EQUIVALENT CIRCUIT OF SWITCHED RELUCTANCE GENERATOR BASED ON DC SERIES GENERATOR

Martin Lipták * — **Valéria Hrabovcová** ** — **Pavol Rafajdus** **

In this paper equivalent circuit of the switched reluctance generator (SRG) will be derived on the base of DC series generator scheme. Equivalent circuit of SRG with the most often employed asymmetric half-bridge power converter (APC) has only one possible current path in the circuit analogous to the equivalent circuit of the DC series generator. Then no-load and short circuit states of the SRG will be defined. The mathematic model of single-phase SRG cooperated with APC will be developed based on the equivalent circuit. The SRG operation will be simulated and then the results verified by measurements on 3-phase 6/4 low power SRG. The generator controlled with DSP will be tested under various load and speed conditions.

K e y w o r d s: SRG, DC series generator, asymmetric half-bridge power converter (APC), equivalent circuit, mathematical model

1 INTRODUCTION

Switched reluctance (SR) machine has various desirable features, which comes from its simple construction. These machines are investigated for their interested properties like the possibility to work by high speed and high temperature because they have no permanent magnets and no winding on the rotor. As it is known, SR machines have doubly salient design of the magnetic circuit. The excitation winding is placed on the stator poles while the rotor is without winding. It follows that the energy conversion is based on reluctance variation. The phase current pulse is controlled and synchronised with the rotor position feedback.

In motoring mode they require a precise control technique. In the last decade also generating mode of SR machine has been investigated and starts to be used. Its control technique can be even simpler than in motoring mode.

For the dynamic simulation an equivalent circuit and then mathematical model has to be established. The purpose of this article is to present new approach for developing of equivalent circuit of SRG based on equivalent circuit of the DC series generator. According this circuit a full mathematical model with 7 equations has been developed from which 4 equations define values for single phase and must be multiplied according the total phase number and 3 equations are common for all phases (see below). SRG operation is presented by simulation as well as measured results on the 3-phase 180W 6/4 SRG with asymmetric half-bridge power converter (APC) controlled by digital signal processor (DSP).

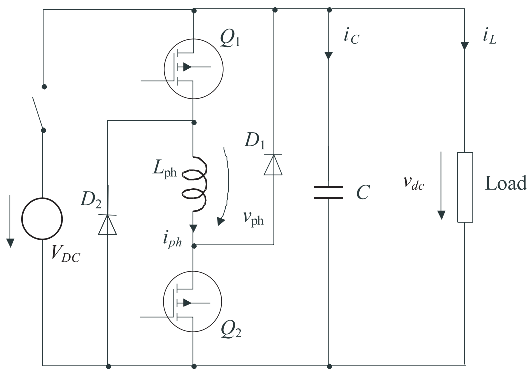

Fig. 1. Simple-phase SRG equivalent circuit

2 EQUIVALENT CIRCUIT

A classical equivalent circuit of SRG with APC usually consists of phase windings, power switches and diodes, main DC bus capacitor, load resistor and mostly also a DC source with series connected diode or a switch for starting excitation [1], [2]. In the Fig. 1 there is such circuit for a single-phase winding. Phase winding with two switches $(\mathrm{Q}_1, \mathrm{Q}_2)$ and diodes $(\mathrm{D}_1, \mathrm{D}_2)$ form a phase leg and their number in complete circuit depends on the total SRG phase number.

The basic voltage equation of one phase of SRG is following:

$$v_{ph} = R_{ph} i_{ph} + L_{ph} \frac{\mathrm{d}i_{ph}}{\mathrm{d}t} + \frac{\mathrm{d}L_{ph}}{\mathrm{d}\theta} \omega \, i_{ph} \tag{1}$$

