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PAPERS

An improved control strategy based sliding mode approach for high-order systems with mismatched disturbances

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This paper develops an improved design of sliding mode control for high-order systems subjected to matched and mismatched disturbances. Unlike most of the literature implementations, that consider the mismatched disturbances as time vanishing disturbances with a known upper bound; the proposed approach works under time non-vanishing of both, the mismatched disturbances and their time derivatives. Furthermore, these disturbances and their time derivatives are bounded by an unknown constant. In contrast to the classical approaches that search for an approximation to the disturbance and then incorporates it into the controller to stabilise the system, the proposed scheme conducts the system output to achieve asymptotic convergence and this is without the need of any exact estimation of the disturbance. Two simulation examples are provided to illustrate the effectiveness of the proposed approach.

K e y w o r d s: mismatched disturbances, sliding mode control, time non-vanishing, high-order systems

1 Introduction

Many engineering systems are affected by various external disturbances that can destroy the control performances and generate undesirable phenomena. Therefore, and in order to handle the effects of the disturbances, researchers developed the sliding mode control strategy (SMC) [1]. This latter was successfully integrated into the control loop of the manufacturing process, thanks to its high tracking performance and robustness. Nevertheless,the conventional SMC loses its performance under the so-called "mismatched disturbance".

The disturbances can be divided into two categories, matched and mismatched disturbances. The matched disturbance corresponds to the case where the disturbance is acting through the same channels as control inputs. This kind of disturbance is common in a large class of linear and nonlinear systems, such as robots [2–5], and aerial vehicles [6–8]. To overcome the effects of this category of disturbance, many control schemes have been proposed, for instance, H_∞ control [9, 10], and sliding mode control [11, 12].

The mismatched disturbance corresponds to the case where the disturbance and the control inputs appear in different channels. This kind of disturbance can be found in many systems like permanent magnet synchronous motor system [13], MAGLEV suspension system [14], and robotics [15]. Developing a controller able to compensate the effects of mismatched disturbances is a challenge to control theory and for SMC design. One of the promising technique that used to tackle this challenge is the combination between SMC and disturbance observer (DO), such as Chen's observer [16] and Yang's observer [17].

Several control laws have been suggested based on the above-mentioned disturbance observers, such as the works [18–21]. Indeed, in [20], and [21], a SMC based self-learning DO scheme was proposed for second-order MIMO systems, where the mismatched disturbance was estimated using a neuro-fuzzy structure. However, a fundamental assumption has been imposed in the aforementioned schemes, which is that the derivative of the mismatched disturbance must be time-vanishing and satisfies $\lim_{t\to\infty} \dot{d}(t) = 0$. Unfortunately, this assumption is not realistic for many practical systems where generally the disturbances and their derivatives are time non-vanishing. Therefore, the application area of the developed controllers is too limited. To overcome this drawback, a few studies have been proposed in the literature. In [22], a modified SMC surface was combined with Yang's DO (EDO-MSMC). Whereas, a similar scheme was combined with Chen's DO in [23]. In both works, the disturbances and their derivatives were assumed to be bounded by an unknown constant. Nevertheless, the tracking error was proven to be bounded and does not converge asymptotically to zero. Another approach was proposed in [24] based on an adaptive SMC and an extended DO (EDO-ASMC). In this work, the authors demonstrated that the tracking error converges asymptotically to zero. However, the upper bound of the disturbance was assumed to be known, which is not a practical assumption. In [25], the disturbance observer was combined with the backstep-

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ping control in order to deal with the time non-vanishing mismatched disturbance. Nevertheless, the stability proof showed that the tracking error converges to a constant different from zero.

It is worth pointing out that, most of the existing SMC methods failed to drive the tracking error to converge asymptotically to zero under time non-vanishing disturbances, where the aforementioned schemes focus on the disturbance observer performance more than the tracking error performance. Indeed, the tracking error converges to a constant instead of zero in most of the suggested controllers. In this paper, an improved sliding mode control is proposed for nonlinear high-order systems subjected to time non-vanishing matched and mismatched disturbances. The proposed controller uses the idea of the disturbance observer to satisfy the required performances and this is without the need of exact estimation of the disturbances. In fact, the proposed scheme avoids the step of improving the disturbance observer performance, and focuses directly on the tracking error performance. This approach will open a new perspective for systems with mismatched disturbance, where other reasonable and practical assumptions can be suggested. The main contributions of this paper are summarised as: (1) Unlike [17, 20, 21,26–28], where the derivatives of the disturbance were assumed to be time vanishing, the proposed controller can work effectively under time non-vanishing of both, the mismatched disturbances and their derivatives. (2) Unlike [24, 29], the proposed SMC does not require to know the upper bound values of the disturbances and their derivatives, which is more practical. (3) Unlike [14, 22, 23, 25, 30, 31], the system output converges asymptotically to zero. Finally, the simulation of two examples is provided to reveal the feasibility and the efficiency of the proposed approach.

