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SLOT FRINGING EFFECT ON THE MAGNETIC CHARACTERISTICS OF ELECTRICAL MACHINES

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Due to the fringing effect, the magnetic flux in the air gap of electrical machines is reduced. This leads to the larger effective air gap length. For this reason, in the design stage of the electrical machines, a larger magnetic flux must be chosen. On the other hand, magnetic loading must be taken smaller than the value corresponding to the real air gap length. Currently, Carter coefficient is applied to compensate the slots effects. This coefficient is calculated with respect to the slots dimensions and air gap length, using Carter formulas and corresponding curves. These curves are taken by solving the two dimensional Laplace equation for voltage, and cannot be accurate (errorless) completely. Nowadays, using the FEM (finite element method) packages of numerical methods, slot effects on the air gap flux distribution are calculated carefully. In this paper using the Ansys package of FE these effects are studied. Using the results of these studies and comparing them with the Carter method, the Carter coefficient is modified.

K e y w o r d s: flux scattering, Carter coefficient, air gap coefficient, finite element

1 INTRODUCTION

Designing of the electrical machines, specially rotating one, involves the solving of some complicated electromagnetic, thermal and mechanical problems. Also the geometrical shape of the rotating machines is complicated and designing and analyzing them is very difficult. Due to the capabilities and flexibilities of the FEM, nowadays FEM is applied to solving such problems. In this method, after the design of the geometry of the model, specifying the material types of different parts of the model and its parameters and physical characteristics, different surfaces of the model are meshed by elements suitable for the proposed analysis. Then applying the excitation and boundary conditions, the model is solved.

In the FEM for analyzing the model, the type of the meshes must be chosen suitably with respect to the analysis type. Elements are triangular or rectangular. If the model geometrical shape is regular, the map meshing can be made, otherwise free meshing is applied.

Slot shape and dimensions effect in the air gap flux scattering are studied by different researchers, such as Carter [8, 9] and Green, and different results are produced. Studying the slot dimensions and their effect on the flux density of the magnetic fields is returned to the beginning of the last century, when the numerical methods were not developed yet. So the results of those methods cannot be accurate completely.

In this paper using the Ansys package, the effect of flux scattering in the air gap between the slots on the magnetic characteristics is studied. The results are compared with the classical methods and finally the modified Carter coefficient is introduced.

2 EFFECT OF FLUX SCATTERING ON THE FLUX DENSITY

In most of the rotating electrical machines (DC or AC), the windings of the stator or rotor are taken into open mouth slots. Slot shapes and their mouth openings depend on the type of the machine and its operation. The quantity of the scattered flux mainly depends on the slot mouth opening, and the slot shape has a small effect. So the slot shapes are chosen rectangular. In the AC machines having sinusoidal flux distribution, the slots number must be chosen greater, and the slot opening smaller. This condition cannot be taken in fact. As shown in Fig. 1, in this condition the flux density of the air gap of the machine is reduced strongly in front of the slots, and distorted from the sinusoidal shape.

In an electrical machine, apart from the effect of the slots opening and flux scattering, having a larger permeability of the iron core, the reluctance of each pole flux is calculated as follows

$$R = \frac{g}{\mu_0 A} \tag{1}$$

where: g – is air gap length,
A – is cross section of each pole.

If the slots effect is considered, the core surface under each pole is reduced and the flux of each pole, having

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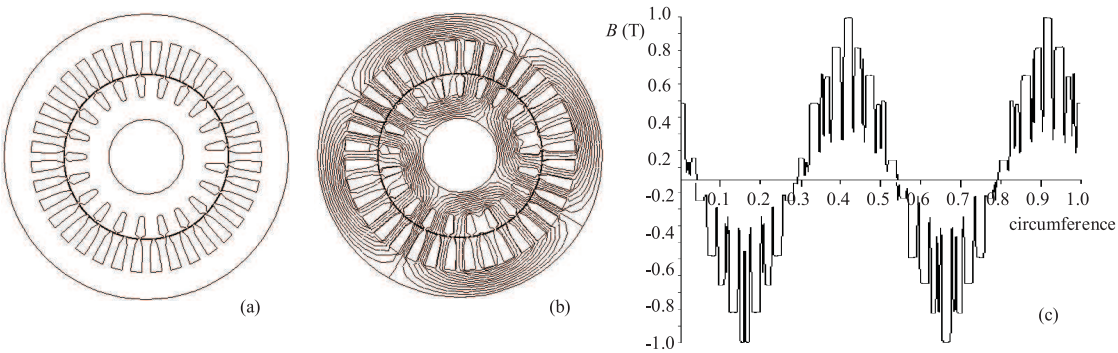

