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PAPERS

Innovation-based fractional order adaptive Kalman filter

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Kalman Filter (KF) is the most widely used estimator to estimate and track the states of target. It works well when noise parameters and system models are well defined in advance. Its performance degrades and starts diverging when noise parameters (mainly measurement noise) changes. In the open literature available researchers has used the concept of Fractional Order Kalman Filter (FOKF) to stabilize the KF. However in the practical application there is a variation in the measurement noise, which will leads to divergence and degradation in the FOKF approach. An Innovation Adaptive Estimation (IAE) based FOKF algorithm is presented in this paper. In order to check the stability of the proposed method, Lyapunov theory is used. Position tracking simulation has been performed for performance evaluation, which shows the better result and robustness.

K e y w o r d s: adaptive estimation, fractional calculus, maximum likelihood estimation, tracking

1 Introduction

Tracking of objects or state estimation is one of the most important research areas for Navigation, Traffic, Surveillance, Intelligent Transport System, Medical field and Economics. The Kalman Filter (KF) is the most widely used state estimator for linear system and gaussian noise. It requires complete prior knowledge of system models and noise parameters. But in practice, it is very difficult to know noise parameter (especially measurement noise) in advance. Due to uncertainty in measuring devices and sensors measurement noise leads to large estimation errors or divergence in KF, which degrades the performance of estimator [13]. To solve the measurement noise uncertainty (R) and KF divergence, many researchers have proposed various Adaptive Kalman Filtering (AKF) algorithms. Mehra [4] first proposed innovation based estimation after that many Innovations based Adaptive Estimation (IAE) proposed by researchers [57]. Hongwei et al. applied Maximum Likelihood Estimation (MLE) on innovation sequence to estimate noise parameters. Xu et al. proposed Optimal Adaptive Kalman Filter (OAKF), in which he fused IAE with Memory attenuation (MA) and used a constant multiplier for calculation of error covariance matrix (P_k). IAE combined with Extended Kalman Filter (EKF), in which estimated covariance of innovation was used for calculation of gain matrix [8]. Improved IAE was proposed [9]; in this a regulator factor was introduced in calculation of gain matrix. To solve the gain divergence problem [10] had shown modified gain Extended Kalman Filter (EKF) in which it is considered that target is moving with constant velocity and noise distribution is Chi-Square. In [11,12] proposed Contour based Unscented Kalman Filter (CUKF). Kaur and Sahambi [13] used Fractional Order Gain Kalman Filter (FOGKF) for vehicle tracking. In FOGKF, conventional Kalman gain is modified by adding fractional derivative of previous conventional Kalman gain, but it degrades the real time performance if measurement noise covariance (R) is not known priori. In this paper, a fractional order Kalman filter combined with IAE is proposed, which introduces the MLE based measurement noise covariance estimation in calculation of Kalman gain. The advantage of proposed method is that even if noise parameters are initially unknown, the proposed Kalman gain will never diverge. The fractional derivative together with MLE helps to improve the accuracy of prediction of Kalman gain through which a new optimal adaptive estimation can be done.

The rest of paper is organized as follows: section 2 present some preliminaries and background. Proposed Innovation-Based Fractional Order Adaptive Kalman Filter (IFOAKF) is presented in section 3. Stability analysis of proposed method is presented in section 4. Simulation results are discussed in section 5. Finally, concluding remarks are presented in section 6.

2 Preliminaries: Innovation based adaptive estimation and fractional calculus

2.1 Innovation based adaptive estimation

A linear system is represented as

$$x_k = A_k x_{k-1} + B_k u_{k-1} + v_k(0, Q_k), \tag{1}$$

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$$z_k = H_k x_k + w_k(0, R_k), \tag{2}$$

where, $x_k \in R^n$ is the state vector at time k, $u_k \in R^m$ is input vector at time k, $A_k \in R^{n\times n}$ is state transition matrices, $B_k \in R^{n\times m}$ is input matrices, $z_k \in R^p$ is actual measurement of x_k at time k, $H_k \in R^{p\times n}$ is output matrices, v_k and w_k are process and measurement noises with zero mean and covariance Q_k and R_k respectively.