2 Problem statement

Consider the second-order system defined as follows

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t)\,, \\ \dot{x}_2 = a(x) + b(x)u + d_2(t)\,, \\ y = x_1\,, \end{cases} \tag{1}$$

where $[x_1, x_2]^\top \in \mathbb{R}^2$ is the state vector, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the measured output, $d_1(t) \in \mathbb{R}$ is the mismatched disturbance, and $d_2(t) \in \mathbb{R}$ is the matched disturbance. It is assumed that $a(x)$, $b(x)$ are known, smooth functions with $b(x) \neq 0$ for all $t \geq 0$ and in the set of all possible x.

The sliding mode surface used in the proposed controller is defined as follows

$$s = \dot{x}_1 + \lambda x_1\,, \tag{2}$$

where λ is a positive constant gain.

Assumption 1. The following inequality holds

$$|d_2| + |\dot{d}_1| + |\lambda d_1| \leq d_{\max}, \tag{3}$$

in which $d_{\max}$ is an unknown constant.

Remark 1. Assumption 1 indicates that the mismatched disturbance and its time-derivative are bounded by a constant that is not required to be known. This assumption is much milder than bounding the disturbance by a known constant or/and assuming its time-derivative equals zero as in [17, 26].

Our object is to design a control law able to drive the system output to zero, despite the influence of the matched and mismatched disturbances.

2.1 Control design

To control second-order nonlinear systems subjected to matched and mismatched disturbances, a SMC law is proposed as follows

$$u = -b^{-1}(x)\big(a(x) + \lambda x_2 + Ks + \hat{d}\,\mathrm{sign}(s)\big), \tag{4}$$

$$\dot{\hat{d}} = \Gamma\,\mathrm{sign}(s)s\,, \tag{5}$$

where the gains K and Γ are positive constants.

Remark 2. The control gains of eqs. (1) and (2) are independent from the upper bounds of the matched and mismatched disturbances which made the implementation much easier and more practical.

2.2 Convergence analysis

Theorem 1. Consider the system described by (1) under Assumptions 1. If the control law given by (4) and (5) is applied, then the system output converges asymptotically to zero, *ie* $\lim_{t\to\infty} y = 0$ and the states remain bounded.

P r o o f . Define a Lyapunov function candidate as

$$V(s, \tilde{d}) = \frac{1}{2}\Gamma s^2 + \frac{1}{2}\tilde{d}^2, \tag{6}$$

$$\tilde{d} = d_{\max} - \hat{d}. \tag{7}$$

The time-derivative of the Lyapunov function gives us

$$\dot{V} = \Gamma s(\dot{x}_2 + \dot{d}_1 + \lambda\dot{x}_1) - \tilde{d}\,\dot{\hat{d}}\,. \tag{8}$$

Therefore

$$\dot{V} = \Gamma s\big(a(x) + b(x)u + d_2 + \dot{d}_1 + \lambda x_2 + \lambda d_1\big) - \tilde{d}\,\dot{\hat{d}}\,. \tag{9}$$

By using Assumption 1, we can get

$$\dot{V} \leq \Gamma s\big(a(x) + b(x)u + \lambda x_2 + d_{max}\,\mathrm{sign}(s)\big) - \tilde{d}\,\dot{\hat{d}}\,. \tag{10}$$

Inserting the control law (4) into (10) yields

$$\dot{V} \leq \Gamma s\big(-Ks + (d_{max} - \hat{d})\,\mathrm{sign}(s)\big) - \tilde{d}\,\dot{\hat{d}}\,. \tag{11}$$

Using (5) leads to the following inequality

$$\dot{V} \leq -\Gamma K s^2. \tag{12}$$

Consequently, $\dot{V}$ is negative semi-definite, therefore, is upper bounded. Then, based on the definition of V, we conclude that s and $\tilde{d}$ are bounded.

Next, let us check the uniform continuity of V. The derivative of $\dot{V}$, using (11) is given by

$$\ddot{V} = -2\Gamma K s\dot{s}\,. \tag{13}$$

$$\dot{s} = -Ks + \tilde{d}\,\mathrm{sign}(s)\,. \tag{14}$$

Hence, $\dot{s}$ is bounded.

