 [1 - 2(\frac{y \cos \theta + z \sin \theta}{\rho(s)})]) [\mathbf{M}] \mathbf{W} \quad (4) \\ & + [\mathbf{K}]_{\Gamma} \mathbf{W} = 0 \end{aligned}$$

$$\begin{aligned} [\mathbf{K}] &= \Sigma_e \int \int_e \{N\} \{N\}^T dy dz \\ [\mathbf{K}]_{\Gamma} &= \Sigma_e \int_{\Gamma} jk \{N\}_{\Gamma} \{N\}_{\Gamma}^T dS \\ [\mathbf{M}] &= \Sigma_e \int \int_e [k^2 \{N\} \{N\}^T - \\ & \frac{\partial \{N\}}{\partial y} \frac{\partial \{N\}^T}{\partial y} - \frac{\partial \{N\}}{\partial z} \frac{\partial \{N\}^T}{\partial z}] dy dz \quad (5) \end{aligned}$$

where  $\mathbf{W}$  is the attenuation function  $\mathbf{W} = (W_y, W_z)$ ,  $N$  is the shape function vector and  $N_{\Gamma}$  is the shape function vector on the computational window edge  $\Gamma$ .

So equation 4 becomes,

$$\begin{aligned} & -j2k[\mathbf{M}] \frac{\partial \{\mathbf{W}\}}{\partial s} + ([\tilde{\mathbf{K}}] - \\ & k^2 \{1 - 2(\frac{y \cos \theta + z \sin \theta}{\rho(s)})\}) [\mathbf{M}] \mathbf{W} = 0 \quad (6) \end{aligned}$$

If the Crank-Nicholson algorithm is now applied to equation (7) in the propagation direction  $s$  the following is obtained,

$$[\mathbf{A}]_i \{\mathbf{W}\}_{i+1} = [\mathbf{B}]\{\mathbf{W}\}_i \quad (7)$$

where,

$$[\mathbf{A}]_i = j2k \sqrt{1 - 2\left(\frac{y \cos \theta + z \sin \theta}{\rho(s)}\right)} [\mathbf{M}]_i + \quad (8)$$

$$0.5\Delta s([\tilde{\mathbf{K}}]_i - k^2\{1 - 2\left(\frac{y \cos \theta + z \sin \theta}{\rho(s)}\right)\}) [\mathbf{M}]_i$$

$$[\mathbf{B}]_i = j2k \sqrt{1 - 2\left(\frac{y \cos \theta + z \sin \theta}{\rho(s)}\right)} [\mathbf{M}]_i - \quad (9)$$

$$0.5\Delta s([\tilde{\mathbf{K}}]_i - k^2\{1 - 2\left(\frac{y \cos \theta + z \sin \theta}{\rho(s)}\right)\}) [\mathbf{M}]_i$$

and  $\Delta s$  is propagation step size and subscript  $i$  and  $(i + 1)$  denotes the quantities related to the  $i$  and  $(i + 1)$  step sizes respectively. The crank-Nicholson scheme is unconditionally stable and the global truncation error is  $O(\delta s)^2$  [12].

### 3 RESULTS AND DISCUSSIONS

#### Simulation Parameters

Cross sectional geometry of a curved rectangular tunnel is shown in Fig. [2]. Tunnel has a rectangular cross section 8m wide and 4m high, which is a typical of actual tunnels. Length of the tunnel is assumed to be 300m. For straight tunnels radius of curvature  $\rho$ , is  $\infty$  while for curved tunnel  $\rho$  is assumed to be 800m. Mesh is generated using Delaunay algorithm [13], total number of elements are 32224 with 384 boundary elements. For better accuracy, lagrange quadratic type elements are used [14]. For this simulation, Gaussian source antenna with a beam width of  $11^\circ$  operating at 950MHZ is used. The microwave transmitting antenna is at the center *ie*  $(x_0, 0, 2)$  while receiving antenna is at  $(x, 0, 2)$  moves horizontally along the length of tunnel.

