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ROBUST GAIN–SCHEDULED PID CONTROLLER DESIGN FOR UNCERTAIN LPV SYSTEMS

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A novel methodology is proposed for robust gain-scheduled PID controller design for uncertain LPV systems. The proposed design procedure is based on the parameter-dependent quadratic stability approach. A new uncertain LPV system model has been introduced in this paper. To access the performance quality the approach of a parameter varying guaranteed cost is used which allowed to reach for different working points desired performance. Numerical examples show the benefit of the proposed method.

K e y w o r d s: LPV systems, Gain-scheduled controller, Robust controller, Parameter-dependent Lyapunov function, Quadratic gain-scheduled cost function, PID controller

1 INTRODUCTION

In real applications a controller must accommodate a plant with changing dynamics. Therefore, controllers based on these models have to be robust in the presence of plant model uncertainty. A practical approach involves scheduling in a family of local controllers in response to the changing plant dynamics [1]. A proposed family of local controllers is implemented using the gain-scheduling approach. The above mentioned gain-scheduled designs are guided by two heuristic rules [2]:

– the scheduling variable should vary slowly, and
– the scheduling variable should capture the plants nonlinearities.

In such cases, the designed gain-scheduled controller should be able to stabilize and guarantee a reasonable performance for all operating conditions. The question remains what happens with a closed-loop system if the developed physical nonlinear model or the model obtained through practical identification is not enough precise? In such a case, frequent in applications, there is a need for robust controller to cope with plant model uncertainty.

Various robust controller design methods for gain-scheduled uncertain plant are available in literature. Robust gain-scheduled controllers design to LPV system can be found in [1], where the authors addressed the problem of interpolating in a set of LTI controllers in order to form a gain-scheduled controller with optimal H_∞ performance. The set of admissible interpolated controllers are framed in terms of the robust controller interpolation criteria. For a special uncertain dynamical system, the robust state feedback stabilization problem in the gain-scheduling can be found in [3]. In this paper it is shown that a possible advantage of the online measurement of the scheduling parameters is that this always allows linear compensators, whose implementation can be easier than that of nonlinear ones. Design of robust gain-scheduled PI controllers for nonlinear SISO process can be found in [2]. The model uncertainty is assumed to be the difference between the nonlinear model and the linear one. In the paper [4] an input-output approach to the gain-scheduled design of nonlinear controllers is presented. A controller formulation inspired by the Youla-Kucera parametrization to propose a controller structure and design approach that allow the gain-scheduling of linear control designs such that a robustly stable nonlinear closed-loop control system is achieved. A robust PID controller is designed in [5].The main feature of the proposed method is that the stability, robustness margin and some performance specification are guaranteed by linear constraints in the Nyquist diagram. The condensing boiler is described by the first order model with a time delay in [6], the problem of attenuation of sinusoidal disturbances with uncertain and arbitrarily time-varying frequencies is solved by synthesis of LPV controller using the L_2 gain method. In [7] the quadratic stability approach is used to design the gain-scheduled controller for each vertex of a plant uncertainty box and the closed-loop system stability is verified by LMI. Other alternative approaches to gain-scheduled controller design can be found in [8–17]. A survey of the gain-scheduled controller design is given in excellent papers [18, 19].

The above short survey implies that in the references there is no systematic procedure for designing a robust PID gain-scheduled controller. This observation motivated us to solve the following research problem: design a PID robust gain-scheduled controller which should guarantee

– stability and robustness properties of a closed-loop system for all scheduled parameters $\theta \in \Omega_s$ and their rate $\dot{\theta}_i \in \Omega_t$, when the uncertain plant parameters π lie in the given polytopic uncertainty box Ω, that is $\pi \in \Omega,\ \theta \in \Omega_s,\ \dot{\theta} \in \Omega_t$,
– for the closed-loop system ensure for all $\pi \in \Omega$, $\theta \in \Omega_s$ and $\dot{\theta} \in \Omega_t$ guaranteed gain-scheduled performance and parameter dependent quadratic stability.

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In this paper the new PID robust gain-scheduled controller design procedure is given.

The paper is organized as follows. Section 2 includes problem formulation of robust PID gain-scheduled controllers design for the original plant uncertainty model and new performance criterion. In Section 3, sufficient robust stability LMI conditions for the structured gain-scheduled controller are given. Respective conditions for robust controller synthesis are in BMI form. In Section 4, the results are illustrated on examples to design a PID robust gain-scheduled controller. The final Section 5 brings a conclusion on the obtained results and possible directions in the gain-scheduled controller design field.

Hereafter, the following notational convention will be adopted. Given a symmetric matrix $P = P^T \in \mathbb{R}^{n\times n}$, the inequality $P > 0$ ($P \geq 0$) denotes the positive definiteness (semidefiniteness) matrix. Symbol $*$ denotes a block that is transposed and complex conjugated to the respective symmetrically placed one. Matrices, if not explicitly stated, are assumed to have compatible dimensions. I denotes the identity matrix of corresponding dimensions.

