

# Exact analytical modeling of magnetic vector potential in surface inset permanent magnet DC machines considering magnet segmentation

Ali Jabbari\*

Surface inset permanent magnet DC machine can be used as an alternative in automation systems due to their high efficiency and robustness. Magnet segmentation is a common technique in order to mitigate pulsating torque components in permanent magnet machines. An accurate computation of air-gap magnetic field distribution is necessary in order to calculate machine performance. An exact analytical method for magnetic vector potential calculation in surface inset permanent magnet machines considering magnet segmentation has been proposed in this paper. The analytical method is based on the resolution of Laplace and Poisson equations as well as Maxwell equation in polar coordinate by using sub-domain method. One of the main contributions of the paper is to derive an expression for the magnetic vector potential in the segmented PM region by using hyperbolic functions. The developed method is applied on the performance computation of two prototype surface inset magnet segmented motors with open circuit and on load conditions. The results of these models are validated through FEM method.

**Key words:** analytical modeling, surface inset PM motor, magnet segmentation, sub-domain method, FEM

## 1 Introduction

Permanent magnet motors are interested in industrial applications due to their high efficiency and power density [1, 2]. An accurate prediction of air-gap magnetic field distribution is necessary in order to calculate machine performance. A variety of techniques including analytical and numerical methods has been conducted to evaluate the magnetic vector potential in electrical machines. Numerical methods like finite element method (FEM) give accurate results and are time consuming specially in first step of design stage. Analytical methods including conformal mapping [3]-[6], Magnetic Equivalent Circuit (MEC) [7-9], sub-domain model [10-26] and slot relative permeance calculation [27-30] are reported to model electrical machines and are useful in first step of performance evaluation and design optimization stage. The sub-domain model is more accurate than the other analytical models [7]. This method is developed based on solution of Laplace and Poisson equations in different regions by applying boundary conditions for electrical machines [10-26].

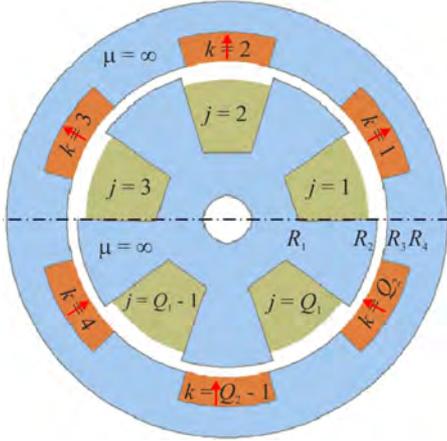
To author's knowledge, a few analytical models are presented to calculate magnetic vector potential in surface inset permanent magnet motors [27-29]. No references in the literature addressing the issue of an analytical model for surface inset magnet segmented machines were found.

The focus of this paper is to develop an analytical model based on resolution of Laplace and Poisson equations in surface inset permanent magnet machines by using the sub-domain method considering magnet segmentation and slotting effects. It is shown that the developed model can effectively estimate magnetic vector potential, magnetic flux density, cogging torque and electromagnetic torque. This model is applied on the performance calculation of two prototypes, *ie* a 2 segmented 5S-2P PM motor and a 3 segmented 5S-2P PM motor. It is shown that the results of analytical model are in close agreement with the results of FEM.

## 2 Problem statement

The geometrical representation of the investigated permanent magnet motor with magnet segmented outer rotor layout is shown in Fig. 1. The machine model is divided into three sub-domains including the armature slots region (domain  $j$ ) which has  $Q_1$  slots, the air-gap region (domain  $I$ ) and the permanent magnet region (domain  $k$ ) which has  $Q_2$  magnets. The machine parameters including the stator yoke radius  $R_1$ , the stator surface radius  $R_2$ , the rotor surface radius  $R_3$ , and the rotor slot radius  $R_4$ .

