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RECURSIVE IDENTIFICATION OF WIENER SYSTEMS WITH TWO–SEGMENT POLYNOMIAL NONLINEARITIES

Jozef Vörös *

The paper deals with the recursive identification of Wiener systems with two-segment polynomial nonlinearities. A special form of Wiener model is considered, which is linear-in-parameters. The proposed algorithm is a direct application of the known recursive least squares method extended with the estimation of internal variable values. Illustrative examples are included.

K e y w o r d s: nonlinear systems, recursive identification, Wiener model, time-varying systems

1 INTRODUCTION

For the subclasses of nonlinear dynamic systems which can be considered as block oriented systems [8] there exist several identification methods using topologically identical models. One of the simplest nonlinear models of this category is the so-called Wiener model consisting of one linear dynamic block and one nonlinear static block. The Wiener models appear in many engineering applications [6], [20], [22], [24] and more approaches were proposed for their identification, *eg*, [1–4], [7], [9], [10–13], [15, 16], [19], [21], [23], [25, 26], [29]. In the parametric form, the linear dynamic blocks of Wiener models are typically described by their transfer functions or in some cases by the FIR models. The characteristics of nonlinear blocks are often approximated by polynomials of proper degree.

To obtain an accurate polynomial fit of given static nonlinearity may cause problems in some situations. With lower-degree polynomials, the approximation error can be quite notable, while, with high degrees, the number of parameters increases [6]. This is the case when the characteristics are strongly asymmetric [14], *eg*, their outputs differ significantly for the positive and negative inputs, respectively, and only the polynomials of higher degree can approximate the nonlinear block characteristics adequately. Therefore it may be reasonable, for such considerably asymmetric nonlinear characteristics, to use descriptions with two distinct maps, *ie*, two-segment polynomial descriptions.

In this paper a special form of the Wiener model, based on a decomposition technique [27], is considered where the nonlinear static block is characterized by a two-segment polynomial approximation [28]. This model is linear-in-parameters and is used in a recursive estimation framework. The proposed algorithm is a direct application of the known recursive least squares method [17, 18] extended with the estimation of internal variables. It enables the estimation of both the parameters of the linear block transfer function and the coefficients of the polynomials approximating nonlinear characteristics using the system inputs, outputs and estimated internal variables. Simulation studies of Wiener system recursive identification are included.

2 WIENER MODEL

The Wiener model is given by the cascade connection of a linear dynamic system followed by a static nonlinearity (Fig. 1). The linear block can be given as

$$B(q^{-1})x(t) = q^{-d}A(q^{-1})u(t) \tag{2.1}$$

where $u(t)$ and $x(t)$ are the inputs and outputs, respectively, $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}

$$A(q^{-1}) = a_0 + a_1q^{-1} + \cdots + a_mq^{-m}\,, \tag{2.2}$$

$$B(q^{-1}) = 1 + b_1q^{-1} + \cdots + b_nq^{-n} \tag{2.3}$$

and d represents the pure delay of the system. If the nonlinear block is characterized by the mapping $C(\cdot)$, then the Wiener model output is

$$y = C(x)\,. \tag{2.4}$$

Let us assume that $C(\cdot)$ is generally an asymmetric two-segment mapping [24]

$$C(x) = \begin{cases} f(x) & \text{if } x \geqslant 0\,, \\ g(x) & \text{if } x < 0\,. \end{cases} \tag{2.5}$$

Introducing the switching function

$$h(x) = \begin{cases} 0 & \text{if } x \geqslant 0\,, \\ 1 & \text{if } x < 0 \end{cases} \tag{2.6}$$

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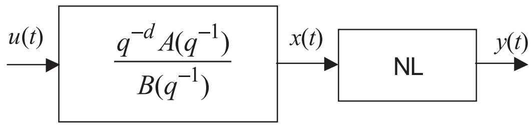

