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OPTIMAL MULTIOBJECTIVE DESIGN OF DIGITAL FILTERS USING TAGUCHI OPTIMIZATION TECHNIQUE

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The multiobjective design of digital filters using the powerful Taguchi optimization technique is considered in this paper. This relatively new optimization tool has been recently introduced to the field of engineering and is based on orthogonal arrays. It is characterized by its robustness, immunity to local optima trapping, relative fast convergence and ease of implementation. The objectives of filter design include matching some desired frequency response while having minimum linear phase; hence, reducing the time response. The results demonstrate that the proposed problem solving approach blended with the use of the Taguchi optimization technique produced filters that fulfill the desired characteristics and are of practical use.

K e y w o r d s: multiobjective filter design, Taguchi optimization technique, magnitude response, minimum linear phase, group delay

1 INTRODUCTION

Digital filters exist in two types: Finite impulse response (FIR) and Infinite impulse response (IIR) or recursive. FIR filters suffer from the problem of high order (hence implementation and performance issues) if strict requirements are imposed at the design stage. Furthermore, IIR filters can have smaller group delay than its equivalent FIR filters [1, 2]. The optimal design of an infinite impulse response (IIR) filter consists in choosing a set of coefficients of the filter to have a frequency response that optimally approximates the desired response [1–22].

Different techniques exist for the design of digital filters. Windowing method; in which the ideal impulse response is multiplied by a window function, is the most popular. There are various kinds of window functions (Butterworth, Chebyshev, Kaiser *etc*), depending on the requirements on ripples in the passband and stopband, stopband attenuation and the transition width. These various windows limit the infinite length impulse response of ideal filter into a finite window to design an actual response. Furthermore, windowing methods do not allow sufficient control of the frequency response in the various frequency bands and other filter parameters such as transition width. The designer always has to compromise between the design specifications [1, 2].

Due to the presence of the denominator of the transfer function, the stability condition of the filter should be taken into account in the optimal design [1, 2–7, 9–18, 11–18, 20, 22, 23], resulting in a constrained optimization problem. Several sufficient conditions [2–4, 6, 10, 12, 18] have been established for the parameterization that represents the filters denominator by a single polynomial. The triangle-based stability conditions [1] are necessary and sufficient and have been incorporated into several design procedures [11, 13, 14] that formularize the filters denominator by cascaded second-order sections (SOSs). In [13], variable transformation is used to convert the finite stability region into the entire coefficient space, such that the original constrained design problem becomes an unconstrained one in the transformed space. However, the transformation increases the nonlinearity of the objective function, which makes it hard to find good (global optimum) solutions in general. In [14], a perturbed stability triangle is proposed to guarantee the SOS to have its zeroes inside a circle of given radius. It is combined with the Gauss-Newton strategy, resulting in an improved design. In [15], the conditions presented for the SOS with zeroes inside a circle of given radius enclose a triangular stability domain and can be easily incorporated into any constrained optimization formulations based on the SOS parameterization. A method that divides the overall design of an IIR filter into successive designs of its second-order sections is presented in [20], where one section is first designed, and then, another section is appended until all sections are designed.

Because of finite word length effects occurring in practical implementations of the designed filters, not only stability of the filter is of great importance but a stability margin is necessary as well. The poles of the transfer function should not lie too close to the unit circle. The sensitivity of pole locations to coefficient quantization increases with decreasing distance from the unit circle. Poles close to the unit circle may considerably enhance quantization noise and increase the maximum amplitude of small scale limit cycle. Consequently, it is desirable to have control over the maximum radius when designing IIR filters.

Linear-phase filters are usually designed as non-recursive (FIR) filters which can have constant group delay over the entire baseband. However, when highly selective filters are required, a very high filter order is needed which makes these filters uneconomical or impractical. To eliminate this problem, attempts have been made to develop