2 PROBLEM FORMULATION AND PRELIMINARIES

Consider a continuous-time linear parameter varying (LPV) uncertain system in the form

$$\begin{aligned} \dot{x} &= \overline{A}(\xi,\theta)x + \overline{B}(\xi,\theta)u \\ y &= Cx, \qquad \dot{y}_d = C_d\dot{x} \end{aligned} \tag{1}$$

where linear parameter varying matrices

$$\begin{aligned} \overline{A}(\xi,\theta) &= A_0(\xi) + \sum_{i=1}^{s} A_i(\xi)\theta_i \ \in \mathbb{R}^{n\times n} \\ \overline{B}(\xi,\theta) &= B_0(\xi) + \sum_{i=1}^{s} B_i(\xi)\theta_i \ \in \mathbb{R}^{n\times m} \end{aligned} \tag{2}$$

$x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^l$ denote the state, control input and controlled output, respectively. Matrices $A_i(\xi)$, $B_i(\xi)$, $i = 0,1,2,\dots s$ belong to the convex set: a polytope with N vertices that can be formally defined as

$$\begin{aligned} &\Omega = \Big\{A_i(\xi), B_i(\xi) = \sum_{j=1}^{N}(A_{ij}, B_{ij})\xi_j\Big\}, \\ &i = 0,1,2,\dots s\,, \quad \sum_{j=1}^{N}\xi_j = 1\,, \quad \xi_j \geq 0 \end{aligned} \tag{3}$$

where s is the number of scheduled parameters; ξ_j, $j = 1,2,\dots N$ are constant or possibly time varying but unknown parameters; matrices A_{ij}, B_{ij}, C, C_d are constant matrices of corresponding dimensions, where C_d is the output matrix for D part of the controller. $\theta \in \mathbb{R}^s$ is a vector of known (measurable) constant or possibly time-varying scheduled parameters. Assume that both lower and upper bounds are available. Specifically

1. Each parameter θ_i, $i = 1,2,\dots s$ ranges between known extremal values
$$\theta \in \Omega_s = \big\{\theta \in \mathbb{R}^s \colon \theta_i \in \langle \underline{\theta}_i, \overline{\theta}_i\rangle\,,\ i = 1,2,\dots s\big\}. \tag{4}$$
2. The rate of variation $\dot{\theta}_i$ is well defined at all times and satisfies
$$\dot{\theta} \in \Omega_t = \big\{\dot{\theta}_i \in \mathbb{R}^s \colon \dot{\theta}_i \in \langle \underline{\dot{\theta}}_i, \overline{\dot{\theta}}_i\rangle\,,\ i = 1,2,\dots s\big\}. \tag{5}$$

Note that system (1), (2), (3) consists of two type of vertices. The first one is due to the gain-scheduled parameters θ with $T = 2^s$ vertices – θ vertices, and the second set of vertices are due to uncertainties of the system – N, ξ vertices. For robust gain-scheduled "I" part controller design the states of system (1) need to be extended in such a way that a static output feedback control algorithm can provide proportional (P) and integral (I) parts of the designed controller. For more details see [20]. Assume that system (1) allows PI controller design with a static output feedback.

To access the system performance, we consider an original scheduling quadratic cost function

$$\begin{aligned} &J = \int_0^\infty J(t)\mathrm{d}t = \int_0^\infty \big(x^TQ(\theta)x + u^TRu + \dot{x}^TS(\theta)\dot{x}\big)\mathrm{d}t \\ &Q(\theta) = Q_0 + \sum_{i=1}^{s} Q_i\theta_i, \quad S(\theta) = S_0 + \sum_{i=1}^{s} S_i\theta_i\,. \end{aligned} \tag{6}$$

The feedback control law is considered in the form

$$\begin{aligned} &u = F(\theta)y + F_d(\theta)\dot{y}_d \\ &F(\theta) = F_0 + \sum_{i=1}^{s} F_i\theta_i, \quad F_d(\theta) = F_{d_0} + \sum_{i=1}^{s} F_{d_i}\theta_i \end{aligned} \tag{7}$$

Matrices F_i, F_{d_i}, $i = 0,1,2,\dots s$ are the static output PI part and the output derivative feedback gain-scheduled controller. The structure of the above matrices can be prescribed.

The respective closed-loop system is then

$$\begin{aligned} M_d(\xi,\theta)\dot{x} &= A_c(\xi,\theta)x \\ M_d(\xi,\theta) &= I - \overline{B}(\xi,\theta)F_d(\theta)C_d \\ A_c(\xi,\theta) &= \overline{A}(\xi,\theta) + \overline{B}(\xi,\theta)F(\theta)C \end{aligned} \tag{8}$$

Let as recall some results about an optimal control of time varying systems [21].

