

NETWORKED ROBUST PREDICTIVE CONTROL SYSTEMS DESIGN WITH PACKET LOSS

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The paper addresses problem of designing a robust output feedback model predictive control for uncertain linear systems over networks with packet-loss. The packet-loss process is arbitrary and bounded by the control horizon of model predictive control. Networked predictive control systems with packet loss are modeled as switched linear systems. This enables us to apply the theory of switched systems to establish the stability condition. The stabilizing controller design is based on sufficient robust stability conditions formulated as a solution of bilinear matrix inequality. Finally, a benchmark numerical example-double integrator is given to illustrate the effectiveness of the proposed method.

Key words: model predictive control (MPC), networked control systems (NCSs), packet-loss, parameter dependent quadratic stability (PDQS), bilinear matrix inequality (BMI)

1 INTRODUCTION

Feedback control systems wherein the loops are closed through real-time networks are called Networked Control Systems (NCSs) [16, 18, 26, 29]. Advantages of using NCSs in the control area include simplicity, cost-effectiveness, ease of system diagnosis and maintenance, increased system agility and testability. However, the integration of communication real-time networks into feedback control loops inevitable leads to challenging problems such as network-induced delays and data packet losses, which can induce instability or poor performance of closed-loop control systems. Therefore, packet loss is one of the most important and special issues of NCSs.

There are two major approaches to accommodate the issue of packet loss in an NCS design. One way is that one first designs the control system without regard to the networks, and then determines a performance level that the networks should satisfy (for example, maximum allowable transfer interval) so that the closed-loop system maintains its performance (for example, stability) when some control and sensor signals are transmitted via the networks [13, 29]. The other approach is to treat the network protocol and traffic as given conditions and designs the control strategies that explicitly take the network-induced issues into account [1, 21, 27, 28]. Under the assumption that the network is modeled as a switch governed by a Bernoulli process, Zhang *et al* [29] proposed a criterion to check whether the NCS is stable at a certain rate of packet losses, and searched for the maximum packet-loss rate under which the overall system remains stable. The method they used derives from the stability analysis for asynchronous dynamic systems. With packet-loss rate known and constant, Seiler and Sengupta [21]

formulated the NCS as a Markovian jump system with two operation modes, and then applied the techniques developed for Markovian jump systems. A dynamic output feedback controller design method was proposed such that the NCS is mean square stable and has H_∞ gain below certain value in terms of linear matrix inequalities (LMIs). Moreover, Yu *et al* [28] modeled the packet-loss process as an arbitrary but finite switching signal. This enables them to apply the theory from switched systems to stabilize the NCS. However, in the framework considered in the references mentioned above, the controller is directly connected to the actuator. That means no packets are dropped in control signals. A general framework was considered in [1], where both sampling signals and control commands are transmitted through the network and may be dropped during the transmissions. The linear quadratic Gaussian control problem was studied based on dynamic programming approach. Xiong and Lam [27] generalized the procedure in [28] to double-sided packet loss, as one of the contributions, and established stability conditions via a packet-loss dependent Lyapunov approach, as another contribution.

In the last two decades, model predictive control (MPC) has been widely adopted in industry as an effective means to deal with multivariable constrained control problems [8, 17]. The idea of MPC is stemmed from employing an explicit model of the plant to be controlled which is used to predict the future state/output behaviour over the finite time horizon. There are some researching results that have been presented in MPC for NCSs. Srinivasaguta *et al* [22] proposed a time-stamped model predictive control algorithm for NCS when random delay is less than one sample time. Liu *et al* [10] proposed using a networked control predictor to take the

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A short version of this paper has been published at 4th IFAC Symposium NMPC'12

latest control value from the predictive control sequence available to deal with random communication time delay problem, and then Mu *et al* [12] presented using a low pass filter to filter the error produced between the delayed plant output measurement and its delayed open-loop model output to improve system robustness. Tang and de Silva [23] proposed a NCS control strategy based on generalized predictive control with the buffering of future control sequence to overcome the transmission delay problems at the controller-to-actuator lines and Tang and de Silva [23] presented the conditions under which the stability of the constrained model predictive control for NCS with random delay can be guaranteed. Li *et al* [9] proposed a stabilizing MPC strategy for NCS with data packet loss between sensor and controller. In [2] the authors presented a packet-based robust MPC approach in a co-design framework for Wireless Networked Control Systems (WNCS) with the round-trip delay. Polytopic description was used to describe uncertainty of system. In [3,4] the author proposed an MPC design method for NCS with double-sided packet loss. A packet-loss dependent Lyapunov function is used for stabilization, and the result is used for synthesizing model predictive control by parameterizing the infinite horizon control moves into a single state feedback law. However, Ding [3] assumes that the actuator information (whether or not the actuator has received new data) can be sent to the controller and in [4] he investigates the case when the actuator information cannot be sent to the controller. One of the ways to overcome the resulting loss-packet problems is the use of prediction based compensation schemes [2,5–7,14]. For example, in [6], the authors presented a networked control scheme which uses a model based prediction and time-stamps in order to compensate for delays and packet dropouts in the transmission between sensor and controller and between controller and actuator, respectively. In order to analyze the properties of the scheme, they introduced the notion of prediction consistency which enables them to precisely state the network properties needed in order to ensure stability of the closed loop.