Fig. 1. a) Two dimensional cut of machine stator and rotor, b) flux lines distribution, c) flux density in the air along the circumference

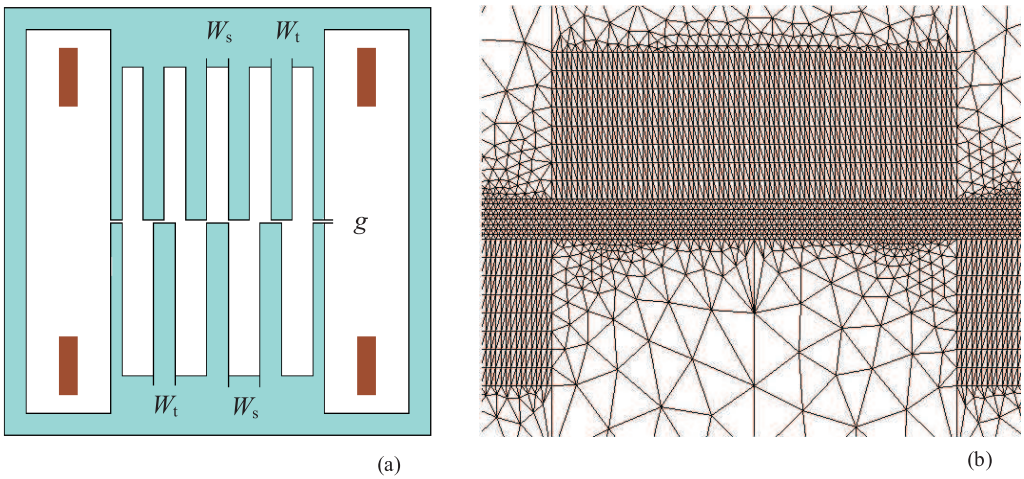

Fig. 2. a) Two-dimensional model of the machine between the two on-lined teeth of stator and rotor, b) free meshed surface of the model

a constant ampere-turn, is reduced. This difference can be applied by inducing the air gap length and the real air gap length g is compensated to an effective air gap length g_e as follows

$$R = \frac{g_e}{\mu_0 A} \,. \tag{2}$$

The ratio of the effective air gap length to the real air gap length is known as the scattering coefficient or gap coefficient k_g. In the design of electrical machines this coefficient is known as the carter coefficient. k_g depends on the slot dimensions and air gap length, and has an analytical function as follows

$$k_g = \frac{g_e}{g} \,. \tag{3}$$

Usually in the analysis of electrical machines, only one pole of the machine is modelled. This reduced the model and the number of equations, and results in reduction of the calculation and finally leads to the fast solving of the problem, still having the accuracy of the results. For this purpose, a two-dimensional model of the machine between the two on-lined teeth of the stator and rotor, as Fig. 2, is chosen for the analysis,

where: W_s– is slot width,
W_t– is tooth width.

Different methods are given in many designing references for specifying and calculating the effective air gap length. Some of these methods are discussed briefly.

Primitive experimental method: If the air gap length is very small with respect to the slot width, the whole flux approximately crosses from the teeth, and then we can suppose the effective surface of each pole approximately equal to the teeth surface. Equation (4) calculates the effective surface as this method.

$$A_e = A\frac{W_t}{W_s + W_t} \,. \tag{4}$$

By replacing A with A_e in equations (1) and (2), the air gap coefficient can be calculated as follows:

$$k_g = 1 + \frac{W_s}{W_t} \,. \tag{5}$$

This method is a purely approximate one because in each tooth there is a small quantity of flux scattering that is not considered in this method.