As a result, we can conclude that $\ddot{V}$ is also bounded.

Since V is lower bounded, $\dot{V}$ is negative semi-definite and $\ddot{V}$ is bounded (*ie* $\dot{V}$ is uniformly continuous), Barbalat's Lemma [32] allows to state

$$\lim_{t\to\infty} s = 0\,; \qquad \lim_{t\to\infty} \dot{x}_1 + \lambda x_1 = 0\,. \tag{15,16}$$

Finally, we can conclude the following

$$\lim_{t\to\infty} x_1 = 0\,. \tag{17}$$

Moreover, from (5), we have $\lim_{t\to\infty} \dot{\hat{d}} = 0$, thus $\hat{d}$ is bounded and converges to a constant.

Remark 3. This work focuses on controller design, where it is concluded that the system output converges asymptotically to zero and this is without the need of any exact estimation of the disturbance. In fact, most of the present schemes are designed to improve the performance of the disturbance observer. As a consequence, the tracking error performance will depend on the disturbance observer performance and the reasonability of its assumptions. In contrast, the proposed control scheme surpasses the step of designing a suitable disturbance observer, and focuses directly on improving the performance of tracking error. This method allows imposing more reasonable and realistic assumptions.

3 High order systems

This section generalises results obtained from second-order to an *nth* order system described as follows

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t)\,, \\ \dot{x}_2 = x_3 + d_2(t)\,, \\ \vdots \quad \vdots \quad \vdots \\ \dot{x}_{n-1} = x_n + d_{n-1}(t)\,, \\ \dot{x}_n = a(x) + b(x)u + d_n(t)\,, \\ y = x_1\,, \end{cases} \tag{18}$$

where $d_1(t), \ldots, d_{n-1}(t)$ are the mismatched disturbances and $d_n(t)$ is the matched disturbance.

The sliding mode surface for the nth order system is defined as follows

$$s = x_1^{(n-1)} + c_{n-1}x_1^{(n-2)} + \cdots + c_2\dot{x}_1 + c_1x_1 = \sum_{i=1}^{n} c_i x_1^{(i-1)}, \quad \text{with } c_n = 1\,. \tag{19}$$

In which, the gains $c_1, \ldots, c_n$ are selected to be positive such that the polynomial $P(s) = s^{n-1} + c_{n-1}s^{n-2} + c_{n-2}s^{n-3} + \cdots + c_1$ is Hurwitz.

Assumption 2. The disturbances satisfy the following inequality

$$\sum_{i=1}^{n-2}\sum_{j=1}^{n-1-i} c_{n-i}|d_{n-i+j}^{(j)}| + \sum_{i=1}^{n} c_i|d_i| \leq d_{\mathrm{Max}},$$

where d_{Max} is an unknown constant.

Remark 4. Similar to the case of second-order system, the matched/mismatched disturbances and their time-derivatives are bounded by an unknown constant. This assumption is practical and can be easily satisfied.

3.1 Control Design

The proposed controller designed for high-order systems under mismatched disturbances is given as follows

$$u = -b^{-1}(x)(a(x) + \sum_{i=1}^{n-1} c_i x_{i+1} + \hat{d}\,\mathrm{sign}(s) + Ks)\,, \tag{20}$$

$$\dot{\hat{d}} \;= \Gamma\, s\,\mathrm{sign}(s), \tag{21}$$

where the gains K and Γ are positive constants.

3.2 Convergence analysis

Theorem 2.. *Consider the high-order system described by* (18) *under Assumption 2. If the control law given by* (20) *and* (21) *is applied, then the system output converges asymptotically to zero, ie* $\lim_{t\to\infty} y = 0$.

P r o o f . We define a Lyapunov function as

$$V(s,\tilde{d}) = \frac{1}{2}\Gamma s^2 + \frac{1}{2}\tilde{d}^2, \tag{22}$$

$$\tilde{d} = d_{\mathrm{Max}} - \hat{d}. \tag{23}$$

By differentiating $V(s,\tilde{d})$ with respect to time, we can obtain

$$\dot{V} = \Gamma s\big(x_1^{(n)} + c_{n-1}x_1^{(n-1)} + \cdots + c_2\ddot{x}_1 + c_1\dot{x}_1\big) - \tilde{d}\,\dot{\hat{d}}\,. \tag{24}$$