Fig. 1. Wiener model

the relation between the inputs and outputs of this nonlinearity can be written as follows:

$$C(x) = f(x) + [g(x) - f(x)]h(x)\,. \tag{2.7}$$

Now assuming that the mappings f and g can be approximated by proper polynomials:

$$f(x) = \sum_{k=1}^{r} f_k x^k\,, \tag{2.8}$$

$$g(x) = \sum_{k=1}^{r} g_k x^k \tag{2.9}$$

the Wiener model output equation can be written as

$$y = \sum_{k=1}^{r} f_k x^k + \sum_{k=1}^{r} p_k x^k h(x) \tag{2.10}$$

where

$$p_k = g_k - f_k\,. \tag{2.11}$$

Finally, after the half-substitution from (2.1) into (2.10) for the separated $x(t)$, assuming $f_1 = 1$, the system output is given in the form

$$y(t) = A(q^{-1})u(t-d) + [1 - B(q^{-1})]x(t) + \sum_{k=2}^{r} f_k x^k(t) + \sum_{k=1}^{r} p_k x^k(t) h[x(t)]\,. \tag{2.12}$$

The output equation (2.12) and the equation (2.1), defining the internal variable $x(t)$, represent the Wiener model with two-segment polynomial nonlinearity, which is linear in all the model parameters. This description includes the single polynomial approximation if $f(\cdot) = g(\cdot)$, *ie*, $p_k = 0$, $k = 1, 2, \ldots, r$.

3 RECURSIVE IDENTIFICATION

The problem with the decomposed form of Wiener model given by (2.12) is that the internal variable $x(t)$ is not accessible for measurement. Therefore an iterative identification method was proposed with the internal variable estimation [28]. The values of internal variable $x(t)$ are recomputed in each iteration using the previous estimates of linear block parameters. This off-line (batch) method can be easily converted into an on-line version.

The Wiener model with polynomial nonlinearity given by (2.12) can be put into a concise form

$$y(t) = \varphi^\top(t)\theta \tag{3.1}$$

where the data vector is defined as

$$\varphi^\top(t) = \varphi^\top(t,\theta) = \big[u(t-d), u(t-d-1), \ldots, u(t-d-m), -x(t-1), \ldots, -x(t-n), x^2(t), \ldots, x^r(t), x(t)h(t), x^2(t)h(t), \ldots, x^r(t)h(t)\big] \tag{3.2}$$

and the vector of parameters is

$$\theta^\top = \big[a_0, \ldots, a_m, b_1, \ldots, b_n, f_2, \ldots, f_r, p_1, p_2, \ldots, p_r\big]. \tag{3.3}$$

The estimates of the parameter vector can be evaluated using the RLS algorithm, minimizing the least-squares criterion based on (3.1), where the data vector $\varphi(t)$ is replaced by $\hat{\varphi}(t)$ with the estimates of the internal variable. The formulae of recursive identification algorithm are as follows:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\hat{P}(t-1)\hat{\varphi}(t)[y(t) - \hat{\varphi}^\top(t)\hat{\theta}(t-1)]}{\lambda + \hat{\varphi}^\top(t)\hat{P}(t-1)\hat{\varphi}(t)} \tag{3.4}$$

$$\hat{P}(t) = \frac{1}{\lambda}\Big[\hat{P}(t-1) - \frac{\hat{P}(t-1)\hat{\varphi}(t)\hat{\varphi}^\top(t)\hat{P}(t-1)}{\lambda + \hat{\varphi}^\top(t)\hat{P}(t-1)\hat{\varphi}(t)}\Big] \tag{3.5}$$

$$\hat{x}(t) = \sum_{i=0}^{m} \hat{a}_i(t-1)u(t-d-i) - \sum_{j=1}^{n} \hat{b}_j(t-1)\hat{x}(t-j) \tag{3.6}$$

$$\hat{\varphi}^\top(t) = \big[u(t-d), u(t-d-1), \ldots, u(t-d-m), -\hat{x}(t-1), \ldots, -\hat{x}(t-n), \hat{x}^2(t), \ldots, \hat{x}^r(t), \hat{x}(t)h(t), \hat{x}^2(t)h(t), \ldots, \hat{x}^r(t)h(t)\big] \tag{3.7}$$

$$\hat{P}(0) = \mu I\,,\ 0 < \mu < \infty \tag{3.8}$$

where the new values (estimates) of internal variables for the data vector $\hat{\varphi}(t)$ in each recursion are computed by (3.6) with the previous estimates of the corresponding parameters. The so-called forgetting factor $\lambda \le 1$ can be effectively applied to reduce the influence of old data, *ie*, both the (old) estimates of internal variables and the system parameters.