LEMMA 1. *Let $V(x,t)$ be a scalar positive definite function such that $\lim_{t\to\infty} V(x,t) = 0$ which satisfies*

$$\min_{u\in\Omega_u}\Big\{\frac{\delta V}{\delta x}A_c(\theta) + \frac{\delta V}{\delta t} + J(t)\Big\} = 0\,. \tag{9}$$

From (9) *obtained control algorithm $u = u^*(x,t)$ ensure the closed-loop stability and on the solution of* (1) *optimal value of cost function as $J^* = J(x_0,t_0) = V(x(0),t_0)$.*

Eq. (9) is known as Bellman-Lyapunov equation and function $V(x,t)$ which satisfies to (9) is Lyapunov function. For a given concrete structure of Lyapunov function the optimal control algorithm may reduce from "*if and only if*" to "*if*" and for switched systems, robust control, gain-scheduled control and so on to guaranteed cost.

DEFINITION 1. Consider a stable closed-loop system (8). If there exists a control law u (7) which satisfies (11) and a positive scalar J^* such that the value of closed-loop cost function (6) J satisfies $J < J^*$ for all $\theta \in \Omega_s$ and ξ_j, $j = 1, 2 \dots N$ satisfying (3), then J^* is said to be a guaranteed cost and u is said to be a guaranteed cost control law for system (8).

Let us recall some parameter dependent stability results which provide basic further developments.

DEFINITION 2. Closed-loop system (8) is parameter dependent quadratically stable in the convex domain Ω given by (3) for all $\theta \in \Omega_s$ and $\dot{\theta} \in \Omega_t$ if and only if there exists a positive definite parameter dependent Lyapunov function $V(\xi, \theta)$ such that the time derivative of Lyapunov function with respect to (8) is

$$\frac{\mathrm{d}V(\xi,\theta,t)}{\mathrm{d}t} < 0\,. \tag{10}$$

LEMMMA 2. *Consider the closed-loop system* (8). *Control algorithm* (7) *is the guaranteed cost control law if and only if there exists a parameter dependent Lyapunov function* $V(\xi, \theta)$ *such that the following condition holds* [21]

$$B_e(\xi,\theta) = \min_u \Big(\frac{dV(\xi,\theta,t)}{dt} + J(t) \Big) \le 0\,. \tag{11}$$

Uncertain closed-loop system (8) conforming to *Lemma* 2 is called robust parameter dependent quadratically stable with guaranteed cost.

We proceed with the notion of multi-convexity of a scalar quadratic function [22].

LEMMA 3. *Consider a scalar quadratic function of* $\theta \in \mathbb{R}^s$

$$f(\theta) = \alpha_0 + \sum_{i=1}^{s} \alpha_i \theta_i + \sum_{i=1}^{s} \sum_{j>i}^{s} \beta_{ij} \theta_i \theta_j + \sum_{i=1}^{s} \gamma_i \theta_i^2 \tag{12}$$

and assume that if $f(\theta)$ *is multiconvex that is*

$$\frac{\partial^2 f}{\partial \theta_i^2} = 2\gamma_i \ge 0\,, \quad i = 1, 2, \dots, s\,.$$

Then $f(\theta)$ *is negative in the hyper rectangle* (4) *if and only if it takes negative values at the vertices of* (4), *that is if and only if* $f(\theta) < 0$ *for all vertices of the set given by* (4). *For decrease the conservatism of Lemma* 3 *the approach proposed in* [22] *can be used.*

In this paragraph for uncertain gain-scheduling system (1) we have proposed to use a model uncertainty in the form of a convex set with N vertices defined by (3). Furthermore, we consider the new type of performance (6) to obtain the closed-loop system guaranteed cost.

3 MAIN RESULTS

This section formulates the theoretical approach to robust PID gain-scheduled controller design for polytopic system (1), (2), (3) which ensures closed-loop system parameter dependent quadratic stability and a guaranteed cost for all gain-scheduling parameters $\theta \in \Omega_s$, and $\dot{\theta} \in \Omega_t$. The main result on robust stability for the gain-scheduled control system is given in the next theorem.

THEOREM 1. *The closed-loop system* (8) *is robust parameter dependent quadratically stable with a guaranteed cost if there exist positive definite matrix* $P(\xi,\theta) \in \mathbb{R}^{n\times n}$, *matrices* N_1, $N_2 \in \mathbb{R}^{n\times n}$ *positive definite (semidefinite) matrices* $Q(\theta)$, R, $S(\theta)$ *and gain-scheduled controller* (7) *such that*

$$\begin{aligned} L(\xi,\theta) = W_0(\xi) + \sum_{i=1}^{s} W_i(\xi)\theta_i + \\ \sum_{i=1}^{s}\sum_{j>i}^{s} W_{ij}(\xi)\theta_i\theta_j + \sum_{i=1}^{s} W_{ii}\theta_i^2 < 0\,, \end{aligned} \tag{13}$$