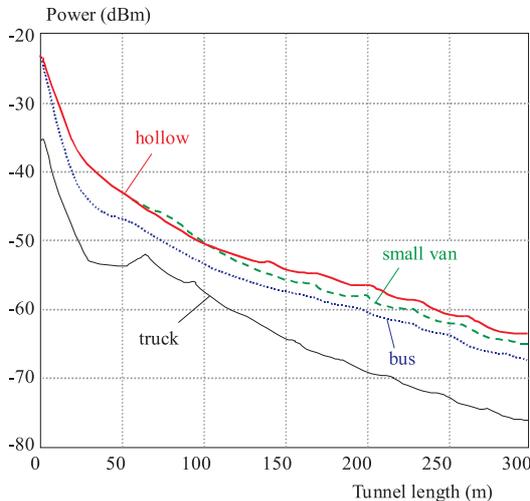


Fig. 3. Received power level vs tunnel length in Straight tunnel

In all simulations dominant mode was transmitted at the entrance of tunnel. Different readings of the power is taken along the length of the tunnel to determine the effect that the vehicles may have on propagation characteristics. The walls of the tunnel are characterised by relative permittivity of  $\epsilon_r = 5.5$  and conductivity of  $\sigma = 0.03$  mho/m while at vehicle boundaries  $\epsilon_r = 1$  and  $\sigma = 4 \times 10^6$  mho/m is assumed. Rotational angle  $\theta$  is assumed to be 0 in all simulations. Three different types of vehicles have been used: small van, bus and a large truck. Typical dimensions of vehicles are shown in Table [1]vehicles.

Table 1. Typical dimensions of vehicles

Type	length (m)	width (m)	height (m)
Small Van	10	0.5	2
Bus	13	1.5	2
Truck	17	2	3

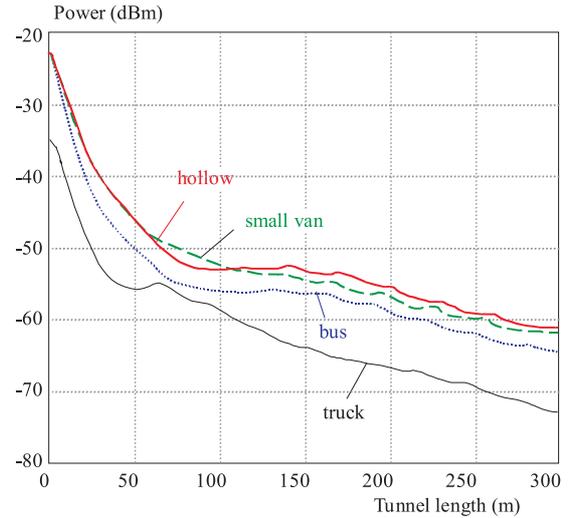


Fig. 4. Received power level vs tunnel length in Straight tunnel

#### Propagation Loss

Propagation loss inside tunnel is affected by a number of parameters including, frequency of transmission, size of tunnel, shape of tunnel, electrical properties of tunnel wall, polarization, size of vehicle, number of vehicles, location of vehicles inside tunnel etc etc. Propagation loss inside tunnel is loss due to tunnel itself and losses due to the presence of obstructions. In this paper propagation loss because of number of vehicles, size of vehicle and its location in a rectangular straight and curved tunnel is examined.

It is well know that the scattering properties of a scatterer depends on the size of scatterer. So different