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methods to design recursive (IIR) filters whose delay characteristics approximate a constant value in the passband. This includes IIR filter design approach that can satisfy both magnitude and phase characteristics simultaneously [24–28]. The design of IIR filters with constant group delay in the passband is also carried out by using allpass structures through evaluation of phase response instead of approximating the group delay directly [29–32]. Some other methods used an indirect approach based on model reduction techniques where a linear-phase FIR filter that meets the required specifications is first designed and then a lower order IIR filter that meets the original amplitude specifications while maintaining a linear-phase response in the passband is obtained [33–35]. Coretlazzo *et al* [36] have achieved the simultaneous design in both magnitude and group delay of IIR and FIR filters based on multiple criterion optimizations. Lutovac *et al* [37] have developed a new design method for elliptic IIR filters that provide the implementation of half of the multiplication constants with few shifters and adders. Sullivan *et al* [38] have proposed the algorithm based on the peakconstrained least-squares optimality criterion for cascaded IIR filters, which can design a filter that has an equalized group delay without the use of all pass filters, and it can simultaneously meet the frequency response magnitude specifications by using all of the filter coefficients available to optimize the filter. Lang [39] has used least square method for designing IIR filter with prescribed magnitude and phase response . This parameterization of the transfer function has been used for designing IIR filters. Gordana Javanovic [40] has proposed a method for the design of IIR notch filters with desired magnitude characteristic, which can be either maximally flat or equiripple. Xi Zhang [41] have proposed a novel method for designing maximally flat IIR filters with flat group delay responses in the passband.

Under these circumstances, evolutionary and meta-heuristic optimization methods find their place. These are referred to as global optimizers while the more familiar, traditional techniques such as conjugate gradient and the quasi-Newtonian methods are classified as local optimizers. The distinction between local and global search of optimization techniques is that the local techniques produce results that are highly dependent on the starting point or initial guess, while the global methods are totally independent of the initial conditions [42]. Though they possess the characteristic of being fast in convergence, local techniques, in particular the quasi-Newtonian techniques have direct dependence on the existence of at least the first derivative. In addition, they place constraints on the solution space such as differentiability and continuity; conditions that are hard or even impossible to satisfy in some situations [42].

Previously, global optimization techniques have been implemented in the design of digital filters. One such approach using neural networks has been described in [43]. Also, use of PSO in the design of frequency sampling finite impulse response (FIR) filter has been described in [44–46]. Differential evolution has been used in the design of digital filters has been implemented in [47–50]. [51, 52] have used Hierarchical Genetic Algorithms to design and optimization of IIR filter structures. Use of Particle Swarm Optimization (PSO) and Genetic Algorithms (GA) in the design of digital filters is described in [53].

In this work, the application of the new optimization technique called Taguchi optimization to the design of digital filters is considered. The purpose is to design a filter that can simultaneously satisfy multiobjective criteria including frequency response and linear phase with the least possible group delay.

2 PROBLEM FORMULATION

Digital filters find their applications in different areas. One area is power system protection where measurement systems involve faulted signals associated with DC decaying signals, harmonic and sub-harmonic components. To eliminate these unwanted components, a digital filter design based on multi-objective optimization technique to satisfy different specifications such as high speed response for a real-time application and frequency domain requirements.

2.1 Digital Filtering Approach

A digital filter based solution is proposed to remove unwanted disturbances using digital filter design techniques. The filter time response must be included in the requirements. The present filtering application imposes different kind of specifications. On one hand, the time domain requirement where both a high speed and accurate system response are needed. On the other hand, the frequency domain requirements (DC, sub-synchronous and harmonic components elimination) which are the magnitude response within small bandwidth including sharp frequency edges as well as an approximately constant group delay in this band are required too. Usually the best optimum value of all the objective functions of this filter design can be obtained for some values of design variables. A compromise or a trade-off between the objective functions must be made to achieve a satisfactory filter design.

The considered recursive digital filter must satisfy three multi-objective functions. These functions are: 1) meet a specified or a desired magnitude response specification; 2) an approximately constant group delay; and 3) a minimum time response or settling time which involves a minimum phase or a group delay. The optimization approach considers the discrete-time transfer function which is formulated on the basis of some desired amplitude response and a stability margin parameter. A norm of the weighted error function is then minimized with respect to the transfer-function coefficients with a prescribed maximum pole radius referred to as stability margin . The stability margin parameter is varied to optimize the filter coefficients which minimizes mainly the magnitude response, satisfies the best approximately constant group delay and the lowest group delay that leads