$$W_{ii}(\xi) \ge 0\,, \quad \theta \in \Omega_s,\ i = 1, 2, \dots, s \tag{14}$$

where we consider parameter dependent Lyapunov matrix

$$P(\xi,\theta) = P_0(\xi) + \sum_{i=1}^{s} P_i(\xi)\theta_i > 0\,. \tag{15}$$

The above matrices (13) *and* (14) *are given as*

$$\begin{aligned}
W_0(\xi) &= \begin{bmatrix} W_{11_0}(\xi) & W_{12_0}(\xi) \\ * & W_{22_0}(\xi) \end{bmatrix}, \\
W_{11_0}(\xi) &= S_0 + C_d^T F_{d_0}^T R F_{d_0} C_d + \\
&\quad N_1^T (I - B_0(\xi) F_{d_0} C_d) + (I - B_0(\xi) F_{d_0} C_d)^T N_1, \\
W_{12_0}(\xi) &= -N_1^T (A_0(\xi) + B_0(\xi) F_0 C) + \\
&\quad (I - B_0(\xi) F_{d_0} C_d)^T N_2 + P_0(\xi) + C_d^T F_{d_0}^T R F_0 C, \\
W_{22_0}(\xi) &= -N_2^T (A_0(\xi) + B_0(\xi) F_0 C) \\
&\quad - (A_0(\xi) + B_0(\xi) F_0 C)^T N_2 \\
&\quad + Q_0 + C^T F_0^T R F_0 C + \sum_{j=1}^{s} P_j(\xi)\dot{\theta}_i, \\
W_i(\xi) &= \begin{bmatrix} W_{11_i}(\xi) & W_{12_i}(\xi) \\ * & W_{22_i}(\xi) \end{bmatrix}, \\
W_{11_i}(\xi) &= S_i + C_d^T (F_{d_0}^T R F_{d_i} + F_{d_i}^T R F_{d_0}) C_d \\
&\quad - N_1^T (B_0(\xi) F_{d_i} + B_i(\xi) F_{d_0}) C_d \\
&\quad - \big[(B_0(\xi) F_{d_i} + B_i(\xi) F_{d_0}) C_d \big]^T N_1, \\
W_{12_i}(\xi) &= -N_1^T (A_i(\xi) + B_0(\xi) F_i + B_i(\xi) F_0) C \\
&\quad - (B_i(\xi) F_{d_0} C_d)^T N_2 + P_i(\xi) \\
&\quad + C_d^T (F_{d_i}^T R F_0 + F_{d_0}^T R F_i) C, \\
W_{22_i}(\xi) &= -N_2^T \big(A_i(\xi) + (B_0(\xi) F_i + B_i(\xi) F_0) C \big)
\end{aligned}$$

$$
\begin{aligned}
&- \left[A_i(\xi) + (B_0(\xi)F_i + B_i(\xi)F_0)C\right]^T N_2 \\
&+ Q_i + C^T(F_0^T RF_i + F_i^T RF_0)C, \\
W_{ij}(\xi) &= \begin{bmatrix} W_{11ij}(\xi) & W_{12ij}(\xi) \\ * & W_{22ij}(\xi) \end{bmatrix} \\
W_{11_{ij}}(\xi) &= C_d^T(F_{d_i}^T RF_{d_j} + F_{d_j}^T RF_{d_i})C_d \\
&- N_1^T(B_i(\xi)F_{d_j} + B_j(\xi)F_{d_i})C_d \\
&- C_d^T(B_i(\xi)F_{d_j} + B_j(\xi)F_{d_i})^T N_1, \\
W_{12_{ij}}(\xi) &= -N_1^T(B_i(\xi)F_j + B_j(\xi)F_i)C \\
&- C_d^T(B_i(\xi)F_{d_j} + B_j(\xi)F_{d_i})^T N_2 \\
&+ C_d^T(F_{d_i}^T RF_j + F_{d_j}^T RF_i)C, \\
W_{22_{ij}}(\xi) &= -N_2^T(B_i(\xi)F_j + B_j(\xi)F_i)C \\
&- C^T(B_i(\xi)F_j + B_j(\xi)F_i)^T N_2 \\
&+ C^T(F_i^T RF_j + F_j^T RF_i)C, \\
W_{ii}(\xi) &= \begin{bmatrix} W_{11_{ii}}(\xi) & W_{12_{ii}}(\xi) \\ * & W_{22ii}(\xi) \end{bmatrix}, \\
W_{11_{ii}}(\xi) &= C_d^T F_{d_i}^T RF_{d_i} C_d - N_1^T B_i(\xi)F_{d_i} C_d \\
&- C_d^T F_{d_i}^T B_i(\xi)^T N_1, \\
W_{12_{ii}}(\xi) &= -N_1^T B_i(\xi)F_i C - C_d^T F_{d_i}^T B_i^T(\xi)N_2 \\
&+ C_d^T F_{d_i}^T RF_i C, \\
W_{22_{ii}}(\xi) &= -N_2^T B_i(\xi)F_i C - C^T F_i^T B_i^T(\xi)N_2 \\
&+ C^T F_i^T RF_i C\,.
\end{aligned}
$$

