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2-D polar curves cloak structures for microwave applications

Muhammad Raza[1*], Oleg Rybin[2], Najma Abdul Rehman,[1] Reeha Shahid[1]

This study investigates the design of electromagnetic devices, specifically wave rotators and cloaks, utilizing a polar rose-shaped structure. By applying the principles of transformation optics, the constitutive parameters – permittivity and permeability tensors – are analytically derived for transforming virtual space in cylindrical coordinates to physical space in Cartesian coordinates. Both integer and fractional indices of the rose configurations are considered, each yielding distinct electromagnetic properties. The analytical findings are validated and supported by numerical simulations conducted using COMSOL Multiphysics. The results highlight the tunability and adaptability of the proposed structures in controlling electromagnetic wave propagation, exhibiting rotational dynamics, cloaking, and absorption phenomena. Potential applications of these rose-shaped structures are briefly discussed.

Keywords: wave propagation, transformation optics, invisible cloak, rose graph, integrated circuits

1 Introduction

The concept of transformation optics traces its roots back to the early 1920s when researchers observed that the gravitational deflection of sunlight around the sun could be explained as the reflection within a suitable dielectric medium [1]. However, it was not until the early 1960s that Dolin, Post, and Lax-Nelso unveiled the foundational principles of transformation optics [2-4]. Unfortunately, these insights were lost for several decades, only resurfacing in the mid-1990s.

Building on this rediscovery, Gordon delved into further exploration of dielectric media and space-time, simulating the latter through an effective 'optical' metric in a hypothetical space [5]. This innovative approach allowed the application of Lagrangian mechanics, traditionally associated with vacuum space-times, to dielectrics. Subsequent research, detailed in [6], discussed the behavior of light in curved space-times. Plebanski extended these ideas, enabling the depiction of elementary gravitational systems as dielectric media within a flat space-time context [7]. Consequently, the groundwork for transformation optics was solidified, directly stemming from these earlier advancements, and culminating in the first formalization of the transformation optics/electromagnetics concept [8, 9]. Although, a new surge of interest in the development of transformation optics/electromagnetics techniques is associated with the explosion of research on metamaterials/metamagnetics [10, 11]. This is because it is practically convenient to use metamaterials/metasurfaces to achieve the cloaking-invisibility phenomenon.

The last two decades were marked by an upsurge in the use of these concepts and their modifications to solve some particular problems of practical importance [12-16]. This allows us to say that this stage of development of theoretical knowledge in the field of the transformation optics currently leads us to the breakthrough of technologies in modern optics. Indeed, recent advances in solving particular problems in mathematical physics using transformation optics approaches have led to the emergence of new solutions for appliance and device designs [17-19]. At the same time, most of these solutions are related to optical range, while the lack of the innovative solutions for the microwave range is becoming more obvious excluding the topics of lenses and antennas [14, 20]. This is especially true for the development of invisibility technologies at microwave frequencies although the broad demand for such research is quite obvious [21].

In this study, we create the theoretical background cloaks with a rose-shaped structures operating in microwave frequency range. The analytic theory is confirmed and supplemented by using the commercial simulation tool COMSOL, that process provides a comprehensive understanding the problem at hand and point the way to understanding its practical value.

2 Main relations

To engineer the material properties of optical devices, employing the cloak invisibility, a transformation of Maxwell's equations and the constitutive relations in initial (virtual) coordinates to equivalent

[1]Department of Mathematics, COMSATS University Islamabad, Sahiwal Campus, Sahiwal, Pakistan
[2]School of Radio Physics Biomedical Electronics & Computer Systems, V.N. Karazin Kharkiv National University, Kharkiv, Ukraine
*E-mail: mraza@cuisahiwal.edu.pk

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(physical) coordinates (using bar notation) brings to the relations [22]:

$$\left.\varepsilon' = \frac{A\varepsilon A^t}{det(A)}, \quad \mu' = \frac{A\mu A^t}{det(A)},\right\} \tag{1}$$

where ε is the (absolute) permittivity tensor, μ is the (absolute) permeability tensor, A is the transformation matrix from virtual coordinates to physical coordinates defined as follows:

$$A = \tilde{g}'^{\frac{1}{2}} \cdot Jac \cdot \tilde{g}^{-\frac{1}{2}}, \tag{2}$$

where Jac is the Jacobian matrix for transitioning between the both polar coordinate systems, $\tilde{g} = [diag(h_1, h_2, h_3)]^2$ is metric tensor in physical polar coordinate system, h_i $(i = 1,2,3)$ are the scale factors.