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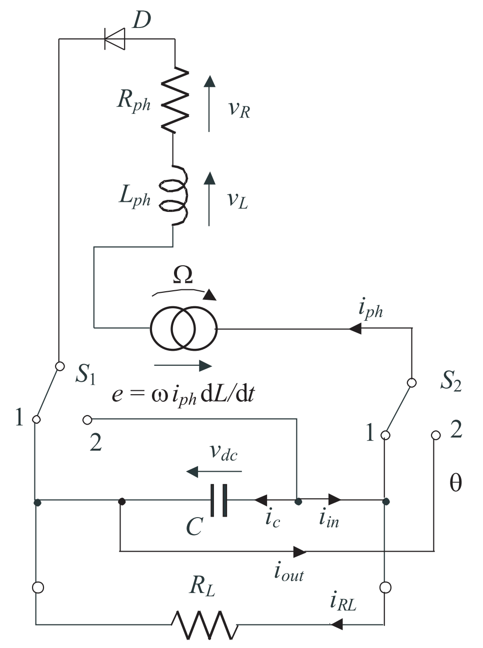

Fig. 2. SRG equivalent circuit derived from the DC series generator

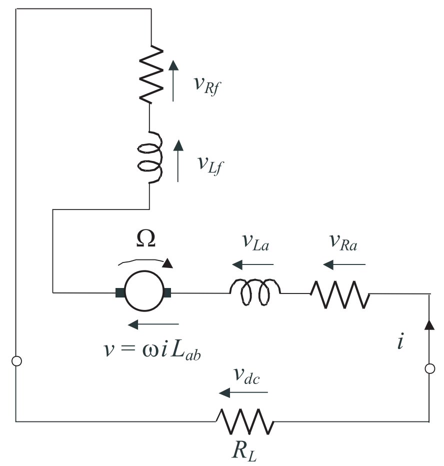

Fig. 3. DC series generator equivalent circuit

where last member on the right side is back EMF [3] which depends on the first derivative of phase inductance with respect to the rotor position, phase current and rotor speed:

$$e = \frac{\mathrm{d}L_{ph}}{\mathrm{d}\theta}\omega\, i_{ph}\,. \tag{2}$$

The generated electrical power is proportional to this back EMF.

The proposed equivalent circuit of single-phase SRG is in Fig. 2 and is derived from an equivalent circuit of the DC series generator (Fig. 3). As it is seen, series dc generator is a self-excited machine in which the field circuit is not complete unless a load circuit is connected to the terminals. For this machine $i_d = i_f = i$ is valid. If load resistor R_L is too large, the terminal voltage of the generator will not built up. It can be said that the common feature of both generators is only one possible current i flowing through their windings. The difference is that whereas in the series generator there are two windings, field and armature ones, a one phase of the SRG has only one winding, acting as a field winding during excitation period and as an armature winding during generating period (see angles θ_{exc} and θ_{gen} in Fig 4). The other difference is that the load circuit of the dc series generator is represented by the resistor R_L, on the terminals of the SRG is applied an RC load represented by the capacitor C and resistor R_L, or in the case of no load condition only capacitor, needed for the energy accumulation of the dc bus. Besides this, the concept of the SRG converter results in the fact that current flows only in one direction. Therefore a diode is added to the SRG equivalent circuit in Fig. 2.

Now look at Figs. 1 and 2: The switches S_1 and S_2 in Fig. 2 substitute the power transistors Q_1 and Q_2 and diodes D_1 and D_2 from Fig. 1. The voltage source e, inductance L_{ph} and resistance R_{ph} represent the members on the right side of equation (1). Phase current, voltage and inductance versus rotor position during one working cycle are shown in the Fig. 4. It is seen that in SRG operation two main periods are defined: excitation period, depicted as angle θ_{exc}, during which phase winding is excited and act as field winding, and generating period, depicted as angle θ_{gen}, during which phase winding act as armature winding in which voltage is generated.

In such a way, in the excitation period θ_{exc} the switches S_1 and S_2 from Fig. 2 are in position 1, the phase current flows from the capacitor C and magnetizes the phase winding. In generating period θ_{gen} the switches are in position 2, phase current flows back to the DC bus and charges capacitor. In the other words: at the instant of θ_{off}, when period of excitation is converted to the period of generating, the switching off of the switches results in sudden change of the current direction in the capacitor. At this instant capacitor starts to be charged. The load resistor is permanently supplied from the DC bus.