Due to space we provide only the outline of the proof. The proof is based on *Lemma*2 and 3. The time derivative of the Lyapunov function $V(\xi,\theta) = x^T P(\xi,\theta)x$ is

$$
\begin{aligned}
&\frac{\mathrm{d}V(\xi,\theta)}{\mathrm{d}t} = \\
&[\,\dot{x}^T \quad x^T\,]\begin{bmatrix} 0 & P(\xi,\theta) \\ P(\xi,\theta) & P(\xi,\dot{\theta}) \end{bmatrix}\begin{bmatrix} \dot{x} \\ x \end{bmatrix} P(\xi,\dot{\theta}) = \\
&\sum_{i=1}^{s} P_i(\xi)\dot{\theta}\,.
\end{aligned} \tag{16}
$$

To isolate two matrices (system and Lyapunov) introducing matrices N_1, N_2 in the following way

$$
[2N_1\dot{x} + 2N_2 x]^T[M_d(\xi\theta)\dot{x} - A_c(\xi,\theta)] = 0 \tag{17}
$$

and substituting (17), (16), $J(t)$ (6) and control law (7) to (11), after some manipulation one obtains

$$
B_e(\xi,\theta) = [\,\dot{x}^T \quad x^T\,]\begin{bmatrix} W_{11}(\xi) & W_{12}(\xi) \\ W_{12}^T(\xi) & W_{22}(\xi) \end{bmatrix}\begin{bmatrix} \dot{x} \\ x \end{bmatrix} \tag{18}
$$

where

$$
\begin{aligned}
W_{11} &= S(\theta) + C_d^T F_d^T(\theta)RF_d(\theta)C_d \\
&+ N_1^T M_d(\xi,\theta) \ + M_d^T(\xi,\theta)N_1, \\
W_{12} &= -N_1^T A_c(\xi,\theta) + M_d^T(\xi,\theta)N_2 + P(\xi,\theta) \\
&+ C_d^T F_d^T(\theta)RF(\theta)C, \\
W_{22} &= -N_2^T A_c(\xi,\theta) - A_c^T(\xi,\theta)N_2 + Q(\theta) \\
&+ C^T F^T(\theta)RF(\theta)C + P(\xi,\dot{\theta})
\end{aligned}
$$

Equation (18) immediately implies (13), which proves the sufficient conditions of *Theorem* 1.

Equations (13) and (14) are linear with respect to uncertain parameter ξ_j, $j = 1,2,\ldots,N$, therefore (13) and (14) have to hold for all $j = 1,2,\ldots N$. For the known gain-scheduled controller parameters, inequalities (13) and (14) reduce to LMI, for gain-scheduled controller synthesis problem (13) (14) are BMI.

R e m a r k 1 . *Theorem* 1 can be used for a quadratic stability test, where Lyapuunov function matrices (matrix) are either independent of parameter ξ_j, $j = 1,2,\ldots,N$ or parameter θ_i, $i = 1,2,\ldots,s$ or both as listed below.

1. Quadratic stability with respect to model parameter variation. For this case one has $P(\theta) = P_0 + \sum_{i=1}^{s} P_i\theta_i$. This Lyapunov function should withstand arbitrarily fast model parameter variation in the convex set (3)
2. Quadratic stability with respect to gain-scheduled parameters θ. For this case $P_i \to 0$, $i = 1,2\ldots,s$ and Lyapunov matrix is $P(\xi,\theta) = P_0(\xi)$. This Lyapunov function should withstand arbitrarily fast θ parameter variations.
3. Quadratic stability with respect to both ξ and θ parameters. Lyapunov matrix is $P(\xi,\theta) = P_0$ and it should withstands arbitrarily fast model and gain-scheduled parameter variation.

4 EXAMPLES

Each example is calculated for three quadratic stability approaches (Remark 1) and for parameter dependent quadratic stability, that is

QS1: Quadratic stability with respect to uncertain model parameter variation. For this case the Lyapunov matrix is in the form

$$
P(\xi,\theta) = P_0 + \sum_{i=1}^{s} P_i\theta_i, \tag{19}
$$

QS2: Parameter dependent quadratic stability. The Lyapunov matrix is given as

$$
\begin{aligned}
&P(\xi,\theta) = P_0(\xi) + \sum_{i=1}^{s} P_i(\xi)\theta_i \\
&P_j(\xi) = \sum_{v=1}^{N} P_{jv}\xi_v, \quad j = 0,1,2,\ldots s, \quad \sum_{i=1}^{N}\xi_i = 1,
\end{aligned} \tag{20}
$$