Improved experimental method: For increasing the accuracy of the above-mentioned method, equation (6) is suggested.

$$k_g = \frac{W_t + W_s}{W_t + k_s W_s} \tag{6}$$

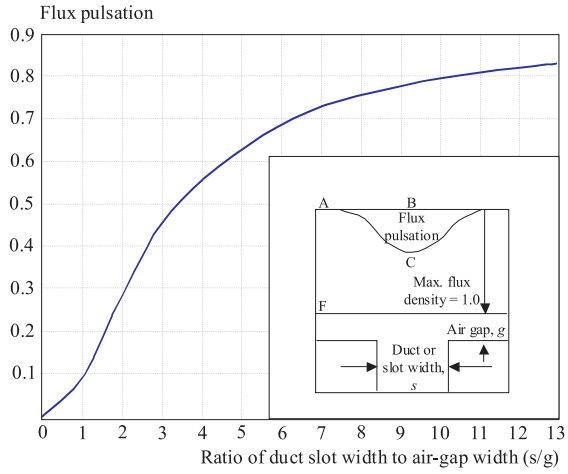

Fig. 3. Carter curve for slot coefficient k_s

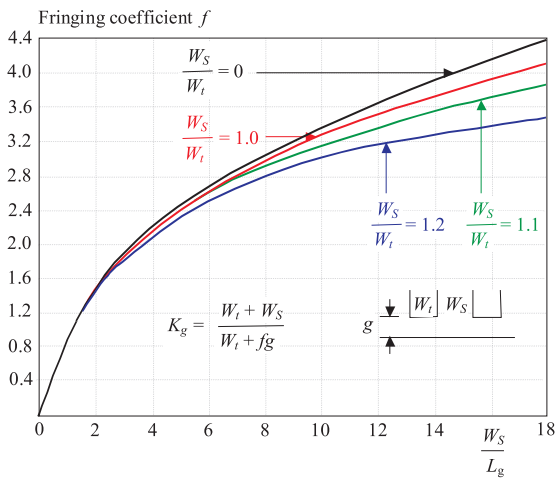

Fig. 4. Green curve for fringing coefficient f versus slot-opening to gap-length ratio

where k_s is known as the slot coefficient and can be calculated as equation (7).

$$k_s = \frac{5g}{5g + W_s}\,. \tag{7}$$

Carter method: F.W. Carter was the first to calculate analytically the flux scattering in an open mouth slot in 1899. He solved the two dimensional Laplace equation (8) for voltage.

$$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} + \frac{\mathrm{d}^2V}{\mathrm{d}y^2} = 0 \tag{8}$$

where V is the electrical potential. He reached equation (9) by plotting the equi-potential surfaces between the slot and pole surfaces, mapping them in the z-plain using Schwarz-Christoffel [1], and calculating the air gap inductance for a slot with an infinite depth and a tooth with infinite width [7].

$$k_s = \frac{\tau_t(5g + W_s)}{\tau_t(5g + W_s) - W_s^2} \tag{9}$$

where τ_t is the tooth pitch and is defined as follows:

$$\tau_t = \frac{W_t}{W_t + W_s}\,. \tag{10}$$

He suggested equation (11) for approximately closed slots [7].

$$k_s = \frac{\tau_t(4.4g + 0.7W_s)}{\tau_t(4.4g + 0.7W_s) - W_s^2}\,. \tag{11}$$

Finally he suggested Fig. 3 curves for k_s, that it is taken using equation (6) [7].

Green method: C. F. Green improved the Carter method for a tooth with finite width with different ratios versus slot width [12] and obtained results very similar to those of Carter. He suggested Fig. 4 curves instead of Fig. 3, with different ratios of W_s/W_t, where f is the scattering coefficient. Inserting this in equation (12), the air gap coefficient or Carter coefficient can be calculated.

$$k_g = \frac{W_t + W_s}{W_t + fg}\,. \tag{12}$$

Currently this curve is applied in machine analytical designing by many researchers [2, 3, 12].