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**Fig. 1.** Schematic representation of a surface inset magnet segmented motor

The angular position of the  $j$ -th armature slot and  $k$ -th stator permanent magnet are defined as

$$\theta_j = -\frac{\alpha}{2} + \frac{2j\pi}{Q_1} \quad \text{with } 1 \leq j \leq Q_1, \quad (1)$$

$$\theta_k = -\frac{\gamma}{2} + \frac{2k\pi}{Q_2} \quad \text{with } 1 \leq k \leq Q_2. \quad (2)$$

### 3 Magnetic vector potential calculation

General solution of Laplace or Poisson equation in each sub-domain is developed in this section. The Laplace equation can be described in polar form as

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} = 0 \quad \text{for } \begin{cases} R_1 \leq r \leq R_2, \\ \theta_1 \leq \theta \leq \theta_2. \end{cases} \quad (3)$$

Replacing  $r$  by  $R_1 e^{-t}$ , one obtains

$$\frac{\partial^2 A}{\partial t^2} + \frac{\partial^2 A}{\partial \theta^2} = 0 \quad \text{for } \begin{cases} \ln \frac{R_1}{R_2} \leq t \leq 0, \\ \theta_1 \leq \theta \leq \theta_2. \end{cases} \quad (4)$$

#### 3.1 Magnetic vector potential in the armature slot sub-domain (Region $j$ )

The Poisson equation in the armature slot sub-domain is given by

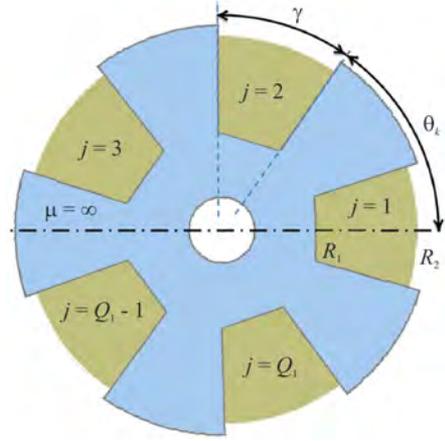
$$\frac{\partial^2 A_j}{\partial t^2} + \frac{\partial^2 A_j}{\partial \theta^2} = -\mu_0 J \quad \text{for } \begin{cases} t_1 \leq t \leq t_2, \\ \theta_j \leq \theta \leq \theta_j + \alpha \end{cases} \quad (5)$$

where  $t_1 = \ln(R_1/R_2)$  and  $t_2 = 0$ .

Neumann boundary conditions at the bottom and at each side of the slot are obtained as

$$\left. \frac{\partial A_j}{\partial \theta} \right|_{\theta=\theta_j} = 0 \quad \text{and} \quad \left. \frac{\partial A_j}{\partial \theta} \right|_{\theta=\theta_j+\alpha} = 0, \quad (6)$$

$$\left. \frac{\partial A_j}{\partial t} \right|_{t=t_2} = 0. \quad (7)$$



**Fig. 2.** Armature slot region (domain  $j$ ) with its boundaries

The general solution of (5) using the separation of variables method is given by

$$A_j(t, \theta) = a_0^j - \frac{1}{2} \mu_0 J_i \left( e^{-t_2 t} + \frac{1}{2} e^{-2t+t_2} \right) + \sum_{h=1}^{\infty} \frac{\cosh \frac{h\pi}{\alpha} (t - t_2)}{\cosh \frac{h\pi}{\alpha} (t_1 - t_2)} a_h^j \cos \frac{h\pi}{\alpha} (\theta - \theta_j) \quad (8)$$

where  $h$  is a positive integer and the coefficients  $a_0^j$  and  $a_h^j$  are determined based on the continuity and interface conditions.

The continuity of the magnetic vector potential between the sub-domain  $j$  and the region  $I$  leads to

$$A_j(t_1, \theta) = A_I(t_4, \theta) \quad \text{for } \theta_j \leq \theta \leq \theta_j + \alpha \quad (9)$$

Interface condition (9) gives

$$a_0^j = \frac{1}{2} \mu_0 J_i \left( e^{-t_2 t} + \frac{1}{2} e^{-2t+t_2} \right) + \frac{1}{\alpha} \int_{\theta_j}^{\theta_j+\alpha} A_I(t_4, \theta) d\theta, \quad (10)$$

$$a_h^j = \frac{2}{\alpha} \int_{\theta_j}^{\theta_j+\alpha} A_I(t_4, \theta) \cdot \cos \frac{h\pi}{\alpha} (\theta - \theta_j) d\theta. \quad (11)$$