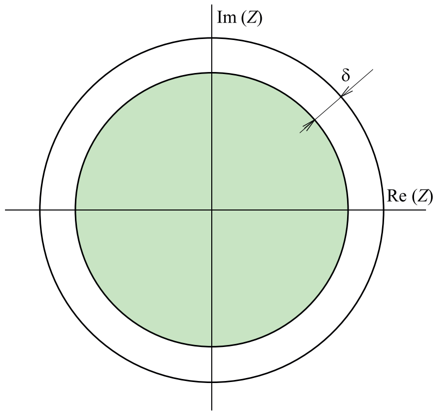

Fig. 1. Stability region and stability margin parameter δ

to minimum settling time or time delay of the system dynamic response

2.2 Filter transfer functions

In the general case an IIR filter can be described by its discrete-time difference equation

$$y[n] + \sum_{i=1}^{2N-1} c_i y[n-1] = \sum_{j=0}^{2M} d_j x[n-j] \tag{1}$$

where $x(n)$ and $y(n)$ are discrete-time input and output signals. Equation (1) can be transformed into the Z-domain and assuming c_i and d_i are real coefficients a second order form transfer function can be obtained, having $2M$ conjugate zeros and $2N$ conjugate poles; called second order sections (SOS), as

$$H(z) = H_0 \frac{\prod_{j=1}^{2M}(a_{0j} + a_{1j}z + z^2)}{\prod_{j=1}^{2N}(b_{0j} + b_{1j}z + z^2)} \tag{2}$$

where a_{ij} and b_{ij} are real coefficients and H_0 is a positive multiplier constant. The polar formulation is also useful and is written as

$$H(z) = H_0 \frac{\prod_{j=1}^{2M}\big(z - r_{aj}e^{j\theta_{aj}}\big)\big(z - r_{aj}e^{-j\theta_{aj}}\big)}{\prod_{j=1}^{2N}\big(z - r_{bj}e^{j\theta_{bj}}\big)\big(z - r_{bj}e^{-j\theta_{bj}}\big)} \tag{3}$$

where r_{aj}, θ_{aj} and r_{bj}, θ_{bj} are the radii and angles of the zeros and poles, respectively.

2.3 Filter stability margin

As poles are moved toward the origin (hence decreasing the pole radius), the system stability margin parameter increases and the system settling time decreases. This action, in fact, brings two required and important properties to the designed system. First, the system time or dynamic response is enhanced as settling time is decreased. Second, the system stability becomes more robust which is a very useful property, particularly in practical implementation. Indeed, the rounding or truncation of the filter coefficients may lead to an unstable implementation if the stability margin is too small. It is therefore desirable to approximate a given response by a transfer function with a prescribed maximum pole radius named stability margin as shown in Fig. 1.

2.4 Magnitude response objective function

The amplitude and the phase responses of a recursive filter is given by

$$H(z) = \big|H\big(e^{j\omega t}\big)\big|\,, \quad \varphi(x,\omega) = \arg\big(H(e^{j\omega t})\big) \tag{4}$$

where ω is the frequency and x is a column vector with $2M + 2N + 1$ components, that is in Cartesian form

$$x = [a_{01}\, a_{11}\, a_{21}\, b_{01}\, b_{11}\, b_{21} \dots b_{2M}\ H_0]^\top \tag{5}$$

or in polar form

$$x = [\rho_a\ \rho_b\ H_0]^\top \tag{6}$$

where

$$\rho_a = [r_{a1}\,,\ \theta_{a1}\,,\ \dots,\ r_{aM}\,,\ \theta_{aM}]^\top \tag{7}$$

and

$$\rho_b = [r_{b1}\,,\ \theta_{b1}\,,\ \dots,\ r_{bN}\,,\ \theta_{bM}]^\top\,. \tag{8}$$

The superscript $\top$ denotes the transpose operation. An approximation error can be formulated as the difference between the actual amplitude response $M(x,\omega)$ and the desired amplitude or magnitude response $M_d(\omega)$ as

$$e(x) = M(x,\omega) - M_d(\omega)\,. \tag{9}$$

By sampling the error function $e(x,\omega_i)$, the actual and the desired amplitude responses $M(x,\omega_i)$ and $M_d(\omega)$ at frequencies $\omega 1$, ω_2, ..., ω_k, the column error vector is

$$[E(x)] = [e_1(x)\,,\ e_2(x)\,, \dots, e_k(x)]^\top \tag{10}$$

where $e_i(x) = M(x,\omega_i) - M_d(\omega_i)$ and $\{\omega_i\colon i = 1,2,\dots,k\}$ is a dense set of frequencies which are distributed over in the pass-band and stop-band of the filter. A weighting or penalty error is included to control portions of the actual filter response curve that are most important to the filter response. This involves modifying the error to the form

$$\bar{e}_i(x) = W(\omega)\big(M(x,\omega_i) - M_d(\omega_i)\big) \tag{11}$$

where $W(\omega)$ is a weighting piece wise constant function over all frequency space, which is assigned a positive value greater than one or less than one to increase or decrease the magnitude approximation in a given band.