From our vision, it seems that there is no previous result on design of robust output feedback MPC for uncertain linear systems over networks with bounded packet loss. Motivated by the above observation, in this paper, we consider the implementation of a robust output feedback linear model predictive control scheme over a network with double-sided packet loss. The main idea of this paper is based on the combination of compensation mechanism in ([2,5–7] and robust model predictive control design approach in [15]). As a result, networked predictive control systems with loss packet are modeled as switched linear systems. This enables us to apply the theory of switched systems to establish the stability condition of networked model predictive control. We suppose that network-induced delays are within the sample time. Packet loss process is considered as arbitrary but bounded by the horizon control of MPC and its contribution is based on [27]. The main goal of this paper is to provide

a robust stability analysis and synthesis robust predictive controller for this scheme with guaranteed cost and PDQS.

The organization of the paper is as follows. Section 2 gives the problem formulation. In Section 3, the robust output feedback predictive controller design method with input constraints using bilinear matrix inequality is presented. The approach of robust constrained networked model predictive control design with guaranteed cost and PDQS is introduced in section 4. In section 5, one benchmark example is solved by using Yalmip BMI solvers to show the effectiveness of the proposed method. Finally, some conclusions are given.

Hereafter, the following notational conventions will be adopted: given a symmetric matrix $P = P^T$, the inequality $P > 0$ ($P \geq 0$) denotes matrix positive definiteness (semi-definiteness). "*" denotes a block that is transposed and complex conjugate to the respective symmetrically placed one. Matrices, if not explicitly stated, are assumed to have compatible dimensions. I denotes the identity matrix of corresponding dimensions. The notation $x(t+k|t)$ will be used to define, at time t k - steps ahead, prediction of a system variable x from time t onwards under a specified initial state and input scenario. Note that $x(t|t) = x(t)$.

1 PRELIMINARIES AND PROBLEM FORMULATION

The framework of NCS considered in the paper is depicted in Fig. 1. Let the polytopic model of the plant to be controlled be described by the following linear discrete time difference equation

$$\begin{aligned} x(t+1) &= A(\xi)x(t) + B(\xi)u(t), \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^l$ denote the state, control input, and output respectively. The matrices $A(\xi)$, $B(\xi)$ belong to convex and bounded set S , which is a polytop with N vertices S_1, S_2, \dots, S_N that can be formally defined as follows

$$\begin{aligned} S := \left\{ A(\xi) \in R^{n \times n}, B(\xi) \in R^{n \times m} : A(\xi) = \sum_{i=1}^N \xi_i A_i, \right. \\ \left. B(\xi) = \sum_{i=1}^N \xi_i B_i, \sum_{i=1}^N \xi_i = 1, \xi_i \geq 0 \right\}. \end{aligned} \quad (2)$$

Matrices A_i , B_i and C are known matrices with constant elements and appropriate dimensions.

Consider the following output feedback predictive control algorithm as the controller of NCS

$$u(t+k) = u(t+k|t) = \sum_{j=k}^{N_u} F_{kj} [y(t+j|t) - w(t+j|t)] \quad (3)$$

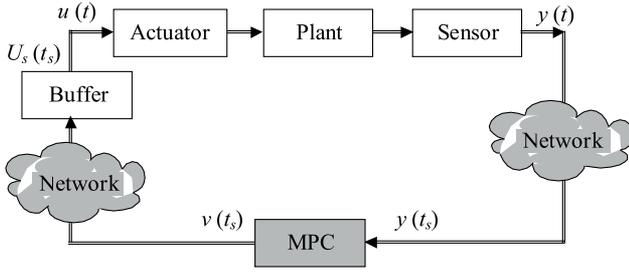


Fig. 1. Configuration of considered networked control system (NCS)

where $k \in \{0, 1, \dots, N_u - 1\}$; $F_{kj} \in R^{m \times l}$ denotes output feedback gain matrices; $u(t+k|t)$, $y(t+k|t)$, and $w(t+k|t) \in R^l$ for $k \geq 1$ denote, respectively, the input, output and desired reference predictions at time instant $t+k$ predicted at time t ($u(t+k|t)$ and $x(t+k|t)$ are initialized by the measurements of the current state $x(t)$). The prediction is carried out over control horizon N_u and prediction horizon $N_y = N_u$. Input constraints are assumed to be

$$\|u_i(t+k)\| \leq \bar{u}_i; \quad i = 1, 2, \dots, m, \quad k = 0, 1, \dots, N_u - 1 \quad (4)$$

where \bar{u}_i is the maximum value of the i -th input control $u_i(\cdot)$.