In the above equations, process noise and measurement noise are uncorrelated to each other

$$\begin{aligned} E[w_k] &= 0 \qquad Cov[w_k w_j] = Q_k \delta_k j,\\ E[v_k] &= 0 \qquad Cov[v_k v_j] = R_k \delta_k j,\\ & \qquad\qquad Cov[w_k v_j] = 0. \end{aligned}$$

Consider that the estimated state is $\hat{x}_k$, mean square error e_k and error covariance matrix P_k at time k is given as

$$e_k = x_k - \hat{x}_k,$$

$$P_k = E\left[e_k e_k^\top\right].$$

From theory of Kalman innovation sequence is an observable parameter, which is given as

$$r_k = z_k - H_k \hat{x}_{k/k-1}, \tag{3}$$

where

$$\hat{x}_{k/k-1}$$

is prior estimate of$\hat{x}_k$. Theoretical covariance is given [9]

$$C_{rk} = E\left[r_k r_k^\top\right] = H_k P_{k/k-1} H_k^\top + R_k. \tag{4}$$

2.2 Maximum likelihood estimation(MLE)

In MLE the relation between covariance and noise variance is given in equation (5), for detailed proof see reference [13]

$$\sum_{j=0}^{k} tr\left\{\left[C_{rk}^{-1} - C_{rk}^{-1} r_k r_k^\top C_{rk}^{-1}\right]\left[\frac{\mathrm{d}R_j}{\mathrm{d}\theta_k} + H_k \frac{\mathrm{d}Q_j}{\mathrm{d}\theta_k} H_k^\top\right]\right\} = = 0 \tag{5}$$

θ_k is an adaptive parameter, in this paper it is diagonal element of Q and R respectively.

2.2.1 Adaptive estimation of unknown measurement noise()

As measurement noise R is unknown or ill defined, so it can be calculated with the help of MLE. It is hard to estimate Q and R simultaneously when both are unknown. So we are assuming that when we are estimating R at that time Q is independent of θ_k and diagonal element of R is linear function of θ_k. The value of R can be calculated as

$$\sum_{j=0}^{k} tr\left\{\left[C_{rk}^{-1} - C_{rk}^{-1} r_k r_k^\top C_{rk}^{-1}\right][I + 0]\right\} = 0, \tag{6}$$

$$\hat{C}_{rk} = \frac{1}{N}\sum_{j=0}^{k} r_j r_j^\top. \tag{7}$$

$\hat{C}_{rk}$ is estimated covariance of innovation sequence, basically it is the average of innovation sequence r_k in window size of N. With the help of equation (4) and (7) estimated, value of Measurement noise covariance (R) is

$$\hat{R}_k = \hat{C}_{rk} - H_k P_{k/k-1} H_k^\top. \tag{8}$$

2.3 Fractional calculus

Grunawald-Letnikov definition is used for computation of fractional feedback Kalman gain in this proposed method [6, 11]. Fractional difference is given as

$$\Delta^\alpha x_k = \frac{1}{h^\alpha}\sum_{j=0}^{k}(-1)^j \gamma_j x_{k-j}, \tag{9}$$

where, α is fractional order (in this paper we assume $\alpha = 0.5$). h is sampling interval (here $h = 1$), k number of samples in signal x and γ_j is gamma function given as

$$\gamma_j = \binom{\alpha}{j} = \left\{\begin{matrix} 1 & \text{for} \quad j = 0\\ \dfrac{\alpha(\alpha-1)\ldots,(\alpha-j+1)}{j} & \text{for} \quad j > 0 \end{matrix}\right\}.$$

3 Innovation based fractional order adaptive Kalman filter (IFOAKF)

In this section, IFOAKF gain is proposed with the help of fractional calculus and MLE. The following step has been involved in calculating of IFOAKF gain:

- Calculate R with the help of MLE as given in section 2.2.1.
- Calculate steady state Kalman gain using estimated value .
- Calculate fractional difference gain of preceding steady state Kalman gain.
- Proposed IFOAKF gain is the algebraic sum of steady state Kalman gain (ii) and fractional gain (iv).