A recursive filter can be designed by finding a point $x = \hat{x}$ in (11) such that

$$\bar{e}_i(x) \approx 0 \text{ for } i = 1,2,\dots,k\,. \tag{12}$$

Such a point can be obtained when solving the optimization problem by minimizing the error function $\bar{e}_i(x)$. The design of a recursive filter that approaches a specified response $M_d(\omega_i)$, can be performed by minimizing the error objective function in terms of L_p norm error which is formulated as:

$$\min_x \mu(x) \text{ subject to } r_{aj} \le 1 - \delta \tag{13}$$

where

$$\mu(x) = \Big\{\sum_{i=1}^{l} W_i(\omega)\big|e_i(x)\big|^p\Big\}^{\frac{1}{p}} \tag{14}$$

and

$$e_{\max}(x) = \max_{1\le i\le l}\big(e_i(x)\big)\,.$$

The p is a positive integer. The Taguchi technique is used to minimize $\mu(x)$ for increasing values of p.

2.5 Group-delay objective function

The group delay is derived from the phase relation, as given in equation (4), and is defined as

$$\tau(x,\omega) = \frac{\mathrm{d}\phi(x,\omega)}{\mathrm{d}\omega} \tag{15}$$

where $\phi(x,\omega)$ is the phase response of the filter,

$$\phi(x,\omega) \arg\bigl(H(e^{j\omega T})\bigr) \,. \tag{16}$$

The group delay for a synthesized recursive filter is desired to be unchanged in the considered region one hand, in the other hand a minimum group delay is another required property needed in certain applications. in the present design, a constant group delay property is considered which is defined as

$$\tau(x,\omega_i) = \tau_c \quad \text{for } \omega_i \in \omega_p \tag{17}$$

where τ_c is constant, and ω_p is the passband region of the filter. In the present application, τ_c is an unknown but can be considered as the mean value over the passband region, which can be determined as

$$\tau_{c,m} = \frac{1}{n_k} \sum_{i=1}^{n_k} \tau\bigl(\overline{x}_m, \omega_i\bigr) \text{ for } \omega_i \in \omega_p \tag{18}$$

where $\overline{x}_m$ is the optimal filter coefficient determined by minimizing the magnitude objective function for an m-th stability margin parameter.

The stability margin parameter is varied for discrete values, from which an optimal constant group delay is determined by minimizing the following objective error function

$$\min_m E_\tau(m) = \frac{1}{\tau_{c,m}} \sum_{i=1}^{n_k} \bigl(\tau(\overline{x}_m,\omega_i) - \tau_{c,m}\bigr) \text{ for } \omega_i \in \omega_p \,. \tag{19}$$

The multi-objective optimization problem is solved by discretizing the stability margin δ_m parameter, the magnitude optimization algorithm is used to generate the corresponding filter coefficients, in which basis the group delay is synthesized where a feasible and optimal solution can be obtained by minimizing the objective function (19). The final multiobjective design is obtained by minimizing the sum combination of the errors in equations (14) and (19).

3 THE TAGUCHI OPTIMIZATION TECHNIQUE

Compared with traditional optimization techniques, Taguchis optimization method is easy to implement and very efficient in reaching optimum solutions. Taguchis optimization method is developed based on the orthogonal array (OA) concept, which offers a systematic and efficient way to select design parameters. In addition, it reduces the number of tests required in the optimization process compared to GA or PSO [54, 55].

Taguchis method was developed based on the concept of the orthogonal array (OA), which can effectively reduce the number of tests required in a design process [54, 55]. It provides an efficient way to choose the design parameters in an optimization procedure.

Before presenting the Taguchi procedure, it is worth understanding what OAs are and how are they generated [54, 55]. Let S be a set of s symbols or levels (the simplest symbols are integers $1, 2, 3\dots$). A matrix A of N rows and k columns with entries from S is said to be an OA with s levels and strength t $(0 < t < k)$ if in every $N \times t$ subarray of A, each t-tuple based on S appears exactly the same times as a row. The notation $\mathrm{OA}(N, k, s, t)$ is used to represent an OA.