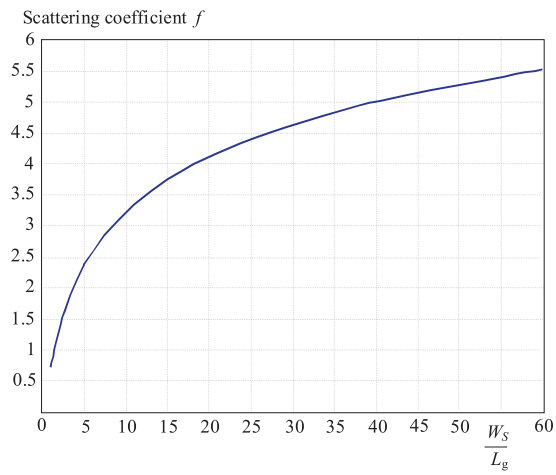

Fig. 5. Flux scattering coefficient f versus slot-opening to gap-length ratio using FE solution

E. Carter based methods: Different equations are suggested for the Carter coefficient based on the Green curve and equation (12). Equation (13) is one of them, which is not explicitly a function of W_s/W_t ratio.

$$k_s = 1 - \frac{2}{\pi}\left(\tan^{-1} y - \frac{1}{\pi}\log\sqrt{1+y^2}\right), \quad y = \frac{W_s}{2g}\,. \tag{13}$$

3 STUDY OF GREEN CURVE USING FEM

Equation (12) is accurate for taking into account the effect of flux scattering. In this part of the paper, the flux scattering coefficient is calculated using FEM. For this purpose, using Fig. 2 model of the machine, for different slot and tooth width and air gap length with constant ampere-turn, the model is solved and the flux crossing the pole, and then k_g is calculated. For this purpose, for any amount of these dimensions, first of all a model is made in Ansys environment, then with a constant excitation it solved and the air gap crossing flux is calculated. The result of each solution is a figure for φ. The Carter coefficient for each dimensions and characteristics can be calculated from the ratio of φ to φ_0 (crossing flux when there is no slot).

The FE packages can solve the non-linear problems, but in this study only the linear electromagnetic solution is made. For calculating of f, the flux scattering coefficient for a finite number of slots in one tooth and for a constant ratio of W_s/W_t, air gap length is varied between 1 mm and 6 mm in 27 stages by steps of 0.5 mm and 0.1 mm, and then the model is solved. By calculating the k_g from equation (12) one curve for f is produced. Then the slot pitch is varied from 0.05 mm to 0.6 mm, and 12 curves with different ratios of W_s/W_t are produced. For having different amounts of W_s/g ratios, the above mentioned process for the different numbers of slots between 1 and 10 in one side of the air gap (stator or rotor only) is repeated. The results of calculations of f for all 120 conditions are the same and plotted in Fig. 5.

In all of the above-mentioned methods, it is assumed that the slots are only in one side of the air gap (stator or rotor only). For the real condition, where slots are on both sides of the air gap, with respect to Carter's suggestions, the total air gap coefficient is the multiplication of two coefficients of the stator and rotor as follows

$$k_g = k_{g1} \times k_{g2} \,. \tag{14}$$

4 COMPARISON OF FE RESULTS AND OTHER METHODS

In this section, the Carter coefficient is calculated using each of the above-mentioned methods for three models, and its percent error based on FEM are calculated and compared.

Model 1: In this model 4 slots in the upper side of the air gap are assumed. Table 1 summarizes the characteristics of this model. The results of the FEM solution are in table 2. Table 3 shows the results of all the above-mentioned methods and percent error of them based on FEM. As this table shows, the best one of these 7 methods is the "accurate experimental" method. Figure 6a shows the flux lines distribution of the first model. In Fig. 6b, the air gap flux density distribution is shown.

Model 2: Five slots in the upper side of the air gap with characteristics as Tab. 4 are considered. Table 5 shows the FEM solution results of this method. Figure 7 shows the flux lines distribution and air gap flux density distribution of the second method. In table 6 the comparison of all the 7 methods is given.