Networks exist between sensor and controller and between controller and actuator. It is assumed that in network transmission there is negligible network-induced time delay (time delay is within sampling time of NCS) or it is treated as a dropout [5, 7], but packet loss may happen. The sensor and the controller only send data at each sampling time, as well as the controller and actuator receive data. If data are lost at one sampling time, at next sampling time network only transmit new data and old data are discarded. The data are transmitted in a single packet. Based on [27], the packet-loss process in this paper is redefined as follows. Let $\mathfrak{S} = \{t_1, t_2, \dots, t_s, t_{s+1}, \dots\}$ a subsequence of $\{1, 2, 3, \dots\}$, denote the sequence of time points of successful data transmissions from the sensor to the actuator, and $l_{p_max} = \max_s(t_{s+1} - t_s - 1)$, $s = 1, 2, 3, \dots$; $l_{p_max} \leq N_u - 1$ be the maximum value of packet-loss number. Then the following concept and mathematical models are introduced to capture the nature of packet losses.

DEFINITION 1. Packet-loss process is defined as

$$\ell = \left\{ \begin{array}{l} l_p(t_s) : l_p(t_s) = t_{s+1} - t_s - 1, t_s \in \mathfrak{S}, \\ 0 \leq l_p(t_s) \leq l_{p_max} \leq N_u - 1 \end{array} \right\} \quad (5)$$

DEFINITION 2. Packet-loss process (5) is said to be arbitrary if it takes values in ℓ arbitrarily from (5).

Note that at time instant $t \in \langle t_s, t_{s+1} \rangle$, if data is not successfully transmitted from the sensor to the controller, the controller will not calculate new control signal for the actuator and as result, the packet-loss occurs.

To overcome the resulting packet-loss problems, we use prediction based compensation schemes from ([2, 5–7]. Instead of a single input, a sequence of predicted future controls $U_s(t_s) = \{u(t_s), u(t_s + 1), \dots, u(t_s + l_p), \dots, u(t_s + l_{p_max})\}$ is submitted and implemented at a buffer device with length l_{p_max} in the actuator. The buffer device is used to store the newest control sequence $U_s(t_s)$ transmitted successfully from MPC to actuator at sampling time $t_s \in \mathfrak{S}$. At time instant $t \in \langle t_s, t_s + l_p(t_s) \rangle$, the packet loss occurs, and control action $u(t)$ corresponding to the current sampling time from control sequence $U_s(t_s)$ in the buffer device will be applied to the actuator. The main goal of this paper is to design a predictive controller (3) with input constraints (4) so that, control action $u(t)$ from control sequence $U_s(t_s)$ robustly stabilizes NCS and ensures input constraints and guaranteed cost of the following cost function (over the infinite optimization horizon) $J = \sum_{t=0}^{\infty} J(t)$, and where

$$J(t) = \sum_{k=0}^{N_y} x^\top(t+k|t) Q_k x(t+k|t) + \sum_{k=0}^{N_u-1} u^\top(t+k) R_k u(t+k), \quad (6)$$

and $Q_k \in R^{n \times n}$, $R_k \in R^{m \times m}$ are positive semidefinite (definite) and definite matrices, respectively for all k ($Q_k = q_k I$, $R_k = r_k I$; $q_k \geq 0$, $r_k > 0$). and I – is the unitary matrix.

3 ROBUST MODEL PREDICTIVE CONTROL WITH INPUT CONSTRAINTS

In this section, we recall some results of the robust MPC design with input constraints from papers [15, 25].

At sampling time $t := t_s \in \mathfrak{S}$, the predicted states of the system (1) for the instant $t+k$, $k = \{0, 1, \dots, N_u - 1\}$ are given by

$$x(t+k+1|t) = A(\xi)x(t+k|t) + B(\xi)u(t+k|t). \quad (7)$$

Let us define stacked vectors with future states and desired references in corresponding forms as follows

$$\begin{aligned} x_f(t) &= [x^\top(t) \dots x^\top(t+N_y-1|t)]^\top, \\ \varpi(t) &= [w^\top(t) \dots w^\top(t+N_y|t)]^\top. \end{aligned} \quad (8)$$

Considering $\nu(t) = [u^\top(t) u^\top(t+1) \dots u^\top(t+N_u-1)]^\top$, state model prediction is obtained as follows

$$A_f(\xi)x_f(t+1) = A_x(\xi)x(t) + B_f(\xi)\nu(t) \quad (9)$$

where

$$A_f(\xi) = \begin{bmatrix} I & 0 & 0 & \dots & 0 & 0 \\ -A(\xi) & I & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -A(\xi) & I \end{bmatrix}$$

$$\in R^{nN_y \times nN_y}$$

$$A_x(\xi) = [A^\top(\xi) \quad 0 \quad \dots \quad 0]^\top \in R^{nN_y \times n} \quad (10)$$

$$B_f(\xi) = \text{diag}\{B(\xi), \dots, B(\xi)\}^\top \in R^{nN_y \times mN_y}$$