3.1 Initialization procedure

The optimization procedure starts with the problem initialization, which includes the *selection of a proper OA* and *the design of a suitable fitness function*. The selection of an $\mathrm{OA}(N, k, s, t)$ mainly depends on the number of optimization parameters. In general, to characterize the nonlinear effect, three levels $(s = 3)$ are found sufficient for each input parameter. Usually, an OA with a strength of 2 $(t = 2)$ is efficient for most problems because it results in a small number of rows in the array [54, 55].

3.2 Design of input parameters

The input parameters need to be selected to conduct the experiments. When the OA is used, the corresponding numerical values for the three levels of each input parameter should be determined.

In the first iteration, the value for level 2 is selected at the center of the optimization range. Values of levels 1 and 3 are calculated by subtracting/adding the value of level 2 with a variable called *level difference* (LD). The *level difference* in the first iteration (LD1) is determined by

$$LD_1 = \frac{\text{Max} - \text{Min}}{\text{Number of levels} + 1} \tag{20}$$

where *Max* is the upper bound of the optimization range and *Min* is the lower bound of the optimization range.

3.3 Conduct Experiments and Build a Response Table

After determining the input parameters, the fitness function for each experiment can be calculated. These results are then used to build a response table for the first iteration by averaging the fitness values for each parameter n and each level m using the following equation

$$F_{av} = \frac{s}{N} \sum_{i, OA(i,n)=m} f_i \,. \tag{21}$$

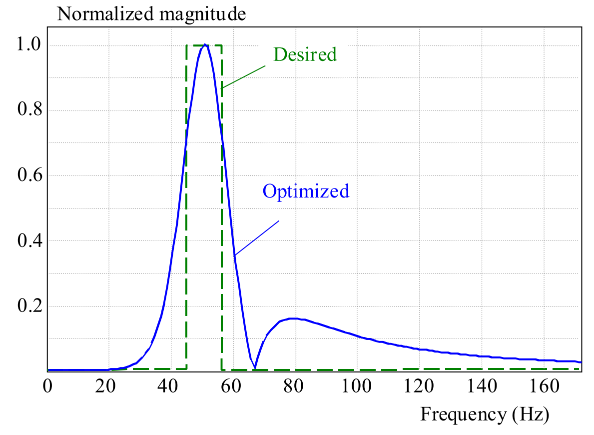

Fig. 2. Magnitude filter response (SOOF)

As an example, consider that parameter x in an N dimensional problem has levels $1, 2$ and 3 as described earlier. With $s = 2$ the fitness values are evaluated based on equation (7) for each level and hence a response table is constructed for each parameter that can be used to choose which level produces the best fitness value (minimum value).

3.4 Identify Optimal Level Values and Conduct Confirmation Experiment

Finding the largest fitness value ratio in each column can identify the optimal level for that parameter. When the optimal levels are identified, a confirmation experiment is performed using the combination of the optimal levels identified in the response table. This confirmation test is not repetitious because the OA-based experiment is a fractional factorial experiment, and the optimal combination may not be included in the experiment table. The fitness value obtained from the optimal combination is regarded as the fitness value of the current iteration.

3.5 Reduce the Optimization Range

If the results of the current iteration do not meet the termination criteria, the process is repeated in the next iteration. The optimal level values of the current iteration are used as central values (values of level 2) for the next iteration. To reduce the optimization range for a converged result, the LD_i is multiplied with a reduced rate (rr) to obtain LD_{i+1} for the $(i+1)^{\mathrm{th}}$ iteration

$$LD_{i+1} = rr \times LD_i = RR(i) \times LD_i \tag{22}$$

where $RR(i)$ is called *reduced function*. When a constant rr is used, $RR(i) = rr^i$. The value of rr can be set between 0.5 and 1 depending on the problem. The larger rr is, the slower the convergence rate.

If LD_i is a large value, and the central level value is located near the upper bound or lower bound of the optimization range, the corresponding value of level 1 or 3 may reside outside the optimization range. Therefore, a process of checking the level values is necessary to guarantee that all level values are located within the optimization range. A simple way is to use the boundary values directly.

3.6 Check the Termination Criteria

When the number of iterations is large, the level difference of each element becomes small from equation (22). Hence, the level values are close to each other and the fitness value of the next iteration is close to the fitness value of the current iteration. The following equation may be used as a termination criterion for the optimization procedure

$$\frac{\mathrm{LD}_i}{\mathrm{LD}_1} < \text{converged value.} \tag{23}$$

Usually, the converged value can be set between 0.001 and 0.01 depending on the problem. The iterative optimization process will be terminated if the design goal is achieved or if equation (23) is satisfied.