Thus

$$\dot{V} = \Gamma s\big(a(x) + b(x)u + \sum_{i=1}^{n-1} c_i x_{i+1} + \sum_{i=1}^{n-2}\sum_{j=1}^{n-1-i} c_{n-i} d_{n-i+j}^{(j)} + \sum_{i=1}^{n} c_i d_i\big) - \tilde{d}\,\dot{\hat{d}}\,. \quad (25)$$

Based on Assumption 2, we can get

$$\dot{V} \leq \Gamma s(a(x) + b(x)u + \sum_{i=1}^{n-1} c_i x_{i+1} + d_{\mathrm{Max}}\,\mathrm{sign}(s)) - \tilde{d}\dot{\hat{d}}\,. \quad (26)$$

Applying the control law (20), gives us

$$\dot{V} \leq \Gamma s\big(-Ks + (d_{\mathrm{Max}} - \hat{d}\,)\,\mathrm{sign}(s)\big) - \tilde{d}\dot{\hat{d}}\,. \quad (27)$$

Using (21) leads to

$$\dot{V} \leq -\Gamma K s^2. \quad (28)$$

Thus, from (28) and according to Barbalat's lemma we obtain

$$\lim_{t\to\infty} s = 0\,. \quad (29)$$

Therefore, (29) yields

$$\lim_{t\to\infty} x_1 = 0\,. \quad (30)$$

Moreover, from (21) we have $\lim_{t\to\infty} \dot{\hat{d}} = 0$, thus $\hat{d}$ is bounded and converges to a constant.

Remark 5. It is worth noting that the sign function may produce some chattering due to its discontinuity. Therefore, smooth functions like saturation function or tangent hyperbolic can be used instead of the sign function.

4 Simulation results

4.1 *Example 1*

Consider the following second-order system described by:

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t)\,, \\ \dot{x}_2 = -x_1 - x_2 + x_2^2 cos(x_1) + u + d_2(t)\,, \\ y = x_1\,, \end{cases} \quad (31)$$

where x_1 and x_2 are the states, u is the control input, $d_1(t)$ is mismatched disturbance, and $d_2(t)$ is the matched disturbance.

To examine the effectiveness of the proposed method in a comparative manner, simulation was also performed for the EDO-MSMC controller [22], and the EDO-ASMC controller [24]. Both schemes are among the few works that assume the boundedness of the mismatched disturbance and its time-derivative. The controller's gains were tuned using the trial and error method until a good and satisfactory tracking performance were obtained.

- The EDO-MSMC controller is designed as follows

$$u = -b(x)^{-1}\big[a(x) + c(x_2 + \hat{d}_1) + \alpha(s)(0)e^{-\alpha t} + k_1 s^* + k_2\,\mathrm{sign}(s^*)\big], \quad (32)$$

with $s^* = s - s(0)e^{-\alpha t}$, and $s = x_2 + cx_1 + \hat{d}_1$.

- The EDO is defined as follows

$$\begin{aligned} \hat{d}_1 &= p_{11} + l_{11}x_1\,, \\ \dot{p}_{11} &= -l_{11}(x_2 + \hat{d}_1) + \dot{\hat{d}}_1\,, \\ \hat{d}_1 &= p_{12} + l_{12}x_1\,, \\ \hat{p}_{12} &= -l_{12}(x_2 + \hat{d}_1)\,, \end{aligned} \quad (33)$$

where, c, k_1, k_2, l_{11}, l_{12}, and α are constants positive gains.

- The EDO-ASMC controller is expressed as follows

$$u = -b(x)^{-1}\big[a(x) + \bar{a}(x) + \tau(\bar{s}, c_2, x_2) + \dot{\hat{d}}_2 + k_1\bar{s}\big], \quad (34)$$

in which, $|\bar{s}| = x_2 + \tau(x_1, c_1, k_1) + \hat{d}_2$, $\tau(\bar{s}, c_2, x_2) = \frac{c_2\bar{s}}{|\bar{s}| + k_2^2\delta_2}$, $\bar{a}(x) = \frac{c_1k_1\delta_1(k_1x_2 + k_1\hat{d}_2 - 2x_1\dot{k}_1)}{(|x_1| + k_1^2\delta_1)^2}$, with $\dot{k}_1 = -\frac{c_1\gamma_1\delta_1|\bar{x}|k_1}{|\bar{x}_1| + k_1^2\delta_1}$, and $\dot{k}_2 = -\frac{c_2\gamma_2\delta_2|\bar{s}|k_2}{|\bar{s}| + k_2^2\delta_2}$.