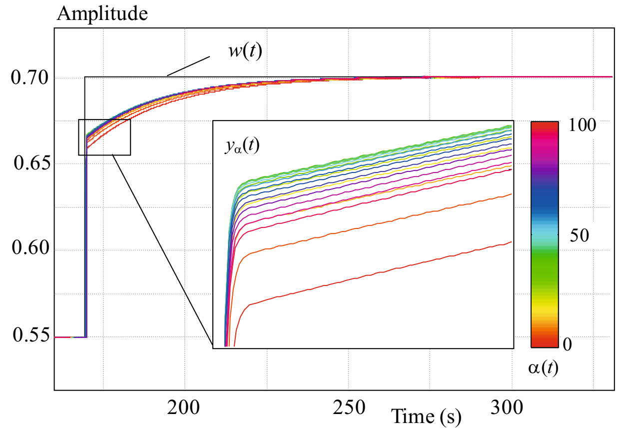

Fig. 1. Simulation results at QS3, $\gamma = 1$, $\alpha \in \langle 0,\ 100\rangle$

QS3: Quadratic stability with respect to gain-scheduled parameters. The Lyapunov matrix is in the form

$$P(\xi,\theta) = \sum_{i=1}^{N} P_i\xi_i, \tag{21}$$

QS4: Quadratic stability with respect to both gain-scheduled and model uncertain parameters. The Lyapunov matrix is

$$P(\xi,\theta) = P_0 \,. \tag{22}$$

Example 1. The first numerical example has been borrowed from [4] with a small modification. Consider a simple linear plant with parameter varying coefficients

$$\begin{aligned} \dot{x}(t) &= \gamma a(\alpha)x(t) + \gamma b(\alpha)u(t),\\ y(t) &= x(t), \quad \text{where:}\\ a(\alpha) &= -6 - \frac{2}{\pi}\arctan\frac{\alpha}{20}\,,\\ b(\alpha) &= \frac{1}{2} + \frac{5}{\pi}\arctan\frac{\alpha}{20}, \end{aligned} \tag{23}$$

$\gamma \in \langle 0.9,\ 1.1\rangle$ being an unknown but constant coefficient and $\alpha \in \langle 0,\ 100\rangle$ a measurable parameter. Let us take 3 working points $\alpha = 0,\ 30,\ 100$ where one obtains two models for $\gamma = 0.9$ and $\gamma = 1.1$ (for each working point). The above models have been recalculated to the form (1), (2), (3). Due to I part controller the extended plant models are

$$A_0 = \begin{bmatrix} -6.4370\gamma & 0\\ 1 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.3130\gamma & 0\\ 0 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.1240\gamma & 0\\ 0 & 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 1.5930\gamma\\ 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1.275\gamma\\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3110\gamma\\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix}, \quad D = 0\,.$$

For parameters $R = rI$, $r = 1$, $Q(\theta) = q_0 I + q_1 I + q_2 I$, $q_0 = 0.1$, $q_1 = q_2 = 0.02$, $S(\theta) = s_0 I + s_1 I + s_2 I$, $s_0 = s_1 = s_2 = 0$, $r_0 = 2000$ $(0 < P(\xi,\theta) < r_0 I)$, $\theta_i \in \langle -1, 1\rangle$, $i = 1, 2$; we have obtained the following PID robust gain-scheduled controller

$$R(s) = R_0(s) + R_1(s)\theta_1 + R_2(s)\theta_2\,.$$

QS1: PENBMI failed
QS2: Closed-loop maximal eigenvalue for $\theta_1 = \theta_2 = 0$ is $\lambda_{\max} = -0.2498$

$$\begin{aligned} R_0(s) &= -1.3667 - 1.2936/s - 0.07s,\\ R_1(s) &= 1.8456 + 0.6652/s + 0.0289s,\\ R_2(s) &= 1.431 + 0.5161/s + 0.028s. \end{aligned}$$

QS3: Closed-loop maximal eigenvalue for $\theta_1 = \theta_2 = 0$ is $\lambda_{\max} = -0.0407$

$$\begin{aligned} R_0(s) &= -9.6682 - 0.5575/s + 0.0375s,\\ R_1(s) &= 0.5187 + 0.0235/s + 0.0019s,\\ R_2(s) &= 1.3277 + 0.0602/s + 0.0049s. \end{aligned}$$

QS4: PENBMI failed

The closed-loop dynamic behaviours for QS3 are given in Fig. 1, where the black line is the setpoint $w(t)$ and the coloured lines are the measured outputs $y(t)$ at $\alpha = 0, 2, 4, \ldots 100$. Another closed-loop dynamic behaviours for QS2, $\gamma = 1$ are given in Fig. 2, where $w(t)$ is the setpoint, $y(t)$ is the system output, $u(t)$ is the controller output, θ_1 and θ_2 are calculated scheduled parameters and α is the exogenous signal.