#### 3.2 Magnetic vector potential in the air-gap sub-domain (Region $I$ )

The Laplace equation in the air-gap sub-domain is given by

$$\frac{\partial^2 A_I}{\partial t^2} + \frac{\partial^2 A_I}{\partial \theta^2} = 0 \quad \text{for } \begin{cases} t_3 \leq t \leq t_4, \\ 0 \leq \theta \leq 2\pi \end{cases} \quad (12)$$

where  $t_3 = \ln(R_2/R_3)$ ,  $t_4 = 0$ .

The general solution of (12) considering periodicity boundary conditions is obtained as

$$A_I(t, \theta) = \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{\cosh n(t - t_4)}{\sinh n(t_3 - t_4)} a_n^I + \frac{\cosh n(t - t_3)}{\sinh n(t_4 - t_3)} b_n^I \right) \cos n\theta + \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{\cosh n(t - t_4)}{\sinh n(t_3 - t_4)} c_n^I + \frac{\cosh n(t - t_3)}{\sinh n(t_4 - t_3)} d_n^I \right) \sin n\theta. \quad (13)$$

The coefficients  $a_n^I$ ,  $b_n^I$ ,  $c_n^I$  and  $d_n^I$  are determined considering the continuity of magnetic vector potential between the internal air-gap sub-domain  $I$  and the region  $j$  and  $k$  using a Fourier series expansion of interface condition (14) and (15) over the air-gap interval.

The continuity of the magnetic vector potential between the internal air-gap sub-domain  $I$  and the regions  $j$  and  $k$  leads to

$$\left. \frac{\partial A_I}{\partial t} \right|_{t=t_3} = h(\theta) = \begin{cases} \left. \frac{\partial A_k}{\partial t} \right|_{t=t_6} & \text{for } \theta_k \leq \theta \leq \theta_k + \gamma, \\ 0 & \text{elsewhere,} \end{cases} \quad (14)$$

$$\left. \frac{\partial A_I}{\partial t} \right|_{t=t_4} = g(\theta) = \begin{cases} \left. \frac{\partial A_j}{\partial t} \right|_{t=t_1} & \text{for } \theta_j \leq \theta \leq \theta_j + \alpha, \\ 0 & \text{elsewhere.} \end{cases} \quad (15)$$

Interface condition (14) gives

$$a_n^I = \frac{2}{2\pi} \int_{\theta_k}^{\theta_k+\gamma} h(\theta) \cos n\theta \, d\theta, \quad (16)$$

$$c_n^I = \frac{2}{2\pi} \int_{\theta_k}^{\theta_k+\gamma} h(\theta) \sin n\theta \, d\theta. \quad (17)$$

Interface condition (15) gives

$$b_n^I = \frac{2}{2\pi} \int_{\theta_j}^{\theta_j+\alpha} g(\theta) \cos n\theta \, d\theta, \quad (18)$$

$$d_n^I = \frac{2}{2\pi} \int_{\theta_j}^{\theta_j+\alpha} g(\theta) \sin n\theta \, d\theta. \quad (19)$$

### 3.3 Magnetic vector potential in the stator permanent magnet sub-domain (Region $k$ )

The Poisson equation in the stator permanent magnet sub-domain is given by

$$\frac{\partial^2 A_k}{\partial t^2} + \frac{\partial^2 A_k}{\partial \theta^2} = -\frac{\mu_0 e^t}{R_3} \left( M_{\theta k} - \frac{\partial M_{rk}}{\partial \theta} \right) \quad (20)$$

for  $t_5 \leq t \leq t_6$ ,  $\theta_k \leq \theta \leq \theta_k + \gamma$

where  $t_5 = \ln(R_3/R_4)$  and  $t_6 = 0$ .