There are some alternatives for the initialization of the above algorithm depending on the first estimates of internal variable $x(t)$. In all the cases the linear block parameters, which determine the internal variable, are included into the estimation process immediately. In the simplest case, the linear model can be considered in the first step. The second alternative is to start the recursion with a quasi-Wiener model in the first step, *ie*, the internal variable is replaced by the output. Other possibilities are to run the recursion for some times (about 10-20 steps) only with the above alternatives and then to continue with the inclusion of the estimates of internal variable. This would correspond with the first step of the iterative method in [28].

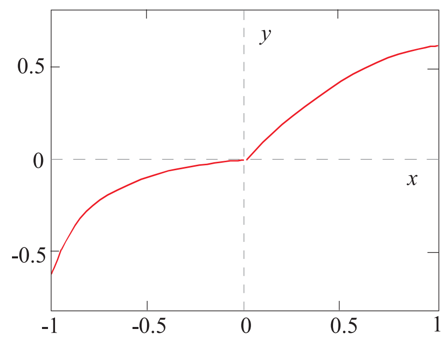

Fig. 2. Example 1: Nonlinearity

The proposed iterative identification method with the internal variable estimation is a modification of the relaxation techniques presented in [8]. Unfortunately there is no general proof of convergence for the identification methods using the block-oriented nonlinear models with internal variable estimation [10], although they are satisfactory for most practical applications.

4 SIMULATION STUDIES

The presented method for the recursive identification of nonlinear dynamic systems using the Wiener model with polynomial nonlinearities was implemented and tested in MATLAB. Several systems were simulated and the estimation of all the model parameters (those of linear and nonlinear blocks) were carried out on the basis of input/output records. In the following, the feasibility of the proposed recursive identification method is illustrated with examples of Wiener systems with strongly asymmetric nonlinearities and with time-varying nonlinearities.

Strongly asymmetric nonlinearities

The first example shows the parameter estimation process for simulated Wiener system with strongly asymmetric characteristic. The linear dynamic block was given by

$$x(t) = 0.5\,u(t-1) + 0.3\,u(t-2) - 0.1\,u(t-3) + 0.5\,x(t-1) - 0.4\,x(t-2)$$

and the nonlinear block was described by the following two-segment polynomial (Fig. 2)

$$f[x(t)] = x(t) - 0.3\,x^2(t) - 0.1\,x^3(t)\,,$$
$$p[x(t)] = -0.9\,x(t) + 0.5\,x^2(t) + 0.8\,x^3(t)\,.$$

The recursive identification was carried out for 1200 samples of uniformly distributed random inputs with $|u(t)| < 0.7$ and generated process outputs $y(t)$. The initial values of all the parameters were chosen zero and the forgetting factor was $\lambda = 0.9$. The process of linear block parameter parameter estimation is graphically shown in Fig. 3 (the top-down order of parameters is a_0, b_2, a_1, a_2, b_1) and that of nonlinear block parameter parameter estimation is shown in Fig. 4 (the top-down order of parameters is p_3, p_2, f_3, f_2, p_1). It can be seen that the model parameter estimates are able to track the true parameters.

As it is generally known the Wiener model is considerably sensitive to the noise because of the output nonlinearity. The proposed approach can also deal with modest noisy systems, nevertheless the convergence of parameter estimation process is slower. For illustration, the estimation process for the above example of Wiener model was tested with additive output noise. The noise was generated as a zero mean Gaussian white noise and the signal to noise ratio (the square root of the ratio of output and noise variances) was SNR $= 500$. Two forgetting factors were used in this example, *ie*, $\lambda = 0.94$ for 200 samples to reduce the influence the old estimates of internal variables at the beginning of recursion and $\lambda = 0.999$ for the rest of data. The results are shown in Figs. 5 and 6.