Using definitions of $x_f(t)$ and $\varpi(t)$, the predictive control algorithm (3) is also obtained in the following form

$$\begin{aligned} \nu(t) &= F_{0x}Cx(t) + F_{0f}C_f x_f(t+1) - F_0\varpi(t) \\ &= F_0C_m\eta(t) - F_0\varpi(t) \end{aligned} \quad (11)$$

where $\eta^\top(t) = [x^\top(t) \quad x_f^\top(t+1)]^\top$, $C_f = \text{diag}\{C, \dots, C\} \in R^{lN_y \times nN_y}$, $C_m = \text{diag}\{C, C_f\}$ and $F_0 \in R^{mN_u \times l(N_y+1)}$:

$$F_0 = \begin{bmatrix} F_{00} & F_{01} & F_{01} & \dots & F_{0(N_u-1)} & F_{0N_u} \\ 0 & F_{11} & F_{12} & \dots & F_{1(N_u-1)} & F_{1N_u} \\ 0 & 0 & F_{22} & \dots & F_{2(N_u-1)} & F_{2N_u} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & F_{(N_u-1)(N_u-1)} & F_{(N_u-1)N_u} \\ \underbrace{F_{0x}} & \underbrace{\dots}_{F_{0f}} & \dots & \dots & \dots & \dots \end{bmatrix} \quad (12)$$

Substituting $\nu(t)$ in the form of (11) into the model prediction (9), the closed-loop model prediction is obtained as follows

$$A_{cf}(\xi)x_f(t+1) = A_{cx}(\xi)x(t) - B_{cf}(\xi)\varpi(t) \quad (13)$$

where

$$\begin{aligned} A_{cf}(\xi) &= A_f(\xi) - B_f(\xi)F_{0f}C_f \\ A_{cx}(\xi) &= A_x(\xi) - B_f(\xi)F_{0x}C \\ B_{cf}(\xi) &= B_f(\xi)F_0 \end{aligned} \quad (14)$$

Assuming that the current state $x(t)$ is known, the above equation implies that, the predicted states $x_f(t+1)$ are initialized by the current state vector $x(t)$ and known reference signal $\varpi(t)$. Because the vector $\varpi(t)$ is independent on state vector $x(t)$ and if vectors $\varpi(t)$ belong to the class of ℓ_2 , then the stability and robustness properties of closed-loop system (13) are not affected by $\varpi(t)$. Therefore, due to Lyapunov function approach, it will be without loss of generality if we set the vector $\varpi(t)$ equal to zero in the case of robust stability analysis or robust controller synthesis.

The cost function (6) can be rewritten as follows

$$J(t) = \eta^\top(t)Q\eta(t) + \nu^\top(t)R\nu(t) \quad (15)$$

where $Q = \text{diag}\{Q_0, \dots, Q_{N_y}\}$, $R = \{R_0, \dots, R_{(N_u-1)}\}$.

Associated with the cost function (15), the guaranteed cost control law is defined as follows.

DEFINITION 3. Consider the system (9). If there exists a control law (11) and a positive scalar J_0 such that for all uncertainties (2), the closed-loop system (13) is asymptotically stable and the closed-loop value of the cost function (15) satisfies $J \leq J_0$, then J_0 is said to be the guaranteed cost and the controller (11) is said to be the guaranteed cost control law.

Finally we recall the well known results from LQ theory.

LEMMA 4 [20]. Consider the discrete-time system (9) with control algorithm (11). Control algorithm (11) is the guaranteed cost control law for the closed-loop system (13) if and only if there exists a Lyapunov function $V_0(t) = x_f^\top(t)P_0(\xi)x_f(t)$ such that the following condition holds

$$\Delta V_0(t) + J(t) \leq 0. \quad (16)$$

Moreover, summarizing (16) from initial time t_0 to $t \rightarrow \infty$, the following inequality is obtained

$$-V_0(t_0) + J \leq 0. \quad (17)$$

Definition 3 and inequality (17) imply that $J_0 = V_0(t_0)$.

THEOREM. The closed loop system (13) is robustly stable with guaranteed cost J_0 and parameter dependent quadratic stability if and only if there exist matrices $H_0 \in R^{nN_y \times n(N_y+1)}$, $P_0(\xi) = P_0^\top(\xi) > 0$ and gain matrix F_0 such that the following bilinear matrix inequality holds [15].

$$W(\xi) \leq 0 \quad (18)$$

where

$$\begin{aligned} W(\xi) &= D(\xi) + A_m^\top(\xi)H_0 + H_0^\top A_m(\xi) + Q + C_m^\top F_0^\top R F_0 C_m, \\ D(\xi) &= \text{diag}\{-P_0(\xi), 0, \dots, 0, P_0(\xi)\} \in R^{n(N_y+1) \times n(N_y+1)}, \\ A_m(\xi) &= [A_{cx}(\xi) \quad -A_{cf}(\xi)] \in R^{nN_y \times n(N_y+1)}. \end{aligned}$$

Consider system (9) where the control $\nu(t)$ is constrained to evolve in the following set

$$\begin{aligned} \Gamma &= \{\nu(t) \in R^{mN_u} : \|\nu_{i_d}\| \leq \bar{u}_i; i_d = i + (j-1)m; \\ & \quad i = 1, \dots, m; j = 1, \dots, N_u\} \end{aligned} \quad (19)$$

To derive sufficient stability conditions for input constraints for (13), we consider that the positive invariant region [19], with respect to closed-loop system motion can be defined by the ellipsoidal Lyapunov function set given by $V_0(t)$ as follows

$$\Omega(P_0(\xi)) = \{x_f(t) \in R^{nN_y} : x_f^\top(t)P_0(\xi)x_f(t) \leq \theta\} \quad (20)$$

where θ is a positive real parameter which determines the size of $\Omega(P_0(\xi))$.