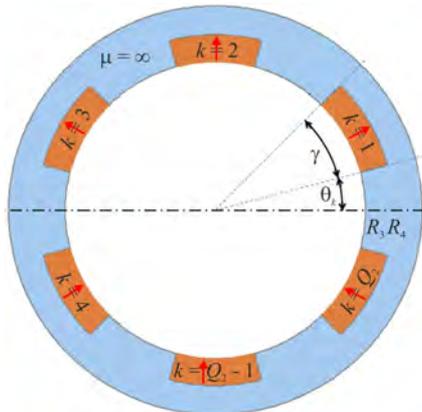


Fig. 3. Permanent magnet region (domain  $k$ ) with its boundaries

The radial and tangential components of radial magnetization for inset design can be expressed as

$$M_{rk} = (-1)^{\lfloor \frac{k}{s+0.5} \rfloor} \frac{B_r}{\mu_0} \quad \text{with } k = 1, 2, \dots, Q_2, \quad (21)$$

$$M_{\theta k} = 0 \quad (22)$$

where  $p$  is number of pole pairs and  $s$  is number of magnet segmentations.

Neumann boundary conditions at the bottom and both sides of the permanent magnet slot are obtained as

$$\left. \frac{\partial A_I}{\partial \theta} \right|_{\theta=\theta_k} = f(t) = R_3 e^{-t} (-1)^{\lfloor \frac{k}{s+0.5} \rfloor} B_r, \quad (23)$$

$$\left. \frac{\partial A_I}{\partial \theta} \right|_{\theta=\theta_k+\gamma} = f(t) = R_3 e^{-t} (-1)^{\lfloor \frac{k}{s+0.5} \rfloor} B_r, \quad (24)$$

$$\left. \frac{\partial A_I}{\partial t} \right|_{t=t_5} = 0. \quad (25)$$

The general solution of (20) is

$$A_k(t, \theta) = a_0^k + R_3 (e^{-(t-t_5)} + t) (-1)^{\lfloor \frac{k}{s+0.5} \rfloor} B_r \left( \theta - \theta_k - \frac{\gamma}{2} \right), \quad (26)$$

$$X_h^k = \begin{cases} \frac{4(-1)^{\lfloor \frac{k}{s+0.5} \rfloor} B_r}{\gamma z^2 (z^2 - 1)} & \text{if } h = 1, 3, 5, \dots \\ 0 & \text{if } h = 2, 4, 6, \dots \end{cases} \quad (27)$$

where  $z = h\pi/\gamma$ ,  $h$  is a positive integer and the coefficients  $a_0^k$  and  $a_h^k$  are determined based on the continuity and interface conditions.

The continuity of the magnetic vector potential between the sub-domain  $k$  and the regions  $I$  leads to

$$A_k(t_6, \theta) = A_I(t_3, \theta) \quad \text{for } \theta_k \leq \theta \leq \theta_k + \gamma. \quad (28)$$

Interface condition (28) gives

$$a_0^k = R_2 (-1)^{\lfloor \frac{k}{s+0.5} \rfloor} B_r \left( \theta - \theta_k - \frac{\gamma}{2} \right) + \frac{1}{\gamma} \int_{\theta_k}^{\theta_k+\gamma} A_I(t_3, \theta) \, d\theta, \quad (29)$$

$$a_h^k = \frac{2}{\gamma} \int_{\theta_k}^{\theta_k+\gamma} A_I(t_3, \theta) \cos z(\theta - \theta_k) \, d\theta. \quad (30)$$

## 4 Performance calculation and model evaluation

### 4.1 Performance computation

The electromagnetic torque is obtained using the Maxwell stress tensor and expressed as