Example 2. Second example has been borrowed from [3]. Uncertain model (1) is given as follows

$$A(\theta) = \begin{bmatrix} 0.1\gamma & \theta_1 + 4\theta_2\\ -1 & 0 \end{bmatrix}, \quad B(\theta) = \begin{bmatrix} 0\\ \gamma\theta_1 + 1.5\theta_2 \end{bmatrix}$$

where $\theta_1 + \theta_2 = 1$, $\theta_i \geq 0$, $i = 1, 2$ and uncertain parameter $\gamma \in \langle 0.9, 1.1\rangle$. Substituting for $\theta_2 = 1 - \theta_1$ and for $\gamma = 0.9$ or $\gamma = 1.1$ one obtains

$$A_{01} = \begin{bmatrix} 0.09 & 4\\ -1 & 0 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} 0.11 & 4\\ -1 & 0 \end{bmatrix},$$
$$A_{11} = A_{12} = \begin{bmatrix} 0 & -3\\ 0 & 0 \end{bmatrix}, \quad B_{01} = B_{02} = \begin{bmatrix} 0\\ 1.5 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0\\ -0.6 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0\\ -0.4 \end{bmatrix},$$
$$C = [1 \quad 1], \quad D = 0$$

For parameters $r = 1$, $\theta_1 \in \langle 0, 1\rangle$, $q_0 = 0.001$, $s_0 = 0$, $q_1 = 0.0002$, $s_1 = 0$, $r_0 = 20000$ the following PID controller is obtained

QS1: Closed-loop maximal eigenvalue for $\theta_1 = \theta_2 = 0$ is $\lambda_{\max} = -0.1483$

$$\begin{aligned} R_0(s) &= -4.2735 - 0.7288/s - 0.521s,\\ R_1(s) &= -29.0575 - 8.6869/s - 15.9056s. \end{aligned}$$

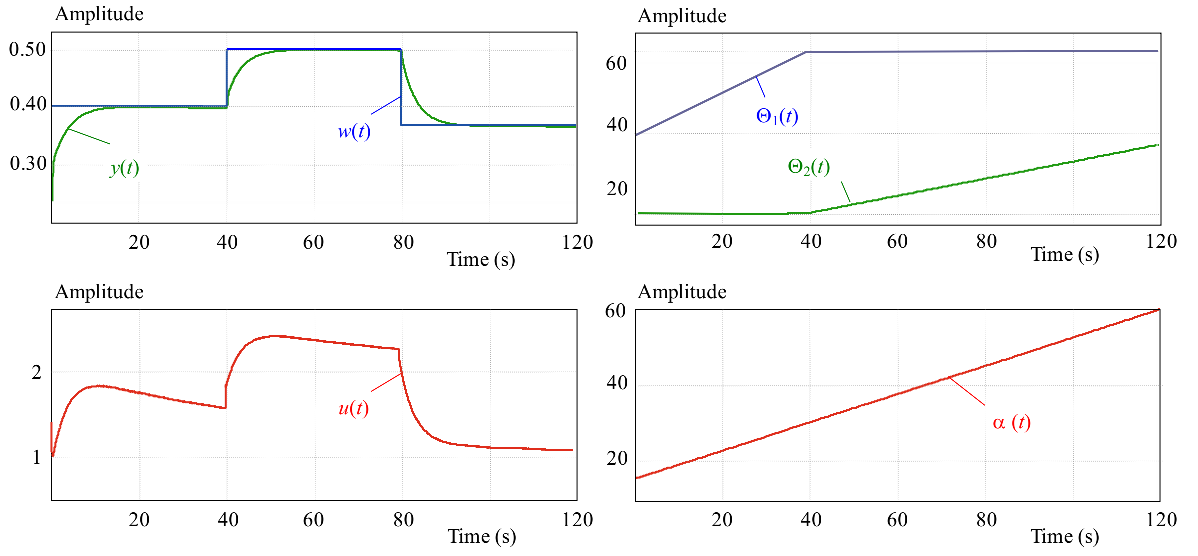

Fig. 2. Simulation results at QS2, $\gamma = 1$, $\alpha \in \langle 0,\ 100\rangle$

QS2: PENBMI failed

QS3: Closed-loop maximal eigenvalue for $\theta_1 = \theta_2 = 0$ is $\lambda_{\max} = -0.1129$

$$R_0(s) = 0.0805 - 0.0466/s - 1.9234s,$$
$$R_1(s) = -0.0649 - 0.4289/s - 15.0506s.$$

QS4: Closed-loop maximal eigenvalue for $\theta_1 = \theta_2 = 0$ is $\lambda_{\max} = -0.0752$

$$R_0(s) = 0.0304 - 0.0405/s - 1.7515s,$$
$$R_1(s) = -0.0763 - 0.5508/s - 19.9456.$$

The closed-loop dynamic behaviours are given in Fig. 3.