Consider $D_{i_d}F_0$ denotes the i_d -th row of matrix F_0 where $D_{i_d} = [0 \dots 0 \quad 1 \quad 0 \dots 0] \in R^{l \times mN_u}$ and define

$$\begin{aligned} \mathcal{L}(F_0) &= \{x_f(t) \in R^{nN_y} : \|D_{i_d}F_0C_f x_f(t)\| \leq \bar{u}_i; \\ & \quad i_d = i + (j-1)m; i = 1, \dots, m; j = 1, \dots, N_u\} \end{aligned} \quad (21)$$

The condition of input constraints reduces to LMI given by the following theorem [25].

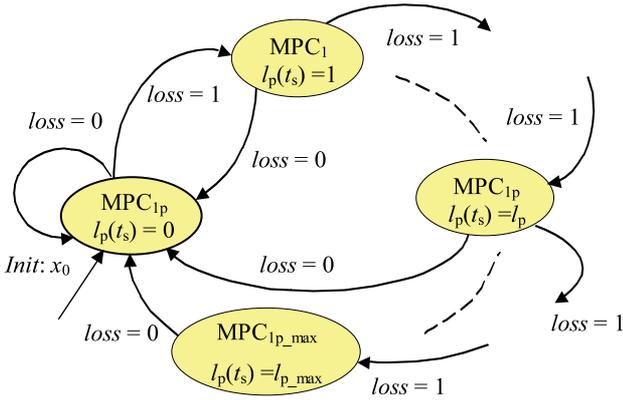


Fig. 2. Schematic representation of a hybrid automaton

THEOREM 6. The inclusion $\Omega(P_0(\xi)) \subseteq L(F_0)$ is for output feedback control equivalent to [25]

$$\begin{bmatrix} P_0(\xi) & * \\ D_{i_d} F_0 C_m & \lambda_{i_d}^0 \end{bmatrix} \geq 0 \quad (22)$$

for all $i_d = i + (j-1)m$; $i = 1, \dots, m$; $j = 1, \dots, N_u$ and $\lambda_{i_d}^0 \in \langle 0, \frac{\bar{u}^2}{\theta} \rangle$.

Without regard to the networks (the packet-loss problem is not considered in the controller design), if conditions (18) and (22) hold, then guaranteed MPC controller (3) is robustly stabilizes uncertain system (1) and guarantees input constraints (4). To fix the packet-loss problem, the compensation mechanism introduced in Section 2 is used. The first l_{p_max} elements of vector $\nu(t)$ are encapsulated into one packet so-called control sequence $U_s(t) = \{u(t_s), u(t_s+1), \dots, u(t_s+l_p), \dots, u(t_s+l_{p_max})\}$ and sent to the buffer device in the actuator. However, control sequence $U_s(t_s)$ obtained from the above MPC controller design (without regard to the networks) can induce instability or poor performance (the guaranteed cost and the input constraints) of the NCS. This problem will be solved in the next section.

4 ROBUST NETWORKED MODEL PREDICTIVE CONTROL DESIGN

In this section, a necessary and sufficient robust stability conditions to robust networked MPC design with guaranteed cost control law for the case of lost packet are presented. Main result of this section on robust networked MPC design can be summarized in Theorem 8.

Due to the compensation mechanism, the control action $u(t)$ corresponding to the current sampling time from control sequence $U_s(t_s)$ in the buffer device will be applied to the actuator at every sampling time $t \in \langle t_s; t_s + l_p(t_s) \rangle$ ($l_p(t_s)$ packets are lost). This process is considered as a switched system with schematic representation of a hybrid automaton in Fig. 2. Hybrid state of the hybrid automaton is $x_h(t) = (l_p(t), x_f(t))$ in which

number packet loss $l_p(t) \in \ell$ is a discrete state, $x_f(t)$ is the state of continuous part. Activity function is defined by closed-loop output feedback model predictive controls MPC_{l_p} in (29).

At sampling time $t := t_s (l_p = 0)$, model predictive control MPC_{l_p} defined in Section 2 is used to compute control sequence $U_s(t)$. If no packet loss at $t+1$, MPC_0 is applied. Otherwise, jump to MPC_1 , it means that one packet lost.

Generally, at sampling time $t := t_s + l_p$ ($1 \leq l_p \leq l_{p_max}$), MPC_{l_p} is applied. If no packet loss at $t+1$, jump to MPC_0 . Otherwise, if $l_p < l_{p_max}$, jump to MPC_{l_p+1} and it means that $l_p + 1$ packets are lost. The predicted states of MPC_{l_p} for the instant $t+k$, $k = 0, \dots, N_u - 1$, are given by

$$\begin{aligned} x(t+k+1|t) &= A(\xi)x(t+k|t) + B(\xi)u(k), \\ u(k) &= \begin{cases} u(t_s + l_p + k) & \text{if } 0 \leq k \leq N_u - l_p - 1, \\ u(t_s + N_u - 1) & \text{if } N_u - l_p \leq k \leq N_u - 1. \end{cases} \end{aligned} \quad (23)$$

Based on equation (3), $u(t_s + l_p + k)$ for $0 \leq k \leq N_u - l_p - 1$ can be rewritten as follows

$$u(t_s + l_p + k) = \sum_{j=l_p+k}^{N_u} F_{(l_p+k)j} C x(t_s + j|t_s). \quad (24)$$

Substituting j by $i + l_p$ to (24), we obtain

$$u(t_s + l_p + k) = \sum_{i=k}^{N_u - l_p} F_{(l_p+k)(l_p+i)} C x(t_s + l_p + i|t_s) \quad (25)$$

for $0 \leq k \leq N_u - l_p - 1$.