As we know that KF working is a two-step process. First step is prediction and second stage is correction, similarly IFOAKF is also a two-step process. The mathematical implementation of proposed IFOAKF is given below.

Predicated state of discrete time system given in equation (1) and (2) is

$$\hat{x}_{k/k-1} = A\hat{x}_{k-1/k-1} + Bu. \tag{10}$$

MLE based prior predicted error covariance ($P_{k/k-1}$) for KF is given as

$$P_{k/k-1} = AP_{k-1/k-1}A^\top + Q_{k-1}. \tag{11}$$

As we could not estimate r and Q simultaneously so in the whole process we consider Q is known and fixed. In correction step, we are first calculating standard Kalman filter gain K_k with the help of $\hat{R}$ which is given as

$$K_k = P_{k/k-1}H_k^\top(H_kP_{k/k-1}H_k^\top + \hat{R}_k)^{-1}. \tag{12}$$

Now using the concept of fractional calculus given in section 3.2, the fractional difference gain f_k of preceding Kalman gain K_k is given as

$$f_k = E\left\{\sum_{j=0}^{k}(-1)^{j+1}\gamma_j K_{k-j}\right\}. \tag{13}$$

Basically fractional difference gain is the mean of fractional derivative of previous Kalman gain. The proposed IFOAKF gain K_{mod} is given by adding equation (12) and (13)

$$K_{\mathrm{mod}} = K_k + f_k. \tag{14}$$

Now the corrected states of linear system and error covariance can be calculated as given below

$$\hat{x}_{k/k} = A\hat{x}_{k/k-1} + K_{\mathrm{mod}}(z_k - H_k\hat{x}_{k/k-1}), \tag{15}$$

$$P_{k/k} = P_{k/k-1}(I - K_{\mathrm{mod}}H_k). \tag{16}$$

4 Stability analysis

In designing of estimator, we have to estimate the state x_k, such that error (e)) will be zero as $k \to 0$ because if $e \to 0$ than estimated state $\hat{x}_k \to x_k$. The dynamics of $\hat{x}_k$ can be written as

$$\dot{\hat{x}}_k = A\hat{x}_k + Bu + K_{\mathrm{mod}}(z_k - H\hat{x}_k). \tag{17}$$

Error $e = x_k - \hat{x}_k$ and its derivative $\dot{e} = \dot{x}_k - \dot{\hat{x}}_k$, on substituting the value of $\dot{x}_k$, $\dot{\hat{x}}_k$ and can be given as

$$\begin{aligned} \dot{e} &= (Ax_k + Bu) - [A\hat{x}_k + Bu + K_{\mathrm{mod}}(z_k - H\hat{x}_k)] \\ &= (A - K_{\mathrm{mod}}H) \dot{} \end{aligned} \tag{18}$$

So equation (18) will be stable only when $(A - K_{\mathrm{mod}}H)$ is Hurwitz matrix. To show that error is asymptotically stable, consider a Lyapunov function

$$V(e) = e'Pe, \tag{19}$$

$$\dot{V}(e) = e'P\dot{e} + \dot{e}'Pe, \tag{20}$$

$$\dot{V}(e) = e'(PA' + AP - 2eK_{\mathrm{mod}}HP)e. \tag{21}$$

According to Lypanouv stability theory, for asymptotic stability $\dot{V}(e) < 0$. Using the following Riccati equation for detail proof sees reference [15]

$$PA' + AP - K_{\mathrm{mod}}HP + S = 0. \tag{22}$$

Then

$$PA' + AP - 2K_{\mathrm{mod}}HP = -S - K_{\mathrm{mod}}HP. \tag{23}$$

After substituting (23) to (21)

$$\dot{V}(e) = -e'(S + K_{\mathrm{mod}}HP)e. \tag{24}$$

From (24), it is clear that error e is asymptotically stable, so the proposed estimator is also stable.

5 Simulation result and discussion

The proposed FOAKF method is applied on Predator-Prey tracking model (1-D), 1-D random motion and 2-D random motion. Tracking of object also depends on motion behavior of the object. For evaluation of proposed method two motion models Constant Velocity (CV) and Constant Turn Rate Velocity (CTRV) model are used.