4 RESULTS AND DISCUSSIONS

The digital filter to be optimized is to be used to eliminate harmonics and sub-harmonics in a power network with a fundamental frequency of 50 Hz. The filter is a bandpass type and is desired to satisfy the magnitude response to ideally pass only frequencies confined in the interval $\{45\,\text{Hz}, 55\,\text{Hz}\}$ while rejecting all other frequency content. The sampling frequency is taken to be 1800 Hz. The filter is of order 10 and is hence composed of 5 cascaded SOSs. At start up, the filter is optimized to match the magnitude response specifications only. Next, more constraints are added to the optimization process including minimum and linear phase and constant group delay to enhance the designed filter performance.

4.1 Single objective design

The purpose of this part is to design a filter which only satisfies the magnitude response described earlier without considering any other performance criteria. The filter is designated single objective optimized filter (SOOF) hereafter. Figure 2 shows both the desired and the optimized magnitude response of the digital filter. It can be noted that the filter has good performance in the sense that it fulfils the requirements of magnitude response. Indeed, the filter response falls exactly within the desired response and it attenuates all other frequencies as the overall Sidelobe level is lower than 16 dB.

However, the optimized filter is not of practical use as it suffers from drawbacks in the dynamic properties. First, as shown in Fig. 3, the time delay is in the order of 100 ms which is not suitable in the present applications as the requirements specify that the time delay should not exceed one cycle (20 ms). In addition, the group delay is not constant as shown in Fig. 4. Furthermore, the phase delay is nonlinear as shown in Fig. 5 and accordingly a non constant group delay. As a result, it is necessary to

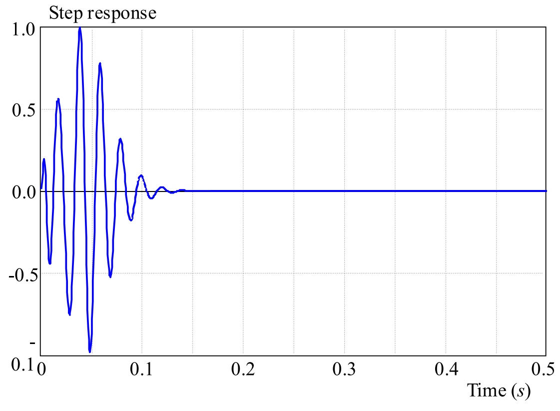

Fig. 3. Step response of the filter and time delay (about 100 ms) (SOOF)

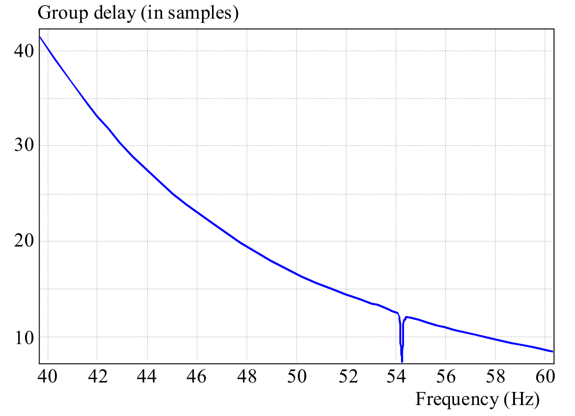

Fig. 4. Group delay response of single objective optimized filter (SOOF)

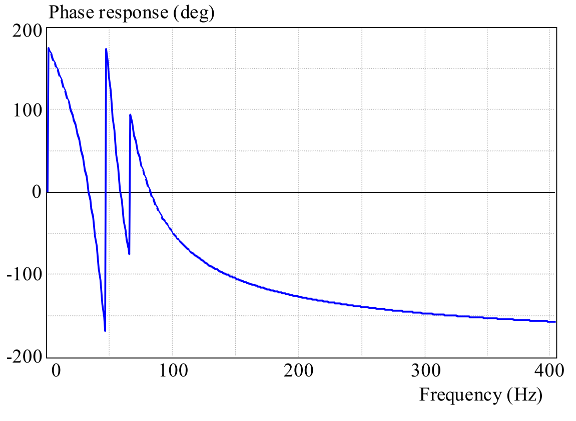

Fig. 5. Phase response of filter with non linear phase or non constant group delay (SOOF)