If the following condition

$$x(t_s + l_p + i|t_s) = x(t_s + l_p + i|t_s + l_p) = x(t + i|t) \quad (26)$$

holds for $0 \leq i \leq N_u - l_p$, then the state model of MPC_{l_p} is obtained in the form of (9) with the following control algorithm

$$\nu(t, l_p) = F_{l_p} C_m \eta(t) = F_{l_p x} C x(t) + F_{l_p f} C_f x_f(t+1) \quad (27)$$

where F_{l_p} is created by rearrange elements of gain matrix F_0 and has the following structure

$$F_{l_p} = \begin{bmatrix} F_{l_p l_p} & F_{l_p(l_p+1)} & \dots & F_{l_p(N_u-1)} & F_{l_p N_u} & 0 \dots 0 \\ 0 & F_{(l_p+1)(l_p+1)} & \dots & F_{(l_p+1)(N_u-1)} & F_{(l_p+1)N_u} & 0 \dots 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & F_{(N_u-1)(N_u-1)} & F_{(N_u-1)N_u} & 0 \dots 0 \\ 0 & 0 & \dots & F_{(N_u-1)(N_u-1)} & F_{(N_u-1)N_u} & 0 \dots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & F_{(N_u-1)(N_u-1)} & F_{(N_u-1)N_u} & 0 \dots 0 \end{bmatrix} \quad (28)$$

$\underbrace{\hspace{10em}}_{F_{l_p x}} \quad \underbrace{\hspace{10em}}_{F_{l_p f}}$

According to mathematical induction (a method of mathematical proof), condition (26) holds for all $1 \leq l_p \leq l_{p_max}$, if we prove that, (26) holds for the cases of $l_p = \{1, 2\}$, and with assumption that (26) holds for $l_p - 1$, it will hold for l_p .

Due to the result of solving difference equation (1), $x(t_s+1|t_s) = x(t_s+1|t_s+1)$ holds for $i = 0$. Using (7) and (23) for $i = 1$, we obtain $x(t_s+2|t_s) = x(t_s+2|t_s+1) = A(\xi)x(t_s+1|t_s) + B(\xi)u(t_s+1)$. Sequentially repeating the previous step for $i = 2, \dots, N_u-1$, it is easily to show that $x(t_s+i+1) = x(t_s+i+1|t_s+1)$. Condition (26) holds for $l_p = 1$.

Repeating all steps and using results in the above proof for $l_p = 1$, we can prove that, condition (26) holds for $l_p = 2$.

Now, (26) is supposed to hold for $l_p - 1$. We have to show that, (26) will hold for l_p . Indeed, for $i = 0$, we have $x(t_s+l_p|t_s+l_p) = x(t_s+l_p|t_s+l_p-1) = A(\xi)x(t_s+l_p-1|t_s+l_p-1) + B(\xi)u(t_s+l_p-1)$. Because (26) holds for $l_p - 1$, then $x(t_s+l_p-1|t_s+l_p-1) = x(t_s+l_p-1|t_s)$. As a result, $x(t_s+l_p|t_s) = x(t_s+l_p|t_s+l_p)$. By the same way for $i = \{1, \dots, N_u - l_p - 1\}$, (26) is true for l_p . The proof of (26) is obtained.

Applying control algorithm (27) into model prediction (9), the closed-loop MPC of MPC_{l_p} is obtained as follows

$$A_{cf}(\xi, l_p)x_f(t+1) = A_{cx}(\xi, l_p)x(t) \quad (29)$$

where

$$\begin{aligned} A_{cf}(\xi, l_p) &= A_f(\xi) - B_f(\xi)F_{l_p f}C_f, \\ A_{cx}(\xi, l_p) &= A_x(\xi) + B_f(\xi)F_{l_p x}C. \end{aligned} \quad (30)$$

Let us define the following parameter-dependent Lyapunov functional candidate for the closed-loop feedback of MPC_{l_p} .

$$\begin{aligned} V_{l_p}(t) &= x_f^\top(t)\bar{P}_{l_p}(\xi)x_f(t), \\ \bar{P}_{l_p}(\xi) &\in R^{nN_y \times nN_y} : \bar{P}_{l_p}(\xi) = \text{diag}\{P_{l_p}(\xi), \dots, P_{l_p}(\xi)\}, \\ P_{l_p}(\xi) &\in R^{n \times n} : P_{l_p}(\xi) = \sum_{i=1}^N \xi_i P_{l_p i}; P_{l_p i} = P_{l_p i}^\top > 0, \\ \xi_i &\geq 0, \sum_{i=1}^N \xi_i = 1. \end{aligned} \quad (31)$$

The cost function for closed-loop feedback for NCS is rewritten as the follows.

$$\begin{aligned} J(t) &= x^\top(t)Q_0x(t) + x_f^\top(t+1)Q_fx_f(t+1) + \\ &\quad (F_{l_p x}Cx(t) + F_{l_p f}C_fx_f(t+1))^\top R \times \\ &\quad (F_{l_p x}Cx(t) + F_{l_p f}C_fx_f(t+1)) = \\ &\quad \eta^\top(t) \left(Q + C_m^\top F_{l_p}^\top R F_{l_p} C_m \right) \eta(t) \end{aligned} \quad (32)$$

where $Q_f = \text{diag}\{Q_1, Q_2, \dots, Q_{N_y}\}$.