The gains $c_1, c_2, \delta_1, \delta_2, \gamma_1$, and γ_2 are positive constants, and the EDO is defined by (33). The simulation study is given in two cases.

Case 1. The matched and mismatched disturbances are imposed at $t = 15$ s, as constants: $d_1 = d_2 = 0.4$. The initial state value is set to $x(0) = [0.5, 0]$. The gains of the controllers are illustrated in Table 1.

Figure 1 depicts the time response of the output for the EDO-MSMC, the EDO-ASMC and the proposed controller. It can be observed that the three methods achieve good convergence performances under constant disturbances. Indeed, these schemes were designed to deal with

Table 1. Control parameters for Example 1

Controller	Parameters	Value
Proposed scheme	λ, K, Γ	$15, 2, 10^7$
EDO-MSMC Scheme	$c, k_1, k_2, \alpha, l_{11}, l_{12}$	$10, 1000, 40, 340, 1900, 400$
EDO-ASMC scheme	$k_1(0), k_2(0), c_1, c_2, \delta_1$	$1.28, 1, 3.5, 2, 0.13$
	$\delta_2, \gamma_1, \gamma_2, l_{11}, l_{12}$	$0.15, 3, 1, 1900, 400$

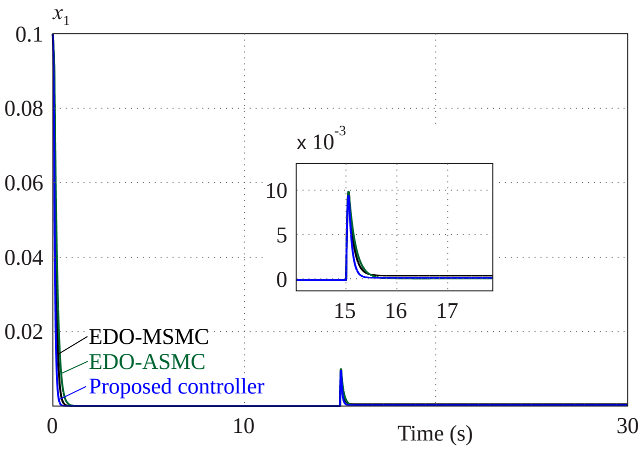

Fig. 1. Output response in the presence of constant disturbance

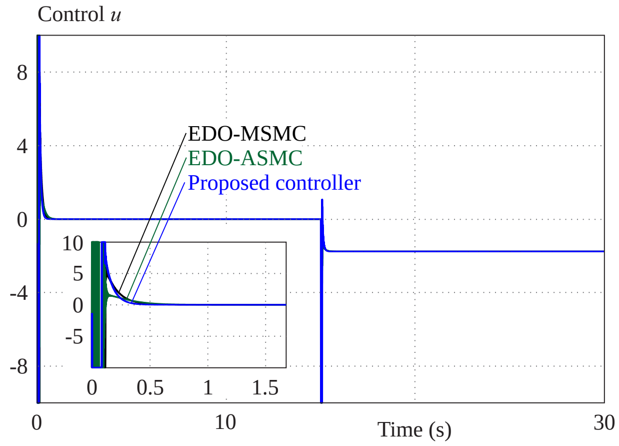

Fig. 2. Control input in the presence of constant disturbance

Table 2. Control parameters for example 2

Controller	Parameters	Value
Proposed scheme	c_1, c_2, c_3, K, Γ	4096000, 76800, 480, 10, 26
EDO-MSMC Scheme	$c_1, c_2, c_3, k_1, k_2,$ $\alpha, l_{21}, l_{22}, l_{41}, l_{42}$	4096000, 76800, 480, 100, 10, 2, 1000, 200, 1000, 200

time non-vanishing disturbances. It can also be seen that at the time-act of the disturbances (t=15 s), the proposed controller has the fastest time response. Figure 2 illustrates the control input of the three schemes. We can see that the proposed method and the EDO-MSMC have similar and smooth control signal. Whereas, the control signal of the EDO-ASMC contains chatters.

Case 2. In this case, the proposed controller performances are evaluated under a time non-vanishing disturbances which are imposed at $t = 15$ s, as follows: $d_1(t) = d_2(t) = 0.15\big(\sin(t) + \sin(2t)\big)$.

Figures 3 and 4 present the time response under a time non-vanishing disturbances of the output and input respectively. It can be seen that the proposed controller can handle effectively the non-vanishing matched/mismatched disturbances. Moreover, the proposed method has better tracking performances with smoother control signal compared to the EDO-MSMC controller, the EDO-ASMC controller.