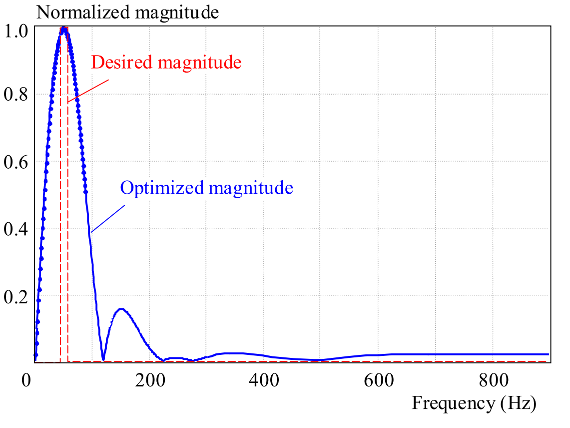

Fig. 6. Magnitude filter response (MOOF)

Table 1. Single-Objective Optimized Filter (SOOF) SOS coefficients and gains

Section	Numerator			Denominator			Gain
1	1	−1.9520345	0.9999998	1	−1.8473812	0.8836000	0.0008977
2	1	−1.9999996	0.9999998	1	−1.9999996	0.9999998	1
3	1	−1.9999996	0.9999998	1	−1.8473812	0.8836000	1
4	1	−1.8851095	0.9999998	1	−1.8473812	0.8836000	1
5	1	1.9999998	0.9999998	1	−1.8473812	0.8836000	1

Table 2. Multi-Objective Optimized Filter (MOOF) SOS coefficients and gains

Section	Numerator			Denominator			0*Gain*
1	1	−1.8293383	0.9999998	1	−1.0914383	0.3540250	0.0124999
2	1	−1.4247566	0.9999998	1	−1.0914383	0.3540250	1
3	1	−1.1469437	0.9999998	1	−1.1899988	0.3540250	1
4	1	0.2279814	0.9999998	1	−1.0914383	0.3540250	1
5	1	−0.4675396	−0.5324602	1	−1.1137364	0.3540250	1

include all the preceding performance criteria into the design process and hence, the problem becomes a multiobjective optimization task. The results are summarized in Table 1 where are presented the filter SOS coefficients and SOS gains.

4.2 Multiobjective filter design

The inclusion of the constant and minimum group delay in the optimization task besides magnitude response criterion produced a filter which satisfies almost all requirements. The filter compromises between these criteria