As for the energy generation principle in excitation period the phase inductance is around its maximum (see Fig. 4) and the phase current in SRG magnetizes the magnetic circuit. In generating period when the phase inductance is decreasing, the back EMF develops the generating current that flows through the diodes back to the DC bus. It is seen in the equivalent circuit that because of diode the SRG cannot be brought to resonance. In each operation cycle in excitation period the phase winding operates as a field winding of the DC generator and then in generating period as an armature winding of the DC generator.

3 NO–LOAD AND SHORT–CIRCUIT STATE

In any source of electrical energy there can be defined two limiting conditions that are no load and short circuit conditions. Because probably no scientist publication till now has dealt with these two limiting operation states of SRG, it would be desirable to define them.

In dc generator, the field winding is excited by means of a constant source of voltage, the rotor is driven by the prime mover and emf induced in the armature winding will appear as a terminal voltage at the open-circuited armature terminals ($R_L = \infty$). The shaft torque is required to overcome the mechanical and other kinds of losses and the torque required by the inertia of the machine.

But SRG has across the terminals a capacitor, which offers better voltage stability. This capacitor has to be connected to the DC bus also in no-load condition. This is difference in comparison with no-load condition of the dc generator. If an ideal capacitor with no leakage resistance would be connected across the terminals, the SRG would come into *ideal no-load state*. That means the phase winding is not excited at all and the DC bus voltage remains on the same rated value. But this would be valid only if the control logic circuits would be supplied from an external DC source.

Because each capacitor has some leakage resistance ($10^6 \div 10^{10}\,\Omega$) and SRG control logic circuits are usually supplied from the DC bus, SRG needs to be excited during the no-load state. Then because of some phase current another part of energy for excitation is converted to the core and winding losses and some mechanical losses also need to be taken into account because of the shaft is driven.

The *real no-load state* is defined as follows: SRG generates energy consumed by the control logic circuits, leakage capacitor resistance, core, winding and mechanical losses what results in no-load phase current needed to keep rated voltage across the terminals.

The short-circuit state can be defined simpler: SRG terminals are short-circuited and therefore $R_L = 0$. In this case SRG cannot operate because it has no supply of excitation energy. This would be similar like DC series generator with short-circuited field winding. After the short-circuit of terminals happen in the SRG, it needs to be re-excited from an external DC source or theoretically by its own remanent magnetism as in conventional DC or AC induction generators.

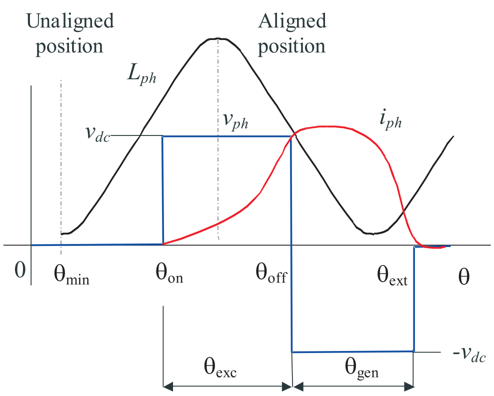

Fig. 4. Typical SRG phase values during one working cycle

4 MATHEMATICAL MODEL

The complete mathematical model has to be determined for the purpose of dynamic simulations. Based on voltage equation (1) the mathematical model with 7 equations has been established for 7 unknown quantities which occur in the equivalent circuit in Fig. 2. The first 4 equations belong to one phase and have to be multiplied by the phase number to get the total number of these kind of equations.

$$\frac{\mathrm{d}i_{ph}}{\mathrm{d}t} = \frac{1}{L_{ph}}\Big[v_{ph} - \Big(R_{ph} + \frac{\mathrm{d}L_{ph}}{\mathrm{d}\theta}\omega\Big)i_{ph}\Big] \tag{3}$$

$$i_{in} = \begin{cases} i_{ph}, & \text{S}_1 \text{ and S}_2 \text{ on)} \\ 0\,, & \text{S}_1 \text{ or S}_2 \text{ off)} \end{cases} \tag{4}$$

$$i_{out} = \begin{cases} i_{ph}, & (\text{S}_1 \text{ and S}_2 \text{ off}) \\ 0, & (\text{S}_1 \text{ or S}_2 \text{ on}) \end{cases} \tag{5}$$

$$v_{ph} = \begin{cases} v_{dc}, & (\text{S}_1 \text{ and S}_2 \text{ on}) \\ -v_{dc}, & (\text{S}_1 \text{ and S}_2 \text{ off and } i_{ph} > 0) \\ 0, & (\text{S}_1 \text{ or S}_2 \text{ on or } i_{ph} = 0) \end{cases} \tag{6}$$