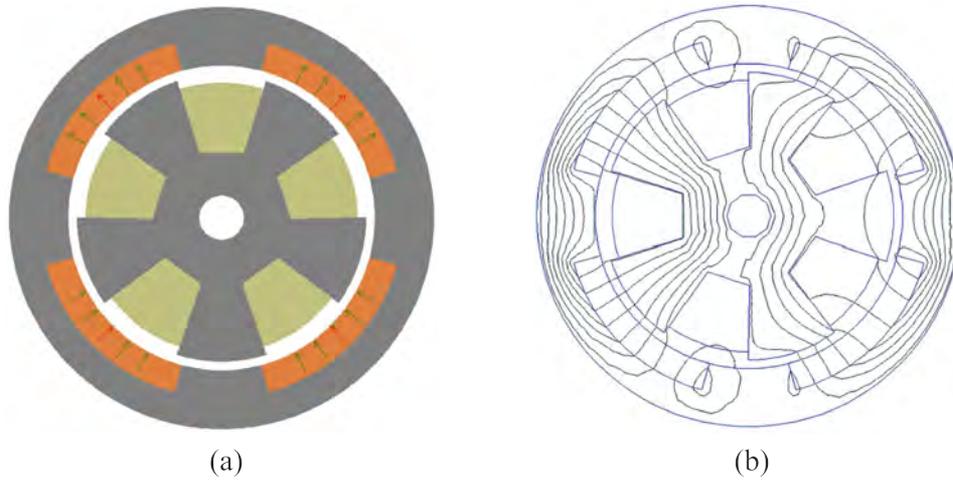
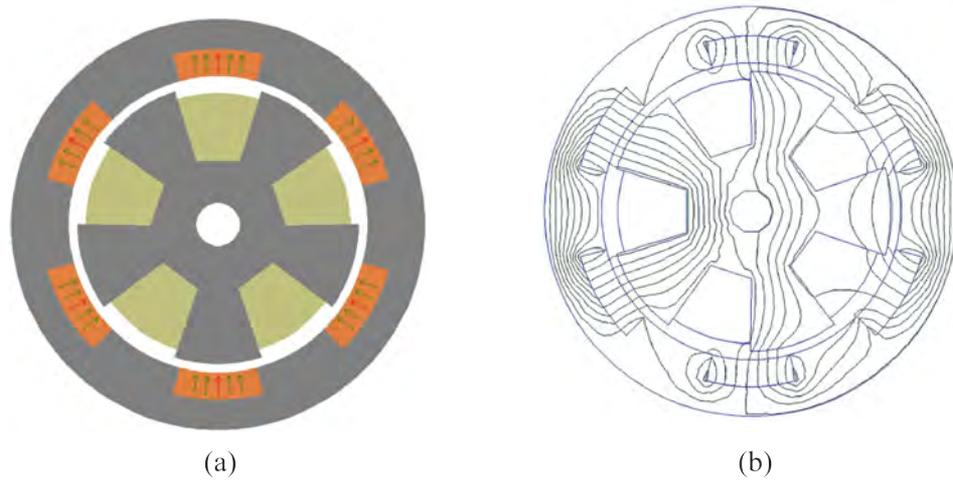
$$T_e = \frac{L_s}{\mu_0} \int_0^{2\pi} B_{I_r}(t_e, \theta) B_{I_\theta}(t_e, \theta) \, d\theta \quad (31)$$

where  $L_s$  is the axial length of the motor and

$$t_e = \ln \frac{R_2}{R_e}, \quad R_e = (R_2 + R_3)/2. \quad (32)$$

**Table 1.** Parameters of the investigated 5s-2p motors

Symbol	Quantity	$S = 2$	$S = 3$
$R_1$	Inner radius of the armature slot	10 mm	10mm
$R_2$	Outer radius of the armature slot	16.4 mm	16.4mm
$R_3$	Inner radius of the stator PMs	17.4 mm	17.4mm
$R_4$	Outer radius of the stator PMs	20.5 mm	20.5mm
$\theta_j$	Angular position of the first slot	17.5	17.5
$\theta_k$	Angular position of the first PM	15	15
$\alpha$	Slot opening angle	37	37
$\gamma$	PMs opening angle	60	30
$p$	Pole pairs-number	12	12
$B_r$	Remanence of the PMs	0.7 T	0.7T
$L_s$	Axial length	35 mm	35mm

**Fig. 4.** Two segmented surface inset PM motor: (a) – the schematic representation, (b) – magnetic flux distribution**Fig. 5.** Three segmented surface inset PM motor: (a) – the schematic representation, (b) – magnetic flux distribution

#### 4.2 Model evaluation

In this section, the proposed analytical model is used to study the open circuit and on load magnetic flux density, open circuit cogging torque and on load electromagnetic torque of two prototype motors. The results of an-

alytical method are then verified by the results of finite element method. The motors parameters are given in Table 1. The schematic representation model of two investigated 5S-2P surface inset PM motors and their corresponding magnetic flux distribution obtained by FEA are shown in Fig. 4 and Fig. 5, respectively.