Applying theory of switched system for discrete system [11, p.129] and using Lemma 4, the switched system

with the schematic representation of hybrid automaton in Fig. 2 is robust stable with guaranteed cost if and only if the following conditions hold

$$\begin{aligned} x_f^\top(t+1)\bar{P}_{i_p}x_f(t+1) - x_f^\top(t)\bar{P}_{i_p}x_f(t) + J(t) &\leq 0, \\ i_p &= \{0, l_p + 1\} \end{aligned} \quad (33)$$

or

$$\begin{aligned} J(t) - x^\top(t)P_{l_p}x(t) + x^\top(t+N_y|t)P_{i_p}x(t+N_y|t) + \\ + \sum_{k=1}^{N_y-1} x^\top(t+k|t)(P_{i_p}(\xi) - P_{l_p}(\xi))x(t+k|t) &\leq 0, \\ i_p &= \{0, l_p + 1\} \end{aligned} \quad (34)$$

Note the switched control algorithm for the case of lost packet is given in Fig. 2.

Necessary and sufficient condition for robust stability with guaranteed cost for the NCS with packet loss is given by the following Lemma.

LEMMA 7. *Control sequence $U_s(t_s)$ is the guaranteed cost control law for the NCS with packet loss if and only if the following condition holds*

$$\begin{aligned} B_{l_p}^{i_p}(\xi) &= A_c^\top(\xi, l_p)(\tilde{P}_{l_p}^{i_p}(\xi) + Q_f)A_c(\xi, l_p) - P_{l_p}(\xi) + \\ &\quad + Q_0 + (F_{l_p x}C + F_{l_p f}C_fA_c(\xi, l_p))^\top R \times \\ &\quad \times (F_{l_p x}C + F_{l_p f}C_fA_c(\xi, l_p)) &\leq 0 \end{aligned} \quad (35)$$

where $i_p = \{0, l_p + 1\}$, $\tilde{P}_{l_p}^{i_p}(\xi) \in R^{nN_y \times nN_y}$ and

$$\tilde{P}_{l_p}^{i_p}(\xi) = \text{diag}\{P_{i_p} - P_{l_p}, \dots, P_{i_p} - P_{l_p}, P_{i_p}\}(\xi). \quad (36)$$

Robust stability condition (35) is not directly applicable due to its numerical complexity. In the following theorem the novel formulation of robust stability condition is developed, which provide LMI for MPC robust stability analysis and BMI for MPC robust design.

THEOREM 8. *Control sequence $U_s(t_s)$ robustly stabilizes the NCS with loss packet process ℓ and ensures the guaranteed cost J_0 , input constraints (4) if and only if there exist matrices $H_{l_p} \in R^{nN_y \times n(N_y+1)}$, $P_{l_p}(\xi) = P_{l_p}^\top(\xi) > 0$, and gain matrices F_{l_p} such that the following bilinear matrix inequality (BMI)*

$$\begin{aligned} W_{l_p}^{i_p}(\xi) &= D_{l_p}^{i_p}(\xi) + A_m^\top(\xi, l_p)H_{l_p}^{i_p} + H_{l_p}^{i_p \top}A_m(\xi, l_p) + \\ &\quad + Q + C_m^\top F_{l_p}^\top R F_{l_p} C_m \leq 0; \\ i_p &= \{0, l_p + 1\} \end{aligned} \quad (37)$$

and the following linear matrix inequality (LMI)

$$\begin{aligned} \begin{bmatrix} P_{l_p}(\xi) & * \\ D_{i_d}F_{l_p}C_m & \lambda_{i_d}^{l_p} \end{bmatrix} \geq 0; \lambda_{i_d}^{l_p} \in \langle 0, \frac{\bar{u}_i^2}{\theta} \rangle; \\ i_d &= i + (j-1)m, i = 1, \dots, m, \\ j &= 1, \dots, N_u - l_p \end{aligned} \quad (38)$$