Using Eqns. (1) leads to spatial variability and anisotropy, creating a cloaking that, combined with the hidden volume, looks like open space when viewed from the outside.

In this study, we theoretically create and characterize a two-dimensional (2-D) a microwave cloaking for the rose-like geometrical device by means of derivation of the appropriate expression for the permittivity and permeability tensors in physical coordinate system. Since it is convenient to represent the rose graph in polar coordinates, the physical coordinate system is also a polar one. However, in order to further validate and extend the results of analytic modelling, numerical analysis by using the commercial Multiphysics simulation tool COMSOL will carry out. That is why, the final result will be presented in Cartesian coordinates. In this way, the final result is to be obtained in terms the so-called auxiliary matrix U [23]:

$$\left.\varepsilon_{car} = \frac{U\varepsilon' U^t}{det\, U}, \quad \mu_{car} = \frac{U\mu' U^t}{det\, U},\right\} \tag{3}$$

where ε_{car} and μ_{car} are the constitutive parameters in the physical Cartesian coordinates.

3 Mathematical modeling

It is known well that the rose graph's polar equation is defined as follows:

$$r = B\cos(n\alpha), \qquad 0 \le \alpha < 2\pi \tag{4}$$

where B is the rose amplitude which influences their length of the rose petals, n $(1 < n \in \mathbb{R})$ is the parameter determines the number of petals. In particular, this study will consider rose with integer $(n = 3,4,8)$ and fractional indices $(n = \frac{1}{3}, \frac{1}{2})$.

Consider a mapping between virtual and physical polar coordinates in the form: $r = r(r', \alpha')$, $\alpha = \alpha'$, $z = z'$. The appropriate expression for the mapping function is given by:

$$r = C\frac{r' - B\cos(n\alpha)}{C - B\cos(n\alpha)}, \tag{5}$$

where C is the outer boundary's radius while B is defined above in Eq. (4). However, B is also the length of the petals given by:

$$r' = \frac{C - B\cos(n\alpha)}{C} r + B\cos(n\alpha).$$

Jacobian matrix Jac is given by:

$$Jac = \begin{bmatrix} \frac{C - B\cos(n\alpha)}{C} & \left(\frac{r - C}{C}\right) B\sin(n\alpha) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The metric tensors $\tilde{g}$ and $\tilde{g}'$ can be computed easily in the form:

$$\left.\tilde{g}^{\frac{-1}{2}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \tilde{g}'^{\frac{1}{2}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r' & 0 \\ 0 & 0 & 1 \end{bmatrix}.\right\} \tag{7}$$

Substituting Eq. (6) and Eqs. (7) into Eq. (2) finally gives:

$$A = \begin{bmatrix} \frac{C - B\cos(n\alpha)}{C} & \frac{r - C}{C} Bn\sin(n\alpha) & 0 \\ 0 & \frac{r'}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

It is already known that the tensors of dielectric permittivity and permeability in physical Cartesian coordinates satisfy the double equality:

$$\varepsilon' = \mu' = \frac{AA^t}{det(A)} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{9}$$

Substituting Eq. (8) into last equality of Eqs. (9) finally gives the effective material parameters essential for achieving desired cloaking effects as specified by the structure's geometry:

$$\left.\begin{aligned}
&a_{11} = \frac{r' - A\cos(n\alpha')}{r'} \\
&\left(1 + \left(\frac{(r' - C)An\sin(n\alpha)}{(C - A\cos(n\alpha'))(r' - A\cos(n\alpha'))}\right)^2\right), \\
&a_{12} = a_{21} = \frac{(r' - C)An\sin(n\alpha)}{(C - A\cos(n\alpha'))(r' - A\cos(n\alpha'))}, \\
&a_{22} = \frac{r'}{r' - A\cos(n\alpha')}, \\
&a_{33} = \frac{r' - A\cos(n\alpha')}{r'}\left(\frac{C}{(C - A\cos(n\alpha'))}\right)^2, \\
&a_{13} = a_{31} = a_{23} = a_{32} = 0.
\end{aligned}\right\} \quad (10)$$