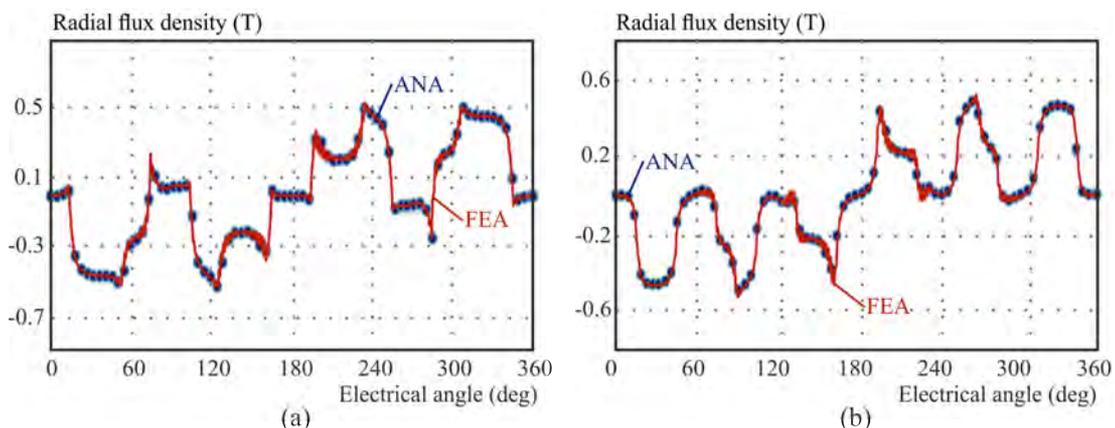


Fig. 6. No load analytical and numerical comparison of radial flux density for: (a) – 2 segment pm motor, (b) – 3 segment PM motor

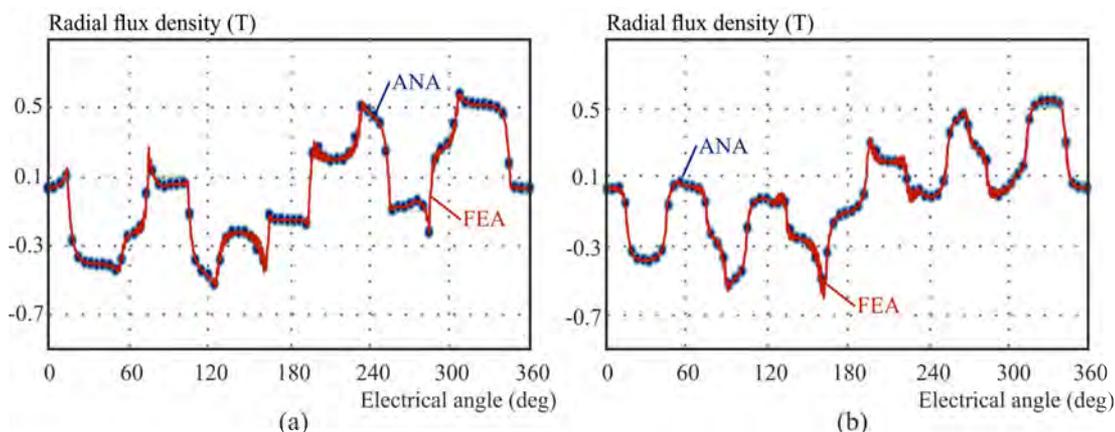


Fig. 7. On load analytical and numerical comparison of radial flux density for: (a) – 2 segment pm motor, (b) – 3 segment PM motor