Equation (3) is phase current differential equation derived from the basic phase voltage equation (1). Next two equations determine so called input current, which is actually phase current during excitation period and the output current is phase current during generating period. Input and output current equations (4), (5) have been added in the previous mathematical model [4]. Equation (6) determines instantaneous value of the phase voltage. Because the power switches and diodes have been replaced by the ideal switches S_1 and S_2 there is no voltage drop on the switch during on state an the switch has infinite resistance during off state. The next 3 equations are common for the whole model:

$$i_c = \sum_{j=1}^{m} i_{out_j} - \sum_{j=1}^{m} i_{in_j} - i_{RL} \tag{7}$$

$$i_{RL} = \frac{v_{dc}}{R_L} \tag{8}$$

$$\frac{\mathrm{d}v_{dc}}{\mathrm{d}t} = \frac{i_c}{C} \tag{9}$$

Equation (7) determines DC bus capacitor current which is a sum of all phase input and output currents and also load current. The last two equations determine load current and DC bus voltage.

Before dynamic simulation static parameters phase resistance R_{ph} and inductance L_{ph} or eventually phase flux linkage ψ_{ph} *vs* rotor position and current have to be implemented to the model. To get L_{ph} values one phase was supplied with DC voltage in each rotor position and

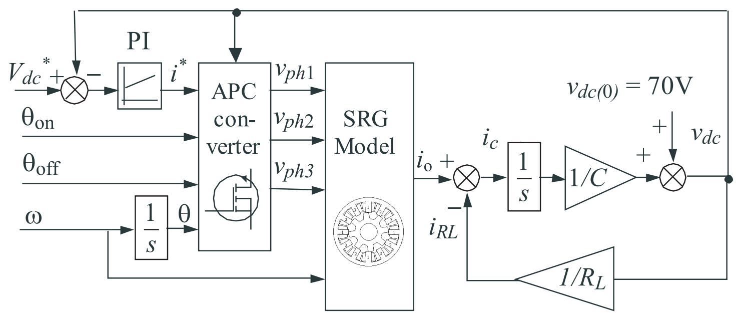

Fig. 5. Block diagram of the simulation model

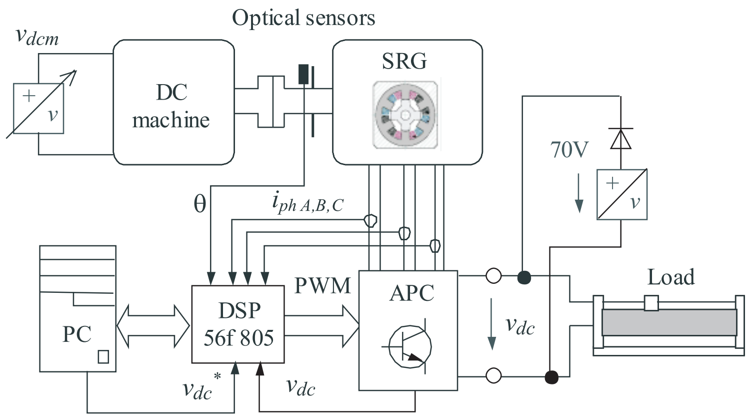

Fig. 6. Block diagram of the 3-phase 6/4 SRG drive set-up

build-up current was scanned by oscilloscope. Then ψ_{ph} and L_{ph} were calculated as follows:

$$\psi_{ph} = \int \big(v_{ph} - R_{ph} i_{ph}\big) \mathrm{d}t \tag{10}$$

$$L_{ph} = \frac{\psi_{ph}}{i_{ph}} \tag{11}$$

Block diagram of the simulation model is in Fig. 5. SRG has been controlled by means of hysteresis current control and commutation instants θ_{on} and θ_{off} have been constants. All phase currents have been calculated independently within the block SRG model. The result of this block is so called "DC link current" i_o which is actually a sum of output currents of all phases minus a sum of input currents of all phases and that is proportional to the generated power:

$$i_o = \sum_{j=1}^{m} i_{out_j} - \sum_{j=1}^{m} i_{in_j} \tag{12}$$

$$P_{out} = i_o v_{dc} \tag{13}$$

5 SRG DRIVE SET–UP

In Fig. 6 there is a block diagram of the experimental setup. The input power on the SRG rotor shaft has been delivered from DC machine, which has been supplied by a regulated DC source v_{dcm}. Black and white disc with 3 optical sensors mounted on SRG's rotor shaft have been used to obtain the rotor position signal. A load resistance and external voltage source are connected across the DC bus. SRG has been controlled by means of the 16 bit fixed point DSP 56f805. Measured phase currents i_{phA}, i_{phB}, i_{phC}, DC bus voltage v_{dc}, required DC bus voltage v_{dc}^* and rotor position θ have been used as input parameters for control strategy. SRG has been regulated for constant output DC bus voltage by means of hysteresis current control equally as it was in simulation model. The details of regulation are presented in [2].

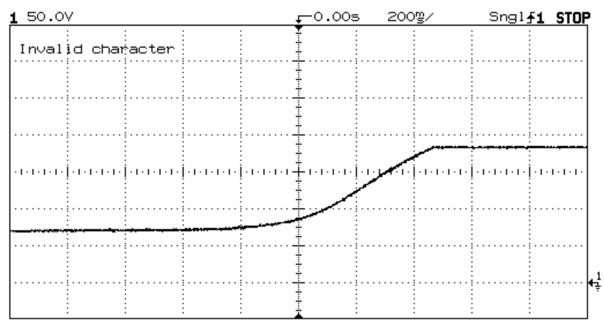

Fig. 7. Starting excitation of the no-loaded SRG, measured v_{DC} raises from 70V to 180V