One can see from Eqs. (10) that the permittivity and permeability tensors (ε' and μ', correspondingly) in physical polar coordinates and are symmetric matrices which is a distinctive property of these matrices in the case of cloaking. As far as mentioned before, the permittivity and permeability tensors in polar/cylindrical coordinates cannot yet be directly entered into COMSOL Multiphysics software for simulation. To convert the above permittivity and permeability tensors into Cartesian coordinates, the calculation of the auxiliary matrix U is to be made. The results of the calculation are as follows:

$$U = \begin{bmatrix} \frac{\partial x}{\partial x'} & \frac{\partial x}{\partial y'} & \frac{\partial x}{\partial z'} \\ \frac{\partial y}{\partial x'} & \frac{\partial y}{\partial y'} & \frac{\partial y}{\partial z'} \\ \frac{\partial z}{\partial x'} & \frac{\partial z}{\partial y'} & \frac{\partial z}{\partial z'} \end{bmatrix}. \quad (11)$$

Taking into account that the rotation by angle α' is counterclockwise (positive direction) about the origin, the functional form is defined as follows:

$$\left.\begin{aligned}
&x = x'\cos\alpha' - y'\sin\alpha', \\
&y = y'\cos\alpha' + x'\sin\alpha', \\
&z = z'.
\end{aligned}\right\} \quad (12)$$

Substituting Eqs. (12) into Eq. (11) gives:

$$U = \begin{bmatrix} \cos\gamma' & -\sin\gamma' & 0 \\ -\sin\gamma' & \cos\gamma' & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (13)$$

Substituting Eq. (13) into Eqs. (3) finally gives permittivity and permeability tensors as:

$$\left.\begin{aligned}
&\varepsilon'_{car} = \mu'_{car} = \begin{bmatrix} \varepsilon'_{r'r'} & \varepsilon'_{r'\gamma'} & \varepsilon'_{r'z'} \\ \varepsilon'_{\gamma'r'} & \varepsilon'_{\gamma'\gamma'} & \varepsilon'_{\gamma'z'} \\ \varepsilon'_{z'r'} & \varepsilon'_{z'\gamma'} & \varepsilon'_{z'z'} \end{bmatrix} \\
&= \begin{bmatrix} \mu'_{r'r'} & \mu'_{r'\gamma'} & \mu'_{r'z'} \\ \mu'_{\gamma'r'} & \mu'_{\gamma'\gamma'} & \mu'_{\gamma'z'} \\ \mu'_{z'r'} & \mu'_{z'\gamma'} & \mu'_{z'z'} \end{bmatrix}, \\
&\varepsilon'_{r'r'} = \mu'_{r'r'} = a_{11}\cos^2\gamma' - 2a_{12}\cos\gamma' sin\gamma' + a_{22}\sin^2\gamma', \\
&\varepsilon'_{r'\gamma'} = \varepsilon'_{\gamma'r'} = \mu'_{\gamma'r'} = \mu'_{r'\gamma'} \\
&= a_{12}(\cos^2\gamma' - \sin^2\gamma') + (a_{11} - a_{22})\cos\gamma'\sin\gamma', \\
&\varepsilon'_{\gamma'\gamma'} = \mu'_{\gamma'\gamma'} = a_{11}\cos^2\gamma' + 2a_{12}\cos\gamma'\sin\gamma' + a_{22}\sin^2\gamma', \\
&\varepsilon'_{z'z'} = \mu'_{z'z'} = a_{33}.
\end{aligned}\right\} \quad (14)$$

Equations (14) will be used in the next paragraph to carry out numerical analysis using COMSOL Multiphysics software.