In order to derive Kalman gain mathematically, mean of noise parameters should be zero and covariances are unity or fixed, but in real scenario noise covariances are variable, we cannot consider it unity for whole process. Later on with the help of MLE technique the measurement noise covariances are estimated, and this is used for calculation of Kalman gain. In this paper when system is linear, CV motion model is used and CTRV is used in nonlinear case. The efficiency of given method is compared with standard KF and FOGKF. All the simulations were performed in MATLAB 2018b.

In Fig.1, simple 1-D predator prey model tracking is present. In this model, a predator tries to catch a flying prey which is moving with constant velocity. From the figure, it is clear that initially when the moving velocity is low than tracking using SKF and proposed method both are good (Nearer to actual position of prey) but as velocity increase, the SKF tracking start diverging but the tracking result of proposed method is still excellent. In Fig. 2 and Fig.3, we find that tracking done by proposed method is far better than various other methods for 1-D

Table 1. RMSE values of various signal

Figure Number	Model	KF	FOGKF	Proposed Method
Fig.1	Predator Prey Model	4.12	-	0.98
Fig.2	1-D CV Motion Model	1.8	1.6	
Fig.3	1-D CRTV Motion Model	0.6	0.5	0.3

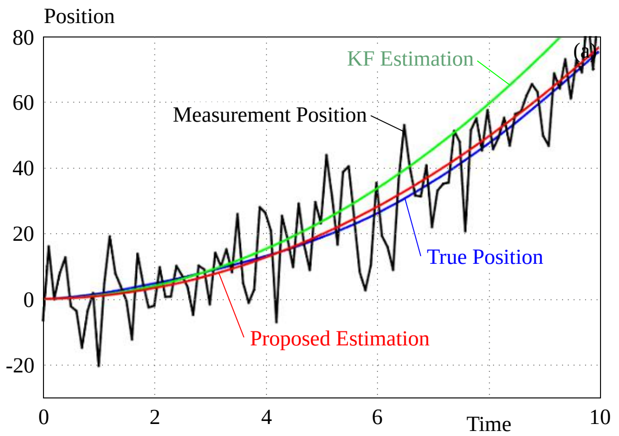

Fig. 1. Predator-prey tracking with various methods using CV motion model

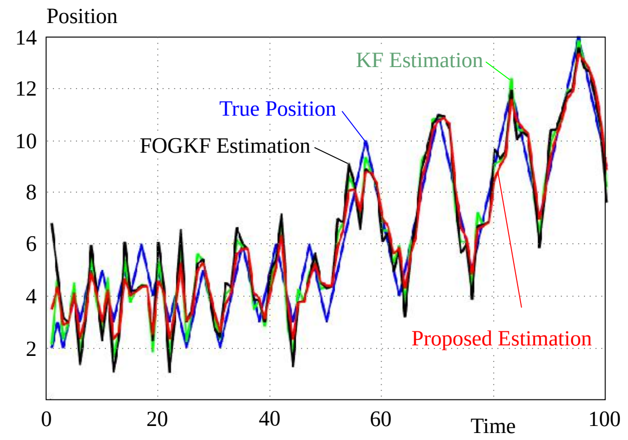

Fig. 2. Random motion 1-d tracking with various methods using CV motion model

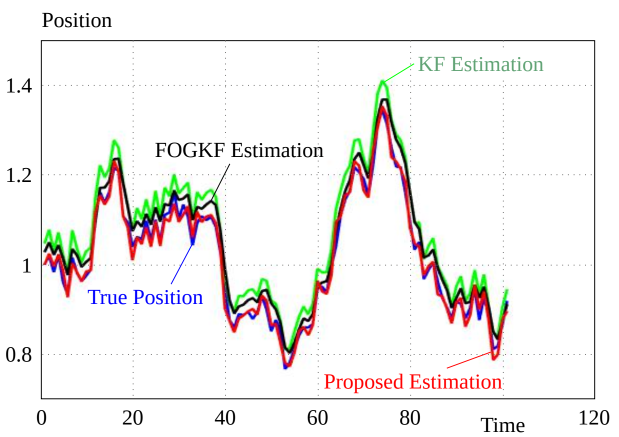

Fig. 3. Random motion 1-d tracking with various methods using CRTV motion model

random motion. The RMSE values of proposed method and others are shown in Table 1.