4.2 Example 2

To evaluate the efficiency of the proposed controller at high-order systems, a flexible joint manipulator is considered in this example.

The state space model of this manipulator is expressed as follows

$$\begin{cases} \dot{x}_1 = x_2 \,, \\ \dot{x}_2 = \frac{ks1}{J11}x_3' - \frac{ks1}{J11}x_1 - \frac{B11}{J11}x_2 - \sin(x_1) \\ \quad \pm\Delta\frac{ks1}{J11}x_1 \pm \Delta\frac{ks1}{J11}x_3' \pm \Delta\frac{B11}{J11}x_2 \,, \\ \dot{x}_3' = x_4' \,, \\ \dot{x}_4' = \frac{ks1}{J12}x_1 - \frac{ks1}{J12}x_3' - \frac{B12}{J11}x_4' \\ \quad +\frac{Kt1}{J11}(u+v) \pm \Delta\frac{ks1}{J12}x_1 \\ \quad \pm\Delta\frac{ks1}{J12}x_3' \pm \Delta\frac{B12}{J11}x_4 \pm \Delta\frac{Kt1}{J11}(u+v) \,, \\ y = x_1 \,, \end{cases} \tag{35}$$

in which the states $[x_1, x_2, x_3', x_4']$ are the link position, the link velocity, the motor shaft position, and the motor shaft velocity respectively. Whereas, u is the control input, v is the external disturbance, and Δ represents the uncertainty. For the detailed description and values of the parameters $ks1$, $J11$, $J12$, $B11$, $B12$, and $Kt1$, please, refer to [22].

The model (35) can be rewritten in the following simple form:

$$\begin{cases} \dot{x}_1 = x_2 \,, \\ \dot{x}_2 = x_3 + d_2(t) \,, \\ \dot{x}_3 = x_4 \,, \\ \dot{x}_4 = a(x) + b(x)u + d_4(t) \,, \\ y = x_1 \,, \end{cases} \tag{36}$$

where $a = k_1k_2x_1 - k_2x_3 - b_2x_4$, $b(x) = k_1\frac{Kt1}{J11}$, $k_1 = \frac{ks1}{J11}$, $k_2 = \frac{ks1}{J12}$, $b_1 = \frac{B11}{J11}$, $b_2 = \frac{B11}{J12}$, $x_3 = k_1x_3'$, $x_4 = k_1x_4'$. The lumped uncertainties are $d_2(t) = -k_1x_1 - b_1x_2 - \sin(x_1) \pm \Delta k_1x_1 \pm \Delta x_3 \pm \Delta b_1x_2$, $d_4(t) = \pm\Delta k_1k_2x_1 \pm \Delta k_2x_3 \pm \Delta b_2x_4 \pm \Delta b(x)u + (b(x) + \Delta b(x))v$.

The proposed controller is compared with the EDO-ASMC, which shows its efficiency in the previous example compared to the EDO-MSMC controller. The simulation is given in two cases.

Case 1 The disturbance is considered constant throughout the process as, $v = 0.8$ and $\Delta = 0.2$. The initial