EXAMPLE 3. Consider the uncertain system (1) [9]

$$A_0 = \begin{bmatrix} 0.2\gamma & -0.8 \\ 0.3 & -1.3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.0 & -0.3\gamma \\ 0.5\gamma & 0 \end{bmatrix},$$
$$B_0 = \begin{bmatrix} 0.4 \\ 0.8\gamma \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}\gamma,\ C = [1 \quad 0].$$

where $\gamma \in \langle 0.9,\ 1.1\rangle$ constant but uncertain parameter $\theta_1 \in \langle 0, 1\rangle$. Despite the simplicity the system with state feedback is not quadratically stabilizable with a fixed gain matrix for $\gamma = 1$. Substituting $\gamma = 0.9$ and $\gamma = 1.1$ we obtain the uncertain plant model (1). For parameters $r = 1$, $q_0 = 0.0001$, $s_0 = 0$, $q_1 = 0.0001$, $s_1 = 0$ and $r_0 = 20000$ the following robust PID controllers are obtained

QS1: Closed-loop maximal eigenvalue for $\theta_1 = \theta_2 = 0$ is $\lambda_{\max} = -0.0222$.

$$R_0(s) = 1.0149 + 0.0172/s - 1.94333s,$$
$$R_1(s) = [-0.4446 + 0.6918/s + 11.845s] \times 10^{-14} \doteq 0.$$

QS2, QS3 and QS4: PENBMI failed

When one changes $s_0 = 0.1$, $s_1 = 0.001$ the new PID controller parameters are obtained for QS1:

$$R_0(s) = 1.0435 + 0.017/s - 1.7061s,$$
$$R_1(s) = [-0.224 + 0.643/s + 10.258] \times 10^{-14} \doteq 0\,.$$

Closed-loop maximal eigenvalue for $\theta_1 = \theta_2 = 0$ is $\lambda_{\max} = -0.0209$. The closed-loop dynamic behaviours are given in Fig. 4.

5 CONCLUSION

A novel design procedure has been proposed for robust gain-scheduled controller design. Several forms of parameter dependent quadratic stability are presented which withstand arbitrarily fast model parameter variation or/and arbitrarily fast gain-scheduled parameter variation. Because of BMI approach the future research should transform BMI to LMI and the obtained design procedure for a polytopic continuous system should be transformed to discrete ones. The proposed approach contributes to the design tools of a robust gain-scheduled controller for uncertain polytopic systems.

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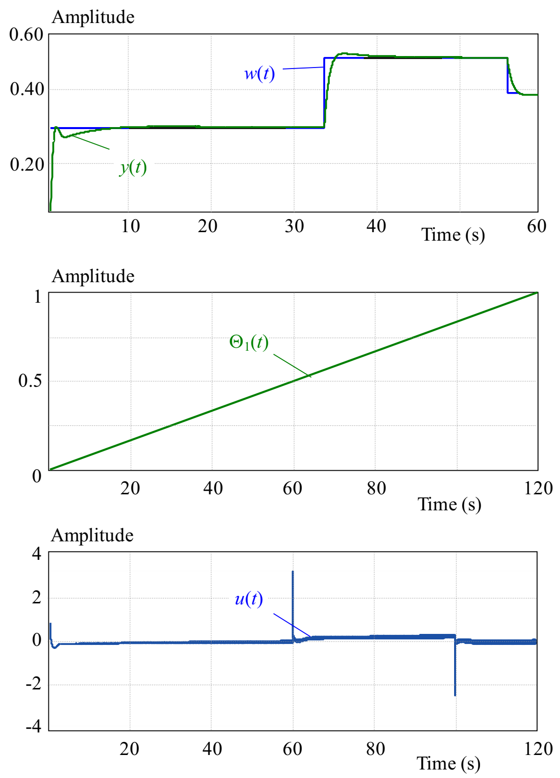

Fig. 3. Simulation results at QS1, $\gamma = 1$, $\theta_1 \in \langle 0, 1 \rangle$

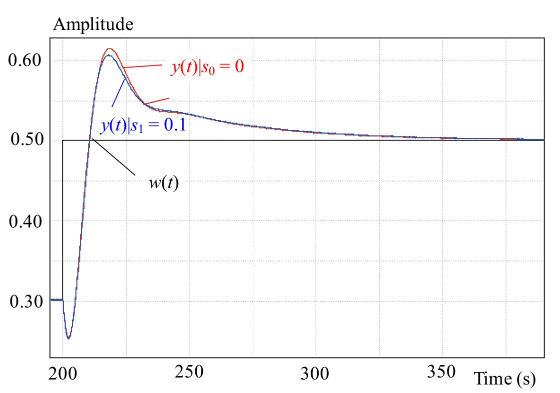

Fig. 4. Simulation results at $\theta_1 = 0$

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