4 Rotational dynamics and cloaking of polar rose structures

In this study, two-dimensional (2-D) simulations are performed for initial wave in the form of a TE-polarized plane EM wave. The simulation simply needs to account for an axial component of permeability and a transverse component of permittivity presented by Eqs. (4) and solves the 2-D cylindrical problem, to which we apply the transformation, in which a perfect electric conductor rose wrapped in an outer shell. The goal is to cause the strong electromagnetic energy reflectors that make up the petals of a perfect electric conductor to scatter electromagnetic energy in all directions thus demonstrating the effect of cloak invisibility. A sheet of uniform current density close to the domain's left edge generates a uniform plane wave. The top and bottom boundaries are perfect magnetic conductors. The initial wave has electric field vector pointing outward and wave vector pointing perpendicular to the plane of the roses.

4.1 Rose structure with integer indices

Consider the cases when $n = 2,\ 3,\ 4$ in Eq. (4). The geometries of problem are depicted in Fig. 1. In the presented geometric configurations, the graph parameters are generated to simulate the polar rose structures with specific dimensions: $B = 0.01$ m, $C = 0.015$ m for case (a), same for case (b), $B = 0.0054$ m, $C = 0.015$ m for case (c); as well as orientation: $\alpha = \pi, \frac{\pi}{4}$ for case (a), $\alpha = \pi, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}$ for case (b), and angle of $\alpha = 360$ for case (c). The frequencies were chosen as follows: 1.8 GHz for case (a), 1.2 GHz and 1 GHz for case (b), 1.2 GHz for case (c). The results of appropriate simulations in the form of normalized intensity distribution of electric field are depicted in Fig. 1.

One can see from Fig.1 that rotational and cloaking phenomena are observed in all the integer-indexed petal structures for given parameters. Furthermore, the mapping simplifies to $r = r'$ along the angular direction $\alpha' = \frac{\pi}{6}$ for the case (a), $\alpha' = \frac{\pi}{4}$ for case (b) and $\alpha' = \frac{\pi}{8}$ for case (c) signifying an identical transformation along this axis. Consequently, no considerable differences in constitutive parameters arise from this transformation. Note that the observed symmetry is completely hidden and is able to be found out due to a using mathematical tool of transformation optics.

The simulations were conducted using COMSOL Multiphysics, which allowed us to model the electromagnetic wave interactions with high precision. A custom-built setup was configured in COMSOL to define the permittivity and permeability tensors derived in the theoretical model. Boundary conditions, including perfect electric and magnetic conductors, were applied at the domain edges to simulate realistic wave behavior. The resulting normalized intensity distributions of the electric field, shown in Fig. 1, illustrate the cloaking and rotation effects achieved by the proposed rose structures.

Another important conclusion is logically coming from Fig. 1(f): a considerable weakening of the EM field intensity at the right wall of the structure indicates that this structure behaves as a directional absorber in such a way that the proposed structure can be used for designing a novel turning element in microwave integrated circuits. Moreover, such absorber-like structures can be also implemented as isolating/ decoupling elements of printed antenna arrays. In this case, it is logical to include at least two such elements in parallel between all adjacent array radiating elements [14].

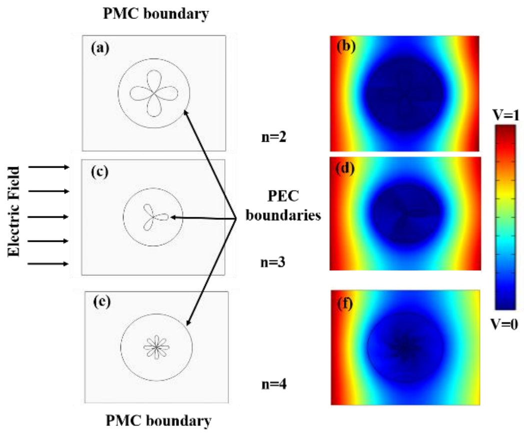

Fig. 1. Simulations results for rose structures with integer indices (n=2, 3, 4) showing rotational and cloaking behavior. These configurations demonstrate electromagnetic scattering patterns, with the electric field intensity decreasing at specific boundaries.