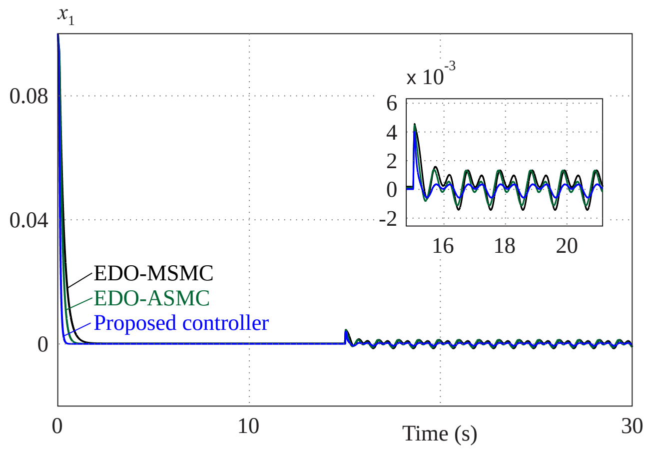

Fig. 3. Response in the presence of time non-vanishing disturbance

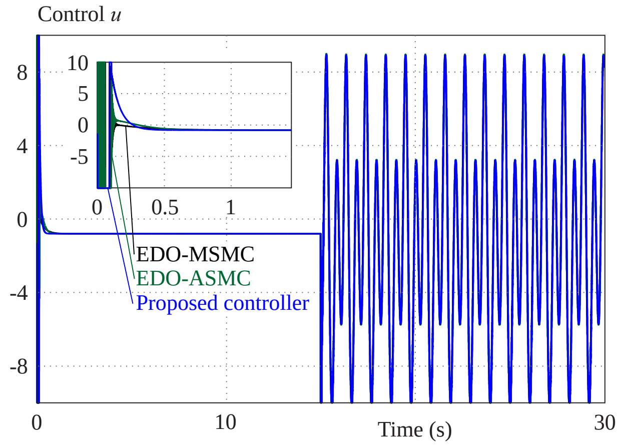

Fig. 4. Control input in the presence of time non-vanishing disturbance

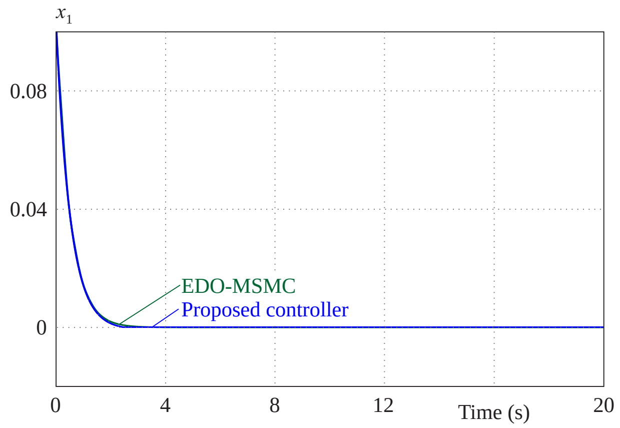

Fig. 5. Response in the presence of constant disturbance

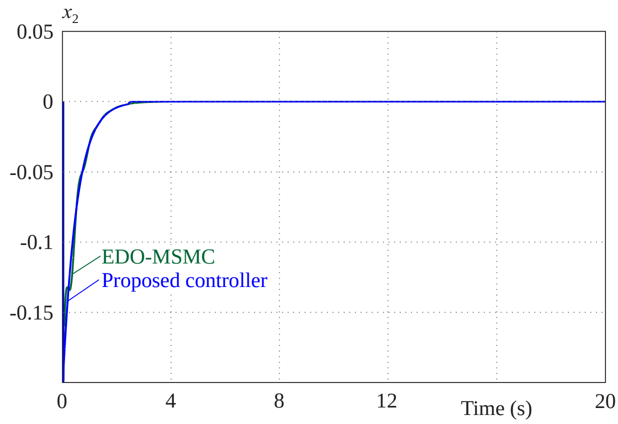

Fig. 6. Evolutions of the state x_2 in the presence of constant disturbance

state value is set to $x(0) = [0.5, 0]$. The gains of the controllers are listed in Table 2.

Figures 5, 6, and 7 depict the time response of the state x_1, x_2 and the input control under the proposed method and the EDO-MSMC method respectively. It is observed that under a constant disturbance the controllers have similar performances, where the states converge asymptotically to zero with a smooth control signal and without any overshoot.

Case 2. In this case, a time non-vanishing disturbance is imposed throughout the process as follows: $v(t) = 0.3\big(1 + 2\cos(16t) + \sin(12t)\big)$.

Figure 8 shows the time response of the output x_1 for the EDO-MSMC and the proposed approach. One can see that under complex mismatched disturbance the EDO-MSMC scheme lost its performance. Indeed, the output kept in a range and did not converge to zero. Whereas, the proposed controller handles the disturbances effectively, where it leads the output to zero at every time a significant disturbance has occurred. Figures 9. and 10 present the time response of the state x_2 and the input control respectively. It is shown that the state has better convergence performances under the proposed scheme compared to the EDO-MSMC. Meanwhile, it is observed that the EDO-MSMC control signal contains many peaks with a larger magnitude than the proposed controller.

Finally, according to these simulation results, the EDO-MSMC, the EDO-ASMC, and the proposed controller can suppress matched/mismatched constant disturbances and provide good convergence performances. However, under non-vanishing mismatched disturbances the EDO-MSMC and the EDO-ASMC exhibit poor convergence performances, where the output stays bounded and does not converge to zero, which coincides with the conclusion of [23]. Moreover, finding the upper bound of the disturbances presents another practical challenge for the EDO-MSMC method. In the contrary, the proposed control law achieves the best convergence performance with a smooth control signal compared to the above-mentioned approaches.