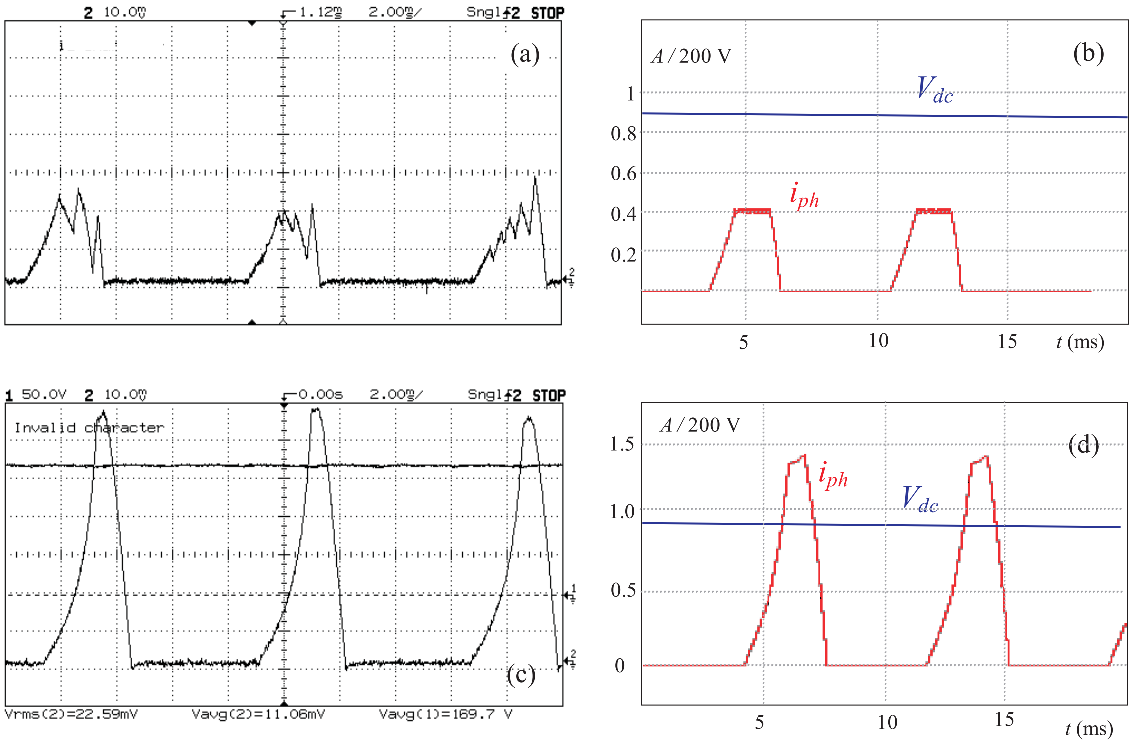

Fig. 8. Measured (a,c) and simulated (b,d) phase current and DC bus voltage, no-load conditions (a,b), by $R_L = 1200\,\Omega$ (c,d) and $n = 2000$ rpm

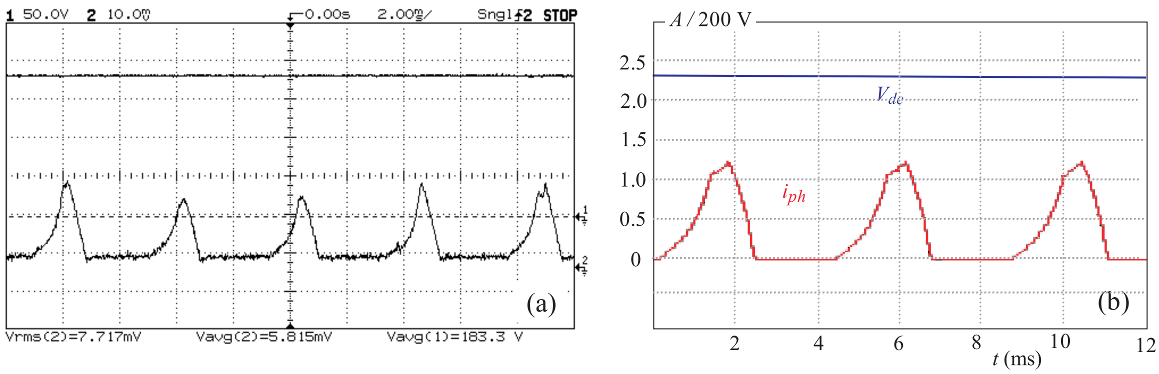

Fig. 9. Measured (a) and simulated (b) phase current and DC bus voltage of no-loaded SRG by 3500 rpm and $V_{dc} = 220$ V

6 MEASURED AND SIMULATED RESULTS

SRG has been started in no load state with the 70V on the DC bus from external source and at the speed 2000 rpm. The voltage has reached the desired value 180 V after 0.7 s (Fig. 7).

Measured and simulated phase current and DC bus voltage profiles are in Fig. 8a–d. The first two pictures represent no-load state where generated power is consumed only for overcoming the losses and supplying the converter control circuits.

When the speed decreased below 1500 rpm the generator performance was very unstable. In this case even weakly loaded generator has had a tendency to field suppression and to voltage drop to zero.

On the other hand SRG has worked better and more stable with higher speed. But high-speed operation has been limited by DC motor because its rated speed was only 3000 rpm. Measured and simulated phase current and DC bus voltage by speed 3500 rpm are in Fig. 9.

Measured and simulated phase voltage v_{ph} and current by speed 2000 rpm and two load conditions are in Fig. 10. By smaller load $R_L = 2200\,\Omega$ phase is switched with 8 kHz PWM with duty cycle $D = 90\,\%$. By higher load $R_L = 1200\,\Omega$, D had to be increased to $98\,\%$ in order to get sufficient generated power for stable SRG operation. With D almost $100\,\%$ this operation has been very close to one-pulse operation. DC bus voltage has been kept only in 150 V in order to fit the whole curves to oscilloscope screen.

It can be declared that measured and simulated results are in a good agreement and the reasons of differences are in not precise mechanical configuration of sensors, optical disc and in neglecting of magnetic circuit saturation.