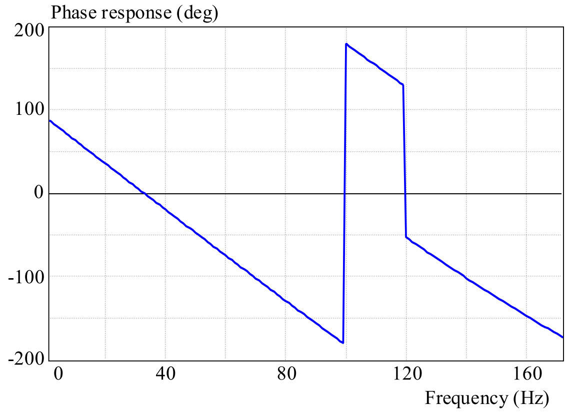

Fig. 7. Optimized filter phase response MOOF

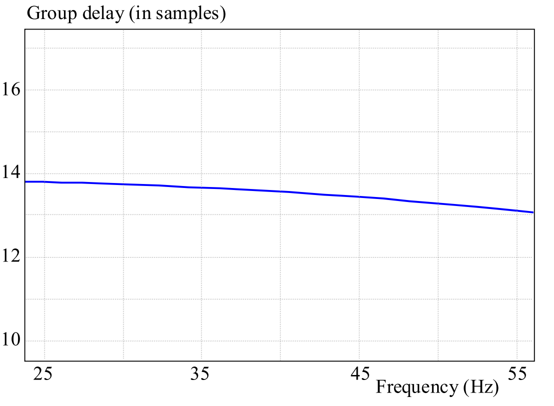

Fig. 8. Group delay response of optimized filter MOOF

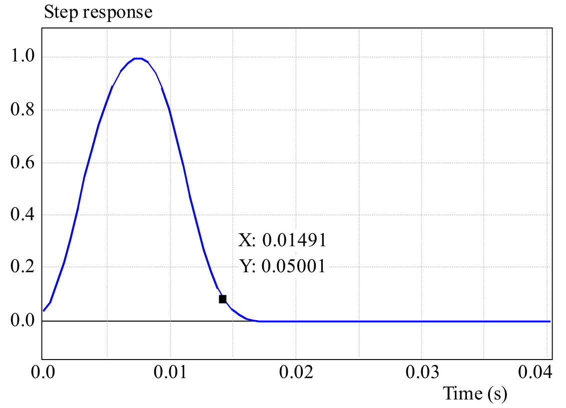

Fig. 9. Optimized filter step response and time delay (14.9 ms) MOOF

to produce the best possible trade-off. The filter is thereafter labeled multiobjective optimized filter (MOOF). The magnitude response of the MOOF is shown in Fig. 6. The filter does not show good performance in terms of this characteristic compared to the SOOF. Indeed, the SOOF bandwidth is better than the MOOF which makes it having better selectivity.

Yet, the MOOF performs better in terms of the other criteria. In terms of phase and group responses, Figures 7 and 8 illustrate the phase and group delay responses. The filter is characterized by a linear phase with an almost constant group delay in the passband. Furthermore, the group delay is minimum producing a small time response as shown in Fig. 9. Indeed, the time delay of the MOOF is about 14.9 ms which conforms to the desired requirements. This is due to the fact that the stability margin or equivalently the pole radii have been taken into account in the optimization process and these latter have been lowered to a value of 0.595 instead of being closer to unity. Table 2 summarizes the SOS coefficients and gains of the MOOF.

To better assess the performance of the filter (MOOF), two tests have been performed. In the first, a step sinusoidal signal of 50 Hz is input to the filter and Fig. 10 shows both filter input and output waveforms. It is seen that the filter output matches exactly the input except for a phase shift and a time delay of less than one cycle. Hence, this filter proves to be practical for high speed measurement systems where the system accuracy is of great importance. In the second test, the previous step sinusoidal signal is corrupted with a DC offset, harmonic and subharmonic components. The subharmonic component is set to 25 Hz and the harmonics to 100, 150 and 200 Hz. Both input and output signals are shown in Fig. 11. As it is clearly seen, the filter succeeded in eliminating the DC and harmonic components and mitigating the subharmonic component in a fast manner witnessed by the short time to start the filtering operation.

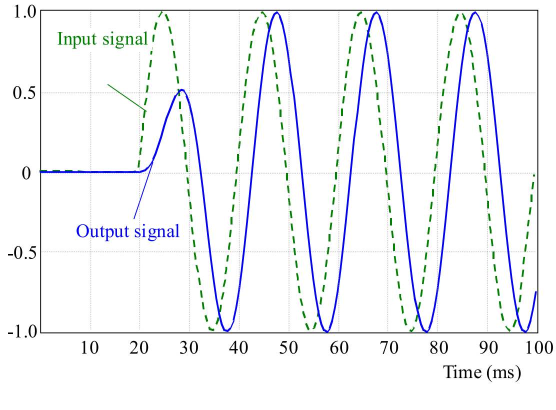

Fig. 10. Test signal: step of sinusoidal signal input and filter output response

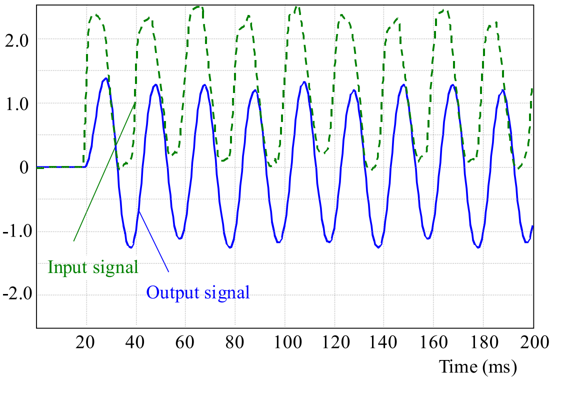

Fig. 11. Second test signal by considering a step sinusoidal signal mixed with harmonic and subharmonic components

5 CONCLUSION

The application of the Taguchi optimization method to design a multiobjective filter digital has been considered in this paper. The objectives of the filter design were to match a desired magnitude response while having a minimum and linear phase. At start up, only magnitude response has been considered in the optimization task. The resulting filter was good in terms of this characteristic while it showed awful dynamic and phase performance. Next, the dynamic properties were included in the optimization algorithm to solve a multiobjective task. The Taguchi optimization method has succeeded in attaining the optimal design in terms of the previous requirements by achieving a compromise between them. The optimized filter has been tested and it showed good performance with required practical characteristics.

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