Table 1. Characteristics of the first model

g (mm)	w_s (mm)	w_t (mm)	W_s/W_t
1	15	10	1.5

Table 2. FEM results of the first model

f	k_g	φ (Wb)	φ_0 (Wb)
3.761	1.81674	0.27667	0.50263

Table 3. Comparison of the 7 methods with FEM for the first model method

	method	k_g	error (%)
1	inaccurate experimental	2.500	37.609
2	accurate experimental	1.818	0.069
3	Carter's first equation	1.023	-43.690
4	Carter's second equation	1.039	-42.810
5	Carter's k_s curve	2.300	26.600
6	Green's f curve	1.953	7.500
7	analytical equation (13)	1.794	-1.252

Table 4. Characteristics of the second model

g (mm)	W_s (mm)	W_t (mm)	W_s/W_t
1	10	10	1

Table 5. FEM results of the second model

f	k_g	φ (Wb)	φ_0 (Wb)
3.250	1.50938	0.33300	0.50263

Table 6. Comparison of the 7 methods with FEM for the second model

	method	k_g	error (%)
1	primitive experimental	2.000	10.087
2	iproved experimental	1.500	-17.435
3	Carter's first equation	1.014	-44.213
4	Carter's second equation	1.018	-43.971
5	Carter's k_s curve	1.852	1.930
6	Green's f curve	1.503	-17.269
7	analytical equation (13)	1.416	-22.058

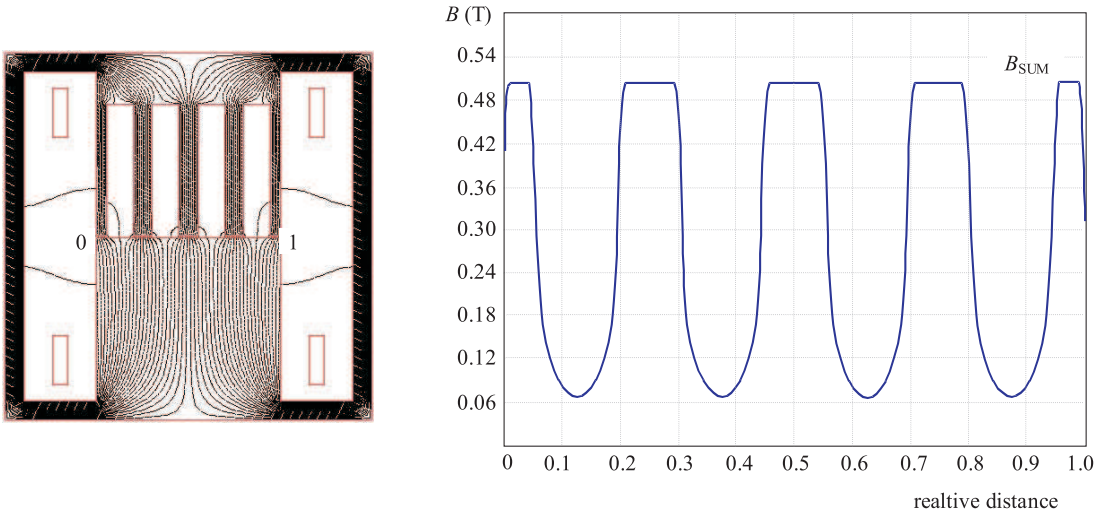

Fig. 6. a) First model, a) flux lines distribution, b) air gap flux density distribution