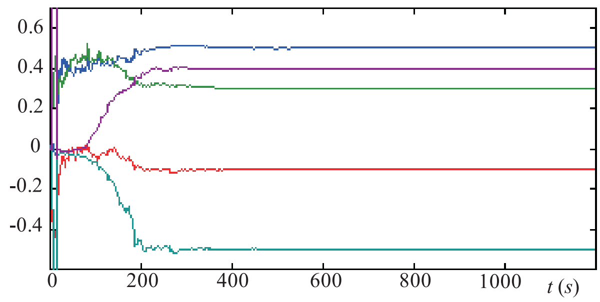

Fig. 3. Example 1: linear block parameter estimates

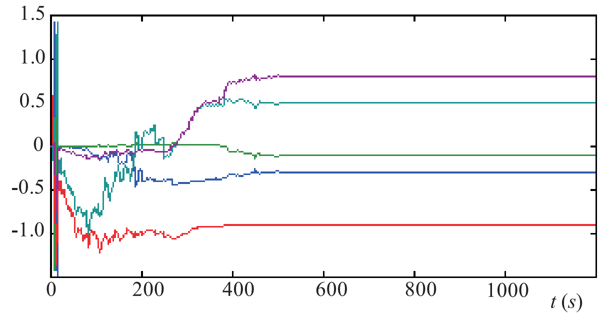

Fig. 4. Example 1: nonlinear block parameter estimates

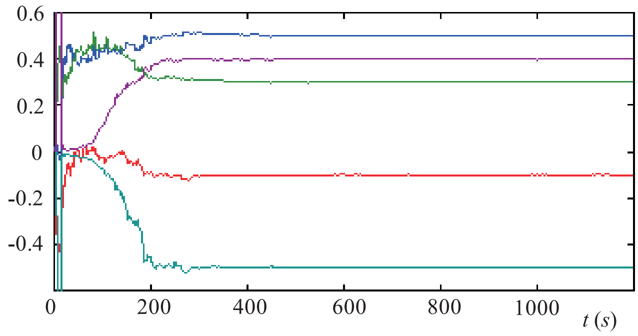

Fig. 5. Example 1 – with noise: linear block parameter estimates

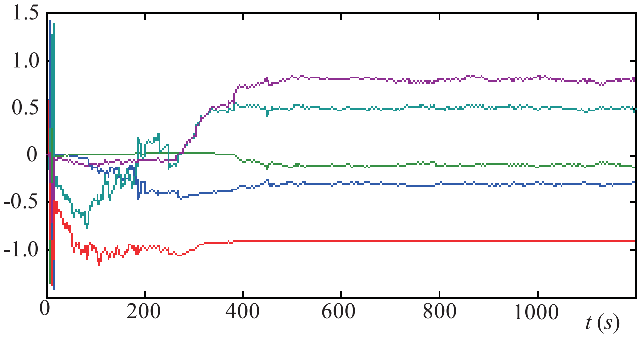

Fig. 6. Example 1 – with noise: nonlinear block parameter estimates

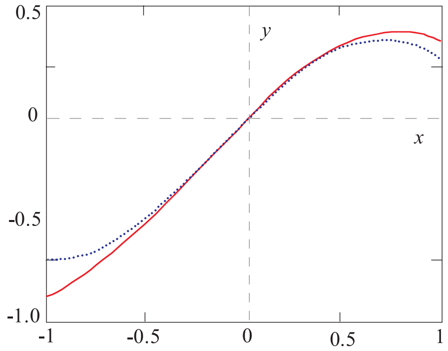

Fig. 7. Example 2: nonlinearity – original (dotted) and changed (full)

Time-varying nonlinearities

As the parameters of nonlinear system descriptions result from mathematical equations by physical modeling, the identified parameters have physical meaning, such as electrical resistance, mechanical friction, etc. Some or all of these parameters can gradually change causing the system to be time-varying. Therefore identification methods are required being able to track changing parameters of such time variant systems.