6 Conclusion

This paper proposes an innovation based fractional order adaptive Kalman filter (IFOAKF) for unknown measurement noise. The proposed method is used for estimating the next states of system. The main contributions of paper are as follows:

- Modification in standard Kalman gain with the help of fractional feedback and MLE
- The importance of MLE based co-variance estimation is that when in data noise co-variances are ill defined or unknown the proposed method first calculates noise covariances than updates it in calculation of Kalman gain and estimation of next state

This algorithm shows high accuracy when R unknown or ill defined. The RMSE of proposed method is improved by 15% in comparison to standard Kalman filter and FOGKF. The proposed method is highly robust in unknown environment, hence the proposed method has better capability to track object in real scenario.

References

[1] M. S. Grewal, A. P. Andrews *Kalman Filtering: Theory and Practice with MATLAB*, John Wiley and Sons, Ltd, 2014.

[2] R. E. Kalman, "A new approach to linear filtering and prediction problems", *ASME, J. Basic Eng.*, vol. 84, no. 1, pp. 35-45, 1960.

[3] A. Yadav, P. Awasthi, N. Naik, and M. R. Ananthasayanam, "A constant gain Kalman filter approach to track maneuvering targets", *2013 IEEE International Conference on Control Applications* (CCA) pp. 562-567, August 2013.

[4] R. Mehra, "Approaches to adaptive filtering", *IEEE Transactions on Automatic Control*, vol. 17, pp. 693–698, October 1972.

[5] A. H. Mohamed and K. P. Schwarz, "Adaptive Kalman filtering for INS/GPS", *Journal of Geodesy*, vol. 73, pp. 193-203, May 1999.

[6] F. Xu, Y. Su, and H. Liu, "Research of optimized adaptive Kalman filtering", *The 26th Chinese Control and Decision Conference* (2014 CCDC), pp. 1210–1214, May 2014.

[7] B. Hongwei, J. Zhihua, and T. Weifeng, "IAE adaptive Kalman filter for INS/GPS integrated navigation system", *Journal of Systems Engineering and Electronics*, vol. 17, pp. 502-508, 2006.

[8] F. Jian-Cheng and Y. Sheng, "Study on innovation adaptive EKF for in-flight alignment of airborne pos", *IEEE Trans- actions on Instrumentation and Measurement*, vol. 60, pp. 1378-1388, April 2011.

[9] W. Gao, J. Li, Y. Zhang, G. Wang, and X. Sun, "Improved innovation-based adaptive estimation for measurement noise uncertainty in SINS/GNSS integration system", *2017 Forum on Cooperative Positioning and Service*, CPGPSiijL, pp. 22-28, May 2017.

[10] S. K. Rao, "Modified gain extended Kalman filter with application to bearings-only passive maneuvering target tracking", *IEE Proceedings - Radar, Sonar and Navigation*, vol. 152, pp. 239-244, August 2005.

[11] S.-K. Weng, C.-M. Kuo, and S.-K. Tu, "Video object tracking using adaptive Kalman filter", *Journal of Visual Communication and Image Representation*, vol. 17, no. 6, pp. 1190-1208, 2006.

[12] Z. Messaoudi, A. Ouldali, and M. Oussalah, "Tracking objects in video sequence using active contour models and unscented Kalman filter", *in International Workshop on Systems, Signal Processing and their Applications*, WOSSPA, pp. 135–138, May 2011.

[13] H. Kaur and J. S. Sahambi, "Vehicle tracking in video using fractional feedback Kalman filter", *IEEE Transactions on Computational Imaging*, vol. 2, pp. 550-561, December 2016.

[14] D. Sierociuk and A. Dzieliadski, "Fractional Kalman filter algorithm for states, parameters and order of fractional system estimation", *International Journal of Applied Mathematics and Computer Science*, pp. 129-140, 2006.

[15] A. Aminifar, "Analysis and Optimization of Embedded Control Systems", *PhD thesis*, Linkau Ping University, 2016.

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