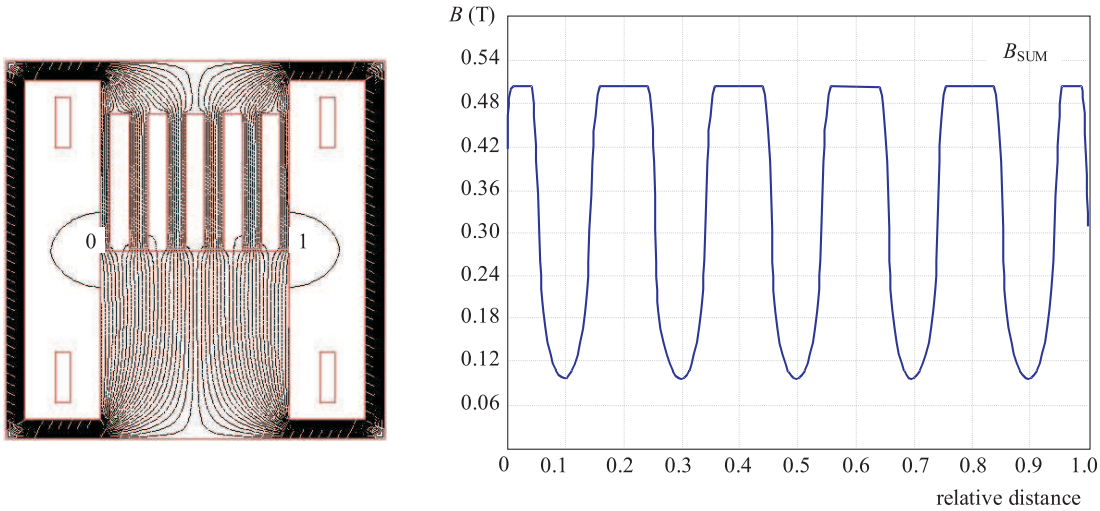

Fig. 7. Second model, a) flux lines distribution, b) air gap flux density distribution

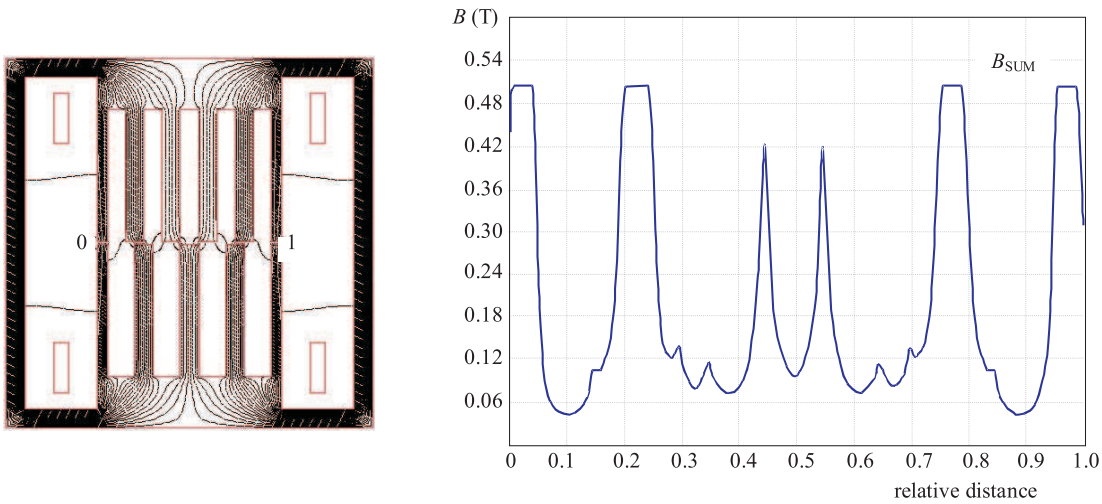

Fig. 8. Third model, a) flux lines distribution, b) air gap flux density distribution

Model 3: On contrary to the two above-mentioned models, in this model the slots are considered on both two sides of the air gap (real condition). Four slots on one side with characteristics as the first model and five slots on the other side with characteristics as the second model. Table 7 represents the FEM solution results. In table 8, the comparison of all the 7 methods is given.

Figure 8 shows the flux lines distribution and air gap flux density distribution of the third model. As shown in this figure, it is clear that the approximate methods and equi-potential mapping method using Schwarz-Christoffel cannot have the same accuracy as the numerical (FE) method. As shown in Tables 3, 6, 8, some of the methods are accurate accidentally for some models with its specific characteristics, but they are not accurate for other models with other characteristics.

Table 7. FEM results of the second model

k_g $(k_{g1} \times k_{g2})$	φ (Wb)	φ_0 (Wb)
2.556	0.19661	0.50263

Table 8. Comparison of the 7 methods with FEM for the third model

	method	k_g	error (%)
1	inaccurate experimental	5.000	95.6
2	accurate experimental	2.727	6.7
3	Carter's first equation	1.037	-59.4
4	Carter's second equation	1.058	-58.6
5	Carter's k_s curve	4.259	66.7
6	Green's f curve	2.935	14.9
7	analytical equation (13)	2.540	-0.6

4 CONCLUSIONS

The flux scattering effect in the air gap of electrical machines due to the existence of the slots, which is applied in the design stage as the Carter coefficient, only depends on the ratio W_s/g, and variation of W_s/W_t, hence W_s/g ratio is constant, not varied. In our accurate study of this effect using numerical methods as FEM, the improved f curve is produced as shown in Fig. 5.

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