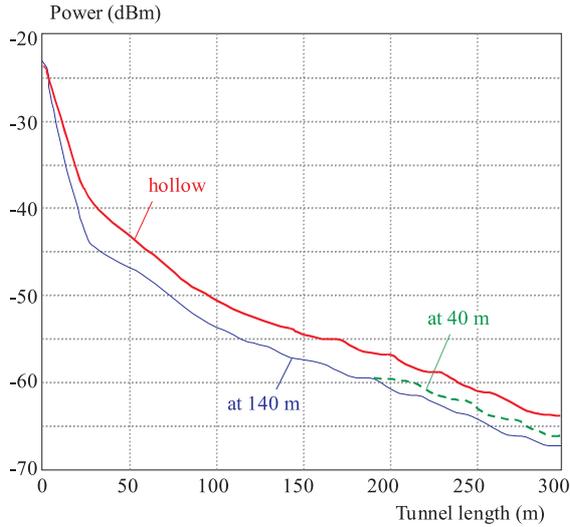


Fig. 5. Received power level vs tunnel length in Curved tunnel

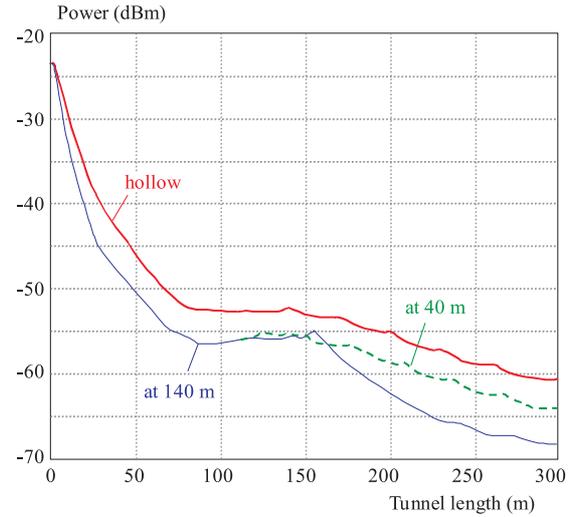


Fig. 6. Received power level vs tunnel length in Straight tunnel

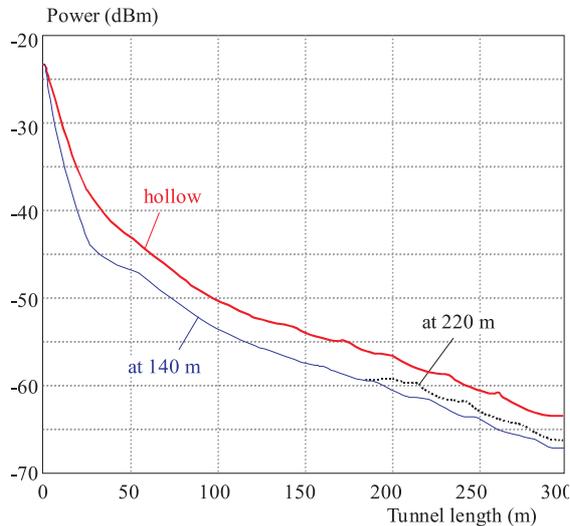


Fig. 7. Received power level vs tunnel length in Curved tunnel

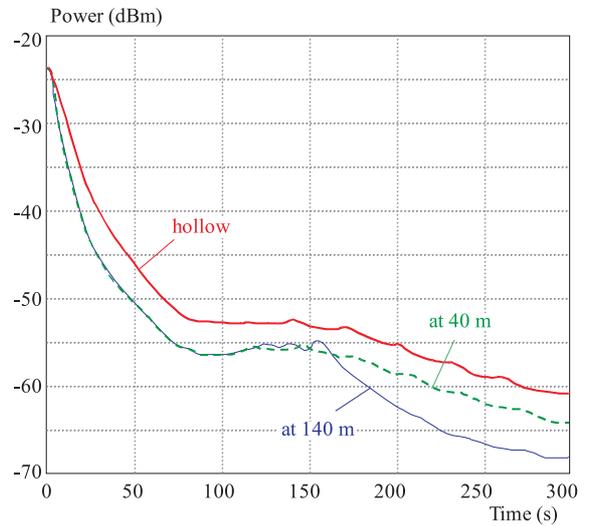


Fig. 8. Received power level vs tunnel length in Straight tunnel

sizes of car are expected to produce different propagation losses. Different vehicles (small van, bus and truck) of dimensions mentioned in Table [1] were placed at 40m from the entrance of tunnel. Results are shown in Fig. 3 size for rectangular straight tunnel and in Fig. 4 size-curve for curved rectangular tunnel. Power level is measured along the length of the tunnel at the center line of the cross section. Presence of vehicles produces an additional loss behind the vehicle in each case. As expected, large vehicles produces more loss than small vehicles both in straight and curved tunnels.

To examine propagation loss due to location of vehicles inside tunnel, a bus of dimensions shown in Table [1] vehicles is placed at various locations in straight and curved tunnel. Power level in dBm versus tunnel length is shown in Fig. 5 and 6. It can be seen from the diagram that power level remains same in stable region and is independent on the location of vehicle for straight tunnel. In curved tunnel, power level in stable region depends on the location of vehicle. This is because of the curvature ef-

fect of tunnel. If the bus is far from the transmitter power loss is more as compare to one close to transmitter.

Next, how the propagation loss changes due to the number of vehicles is examined. Power level is calculated for hollow tunnel, with one bus and with three buses all placed at 40m away from the entrance of the tunnel. From Fig. 7 and 8 it is clear that power level changes because of the number of vehicles and produces additional loss which is quite prominent in stable region. Value of propagation loss depends on many factors, but it almost increases in proportion, provided identical vehicles are used.

## 5 CONCLUSION

An efficient finite element based V-PEM is capable of modelling VHF/UHF radio communications in straight and curved tunnels beyond the limits of existing methods. Propagation loss inside tunnels is dependent on a number of parameters which can be categorized in two

domains: Losses due to tunnel and due to obstructions. Power level at the center of tunnel for different vehicles has been calculated. It is shown that propagation loss is dependant on size and number of vehicles. A fully vectorial finite element base propagation model inside tunnels blocked by vehicles have been proposed in this paper. The results of this paper will contribute in modelling and characterisation of future communication channels in tunnels.

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**Kamran Arshad** was born in Islamabad, Pakistan on 18th of November 1978. He received BE degree in Electrical Engineering from NED University of Engineering and Technology, Pakistan in 2000 with distinction and an MS degree in Electrical Engineering from King Fahd University, Dhahran, Saudi Arabia in 2003. He is currently a PhD research student in School of Computing Science, Middlesex University, London UK His main research interests are in wireless communication, development of future wireless networks, radio wave propagation modelling and computational electromagnetics. Kamran Arshad is an author of more than 13 papers published in international journals and conference proceedings and received two best student paper awards. Kamran Arshad is listed in Marquis Who'sWho in the World as well as in Who'sWho in Science and Engineering.



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