The second example shows the parameter estimation process for simulated Wiener system with time-varying two-segment nonlinearity. The linear dynamic block was given by

$$x(t) = 0.5\,u(t-1) + 0.4\,u(t-2) + 0.2\,x(t-1) - 0.35\,x(t-2)$$

and the nonlinear block was described by the following two-segment polynomial (the dotted line in Fig. 7)

$$f[x(t)] = x(t) - 0.4\,x^2(t) - 0.3\,x^3(t)$$
$$p[x(t)] = 0.1\,x(t) + 0.4\,x^2(t) - 0.1\,x^3(t)$$

The changes of nonlinear block parameters occurred slowly and gradually to

$$f[x(t)] = x(t) - 0.4\,x^2(t) - 0.2\,x^3(t)$$
$$p[x(t)] = 0.1\,x(t) + 0.4\,x^2(t)$$

in the time interval $t \in (1000, 2000)$ (see the full line in Fig. 7). The recursive identification was carried out for 3000 samples of uniformly distributed random inputs with $|u(t)| < 0.5$ and generated process outputs $y(t)$. The initial values of all the parameters were chosen zero and the forgetting factor was $\lambda = 0.9$. The process of linear block parameter parameter estimation is shown in Fig. 8 (the top-down order of parameters is a_0, a_1, b_2, b_1) and that of nonlinear block parameter parameter estimation is shown in Fig. 9 (the top-down order of parameters is p_2, p_1, p_3, f_3, f_2), where the changes of polynomial parameters $f_3(t)$ and $p_3(t)$ can be seen.

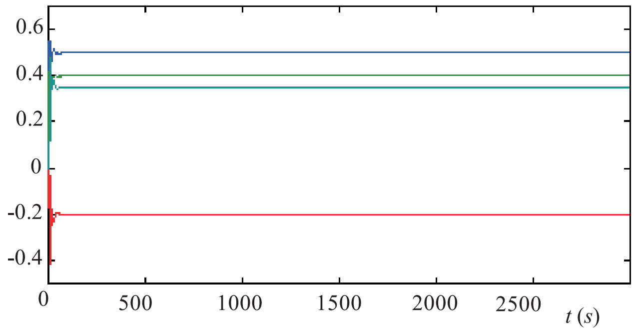

Fig. 8. Example 2: linear block parameter estimates

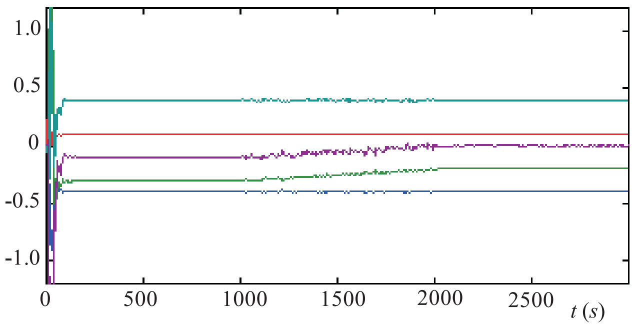

Fig. 9. Example 2: nonlinear block parameter estimates

5 CONCLUSION

Recursive identification methods are important for the property that they can be computed in real time, and hence the estimates of the model parameters can follow the true ones and their eventual changes. The proposed form of Wiener models with general two-segment polynomial nonlinearities seems to be appropriate for the recursive identification of a broad subclass of nonlinear dynamic systems. As all the model parameters appear explicitly in the model description and are identical with those of linear and nonlinear blocks, respectively, several approaches to the adaptive control using linear models can be adopted for the control of this subclass of nonlinear systems [5].

Identification of nonlinear dynamic systems is a very difficult problem and no approach can be recommended as being universal. Although several identification methods using block-oriented models are available, further research is needed, especially in connection with adaptive control algorithms.

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