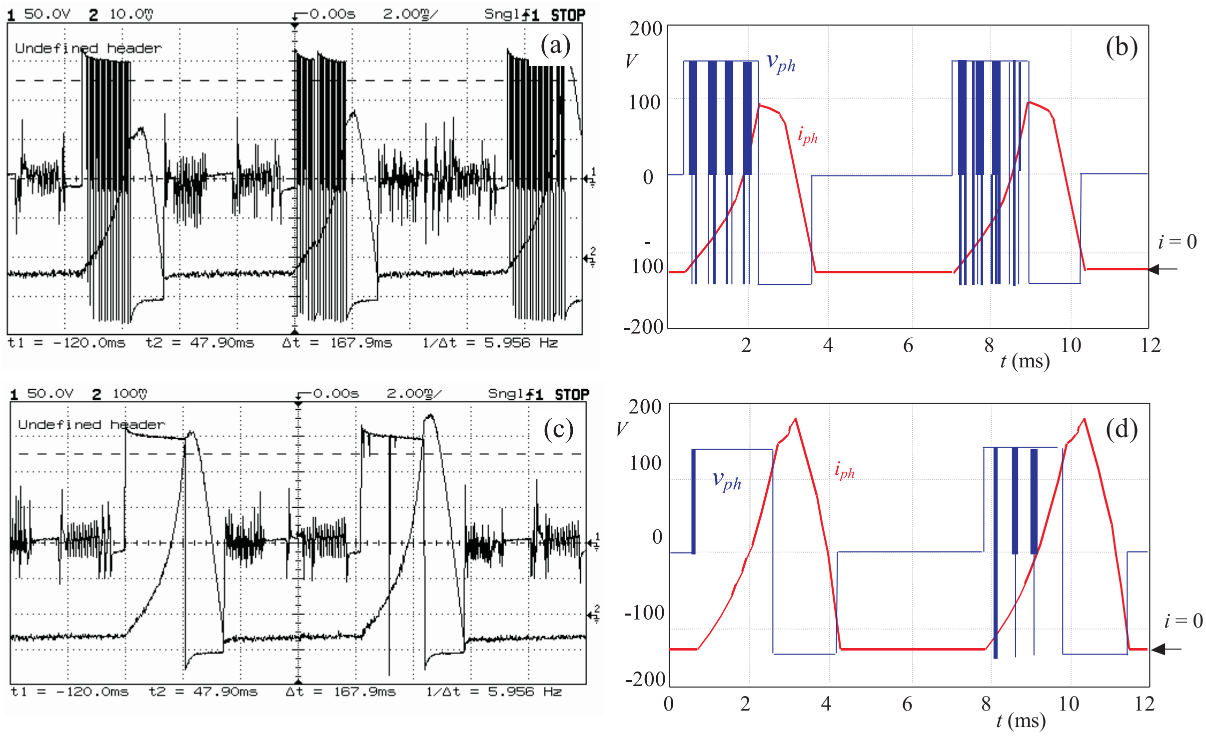

Fig. 10. Measured (a,c) and simulated (b,d) phase current and voltage of SRG by 2000 rpm and $R_L = 2200\,\Omega$ (a,b) and $R_L = 1200\,\Omega$ (c,d)

7 CONCLUSIONS

This paper is focused on the SRG equivalent circuit development, mathematical model and its simulation and measurement. The equivalent circuit has been established by means of a new approach based on equivalent circuit of DC series generator. Behaviour of SRG winding and magnetic circuit has been explained as very similar as behaviour of field and armature winding in DC series machine. Full mathematical mode for one SRG phase with most used APC type converter, which includes 7 equations for all voltages and currents in the equivalent circuit, is presented. Simulation model as well as corresponding experimental set-up of 3-phase 6/4 SRG controlled by means of hysteresis current control has been established and simulation and measured results of phase current and voltage values and DC bus voltage have been presented and compared. Measured and simulated results are in a good agreement and the reasons of differences are in not precise mechanical configuration of sensors, optical disc and in neglecting of magnetic circuit saturation.

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