4.2 Rose structure with fractional indices

Consider the case when $n = \frac{1}{3}, \frac{1}{2}$ in Eq. (4). The geometries of problem is depicted in Fig. 2(a,c). In this geometric configuration, we numerically analyze the polar rose structures at the frequency of 1.4 GHz for case (b) and 1.5 GHz for case (d) with the following dimensions: $B = 0.0145$ m, $C = 0.0185$ m for case (b), same for case (d); and orientation: any value of the angle α for case (b), $\alpha = \frac{\pi}{4}, \frac{\pi}{3}$ for case (d). The result of appropriate simulation in the form of normalized intensity distribution of electric field is depicted in Fig. 2(b,d).

One can observe from Fig. 2(b,d). that the initial wave passes through the cloak and rotation phenomena for given parameters for both the fractional-indexed petal structures considered in Fig. 2 but no considerable absorption phenomenon was observed.

One can observe from Fig. 2(b,d) that the initial wave passes through the cloaks at a given frequency without experiencing either rotation or absorption effect.

Thus, summarizing the above analysis, we can conclude that the considered structures with integer index are more promising in terms of practical applications than structures with fractional indexes.

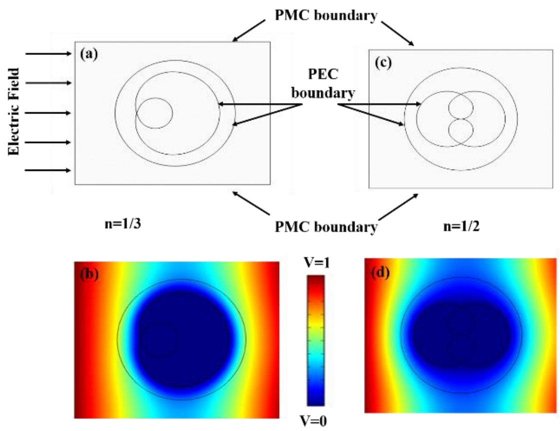

Fig. 2. Simulation results for rose structures with fractional indices (*n*=1/3, 1/2) at selected frequencies. Observed cloaking effects allow the wave to pass through without noticeable rotation or absorption, highlighting distinct characteristics of fractional-index structures compared to integer-indexed designs.

This is at least partially due to the fact that there are no considerable differences in constitutive parameters arise from this transformation ($r = r'$) for the case of the structures with integer index. Indeed, a symmetry, for example, is widely used in the design of a number of microwave devices. Indeed, the modes of a system/structure can be classified by its symmetry. Identification of the underlying structure of the modes inspires our understanding: the electronic structure of solids would be far less advanced without the aid of symmetry [24]. This is because it is difficult to realizing multifunctionality of devices in the absence of symmetry.

At the same time, the considered structures do not provide multi-frequency cloaking, which was previously considered as a serious drawback from the application point of view [25]. In fact, it is challenging to design structures with multi-frequency cloaking that simultaneously exhibit masking, rotation, and absorption phenomena, though such a combination may become feasible in the future.

Notice that there are no explicit limitations regarding the frequency range of the analytical results obtained in the study. That is why the results presented in this paper could inspire the development of similar structures within the optical frequency range.

5 Conclusion

Using the mathematical tool of transformation optics, the constitutive parameters (permittivity tensor and permeability tensors) for a cloak invisibility of 2-D structures containing perfectly conducting rose-shaped elements were derived in this study. The case of an initial plane TE-wave considered at different GHz frequencies for different numbers of the petals of the rose elements using the above tensors and commercial software, COMSOL Multiphysics. Carrying out the appropriate numerical analysis, reveals the sensitivity of the results to varying parametric values which can lead to the simultaneous occurrence of such effects as cloaking and of rotation and/or absorption of the initial EM wave at some its frequency and some sets of the geometrical parameters of the rose elements.

The aforementioned tunability of the considered structures can create elements of microwave integrated circuits such as novel wave rotators, absorbers, turning elements and isolating/decoupling elements of printed antenna arrays. Furthermore, we can explore design optimizations for multi-frequency appli-cations, enlarging the scope of optical and microwave cloaking technologies.

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