5 Conclusion

In this paper, an improved control strategy based on sliding mode control has been developed for a class of high-order systems subjected to matched and mismatched disturbances. The system output has been

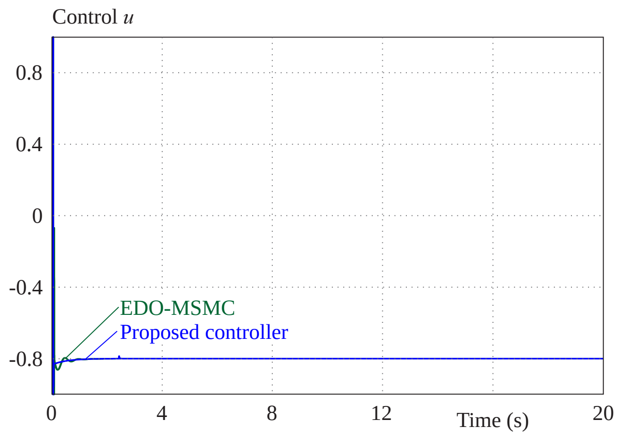

Fig. 7. Control input in the presence of constant disturbance

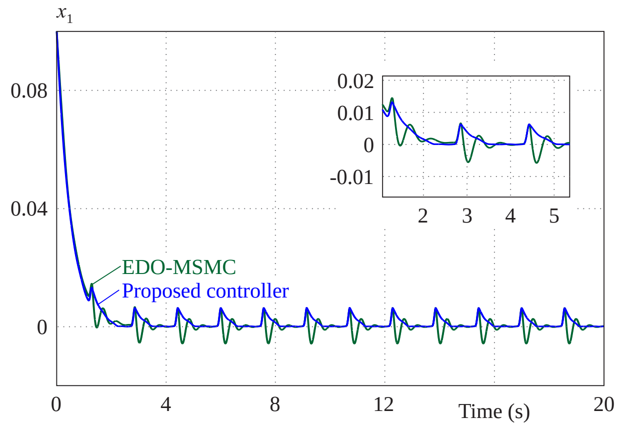

Fig. 8. Response in the presence of time non-vanishing disturbance

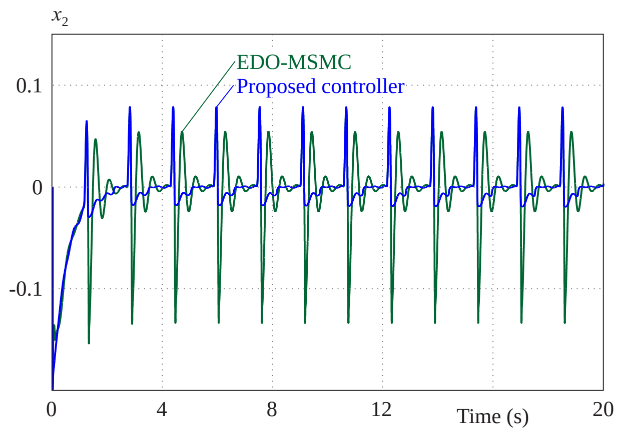

Fig. 9. Evolutions of the state x_2 in the presence of time non-vanishing disturbance

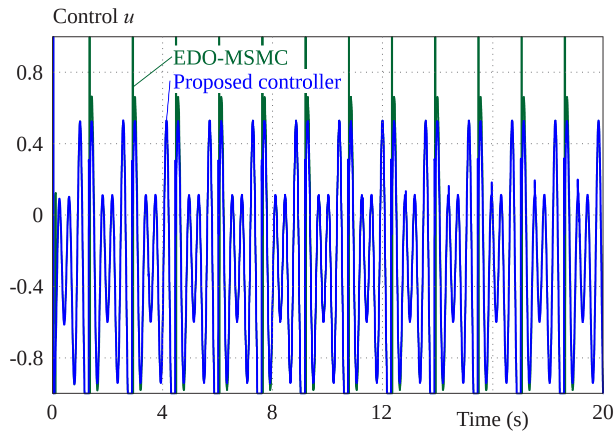

Fig. 10. Control input in the presence of time non-vanishing disturbance

proven to converge asymptotically to zero using Lyapunov analysis. Simulation results have shown the efficiency of the proposed controller over the EDO-MSMC, and EDO-ASMC controllers, where better convergence performances have been achieved with a smooth control signal. The proposed scheme does not require any a priori knowledge on the disturbances, nor their upper bounds which makes it highly suitable for industrial applications.

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