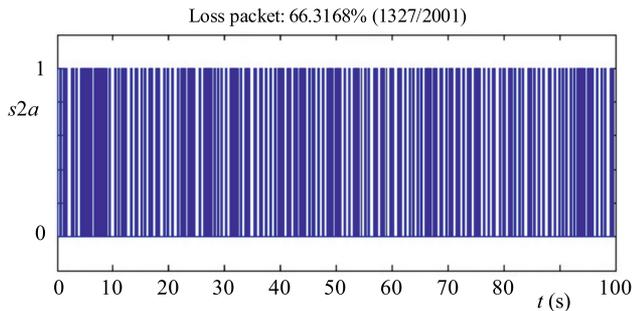


Fig. 3. The status of data transmission in the network

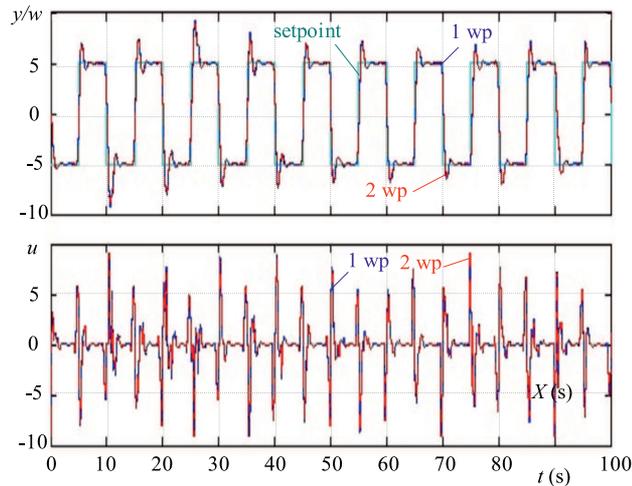


Fig. 4. Simulations of output and input control signals at two operating points

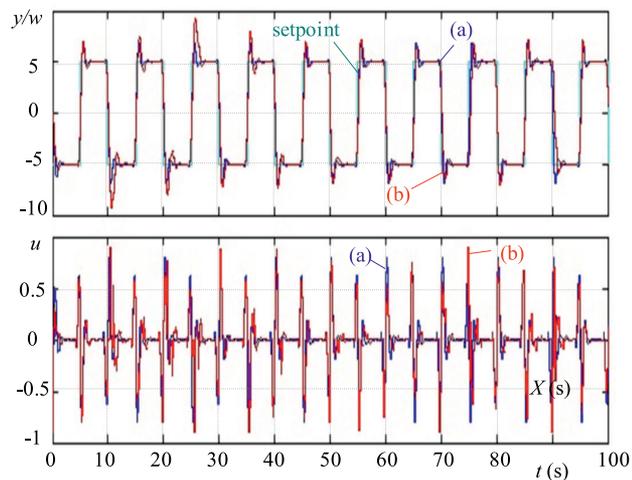


Fig. 5. Comparison between the case of packet loss and no packet loss in the network: (a) – no loss packet, (b) – loss packet

Our simulations are based on the framework in Fig. 1. The status of data transmission in the network is shown in Fig. 3, where $s2a = 1$ ($s2a = 0$) indicates that the data is (is not) successfully sent from the sensor to the actuator. We can see that, only 1327 of 2001 packets were successfully transmitted via network; it means that 66.318 % of packets were lost. Fig. 4 shows simulations of output and input control signals at two operating points 1.wp and 2.wp. It shows that networked MPC is robustly

stable and guarantees input constraints. In Fig. 5, there is comparison between the case of packet loss and no packet loss in the network at the first operating point. Comparison between the results obtained by design method without considering packet loss and proposed method at the first operating point is shown in Fig. 6. It shows that, the design method without considering packet also stabilizes NCS, but gives less performance than the proposed method.

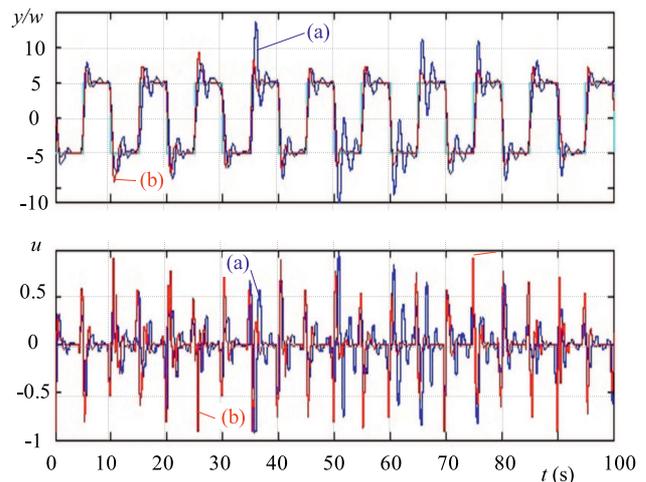


Fig. 6. Comparison between the case of design method without considering packet loss and the proposed method: (a) – without considering packet loss, (b) – considering packet loss

6 CONCLUSION

The stabilization of networked predictive control system with packet-loss was studied in this paper. The packet-loss process is arbitrary and bounded by the control horizon of model predictive control. Networked predictive control systems with packet loss are modeled as switched linear systems. This enables us to apply the theory of switched systems to establish the stability condition of networked predictive control systems. The stabilizing controller design is based on sufficient robust stability conditions formulated as a solution of bilinear matrix inequality BMI. The effectiveness of the proposed method was illustrated by a numerical example and simulations.

Acknowledgements

The work has been supported by Grant 1/1241/12 of the Slovak Scientific Grant Agency.

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Received 14 April 2013

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