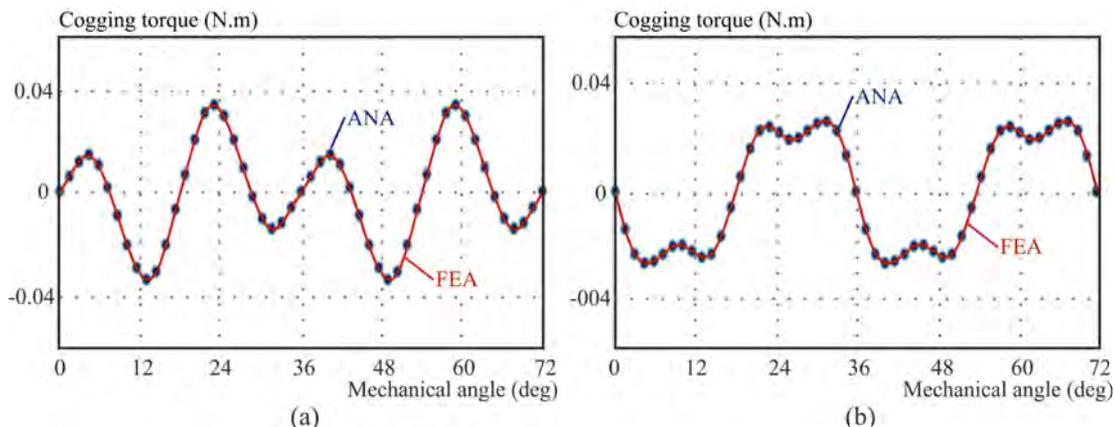


Fig. 8. Open circuit analytical and numerical comparison of cogging torque for: (a) – 2 segment pm motor, (b) – 3 segment PM motor

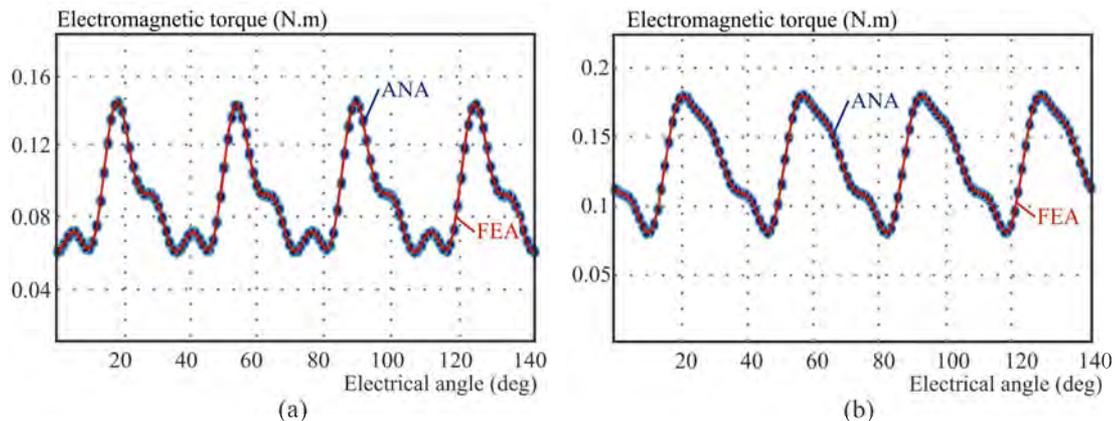


Fig. 9. On load analytical and numerical comparison of electromagnetic torque for: (a) – 2 segment pm motor, (b) – 3 segment PM motor

2D finite element method is applied on performance calculation of the two magnet segmented and three magnet segmented surface inset permanent magnet motors. A comparison of open circuit and on load analytical and numerical results of radial flux density, cogging torque and electromagnetic torque in the investigated motors are shown in Fig. 6, Fig. 7, Fig. 8 and Fig. 9, respectively.

## 5 Conclusion

An exact analytical model for performance prediction in surface inset permanent magnet machines considering slotting effects and magnet segmentation has been developed in this paper. Fourier analysis method based on sub-domain method is applied to derive analytical expressions for calculation of magnetic vector potential, magnetic flux density, cogging torque and electromagnetic torque in surface inset permanent magnet machines. This model is applied for performance computation of two prototype motors and the results of proposed model are verified thanks to FEM results.

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