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LINK ELEMENT WITH VARYING ELECTRIC CONDUCTANCE

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The aim of this article is to derive an electrical two nodal link element with a continuously varying cross-section and electrical conductivity along its longitudinal axis. The element matrix and load vector contain transfer constants, which depend on the variation of the above-mentioned parameters. These transfer constants can be determined by a simple numerical algorithm. The accuracy and efficiency of the new finite element is verified on 1D steady state analysis of electrical current conduction problems.

K e y w o r d s: finite element method, varying electric conductance, functionally graded materials

1 INTRODUCTION

At present a 1D electric link element with a constant cross-section and constant electrical conductivity along its axis is implemented to commercial FEM software (*eg* ANSYS, [1]). The variation of electrical conductance can be a result of the powder metallurgy process with a non-uniform composition of two or more different materials (functionally graded materials), or by non-homogenous temperature distribution along the link. The cross-sectional area variation of the conductor causes the variation of electrical conductance, too.

Electrical FEM analysis of the conductors with varying conductance can be made by the above-mentioned link elements with constant average conductance (obtained from element nodal values), or using solid elements, but the results obtained with such elements depend on the mesh density very strongly [2], and input data preparation is therefore very time-consuming. To avoid these drawbacks, a new two nodal link element with variation of the cross section and/or electrical conductivity along the longitudinal link axis has been derived in this article using the shape functions which contain transfer constants. The transfer constants depend on the conductance variation and could be solved by a simple numerical algorithm [3, 4].

Firstly, we will determine the electric conductance matrix of the link element with a continuously varying cross-sectional area and electrical conductivity. Then, the results of numerical solution obtained using our new element will be compared with the analytical ones and with the results obtained using the ANSYS LINK68 element solution. Finally, the analysis of a chosen current path will be performed using our new link element. The Joule heat will be computed using our derived relations. The effectiveness and accuracy of this new element will be discussed and evaluated.

2 DERIVATION OF ELECTRIC CONDUCTANCE MATRIX

The electric conductance of a link element for 1D electrical current conduction is a product of the cross-section area and electrical conductivity. We assume a continuous variation of the two parameters along the longitudinal link axis — Fig. 1.

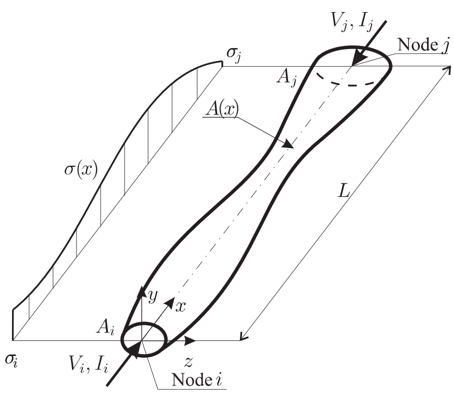

Fig. 1. Link element with variation of cross-section and electrical conductivity.

2.1 Cross-sectional area variation

The variation of the cross-section will be assumed as the polynomial

$$A(x) = A_i \eta_A(x) = A_i \Big(1 + \sum_{k=1}^{s} \eta_{Ak} x^k\Big) \tag{1}$$

where A_i is the cross-section at node i and the polynomial $\eta_A(x)$ expresses the variation of the cross-section along the element. The order of the polynomial is s. Constants η_{Ak}, where $k = 1, \dots, s$, and the order s of polynomial depend on the cross-section variation.

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2.2 Electrical conductivity variation

The continuous variation of electrical conductivity of the link element in the polynomial form is assumed as

$$\sigma(x) = \sigma_i \eta_\sigma(x) = \sigma_i \Big(1 + \sum_{k=1}^{r} \eta_{\sigma k} x^k\Big) \tag{2}$$

where σ_i is the electrical conductivity at node i, and $\eta_\sigma(x)$ is the polynomial of varying conductivity along the element — see Fig. 1. Its constants $\eta_{\sigma k}$, where $k = 1, \dots, r$, and the order r of polynomial depend on the conductivity variation.

2.3 Electric conductance variation

The varying electric conductance means the product of the varying electric conductivity and varying cross-sectional area (Fig. 1), and is given by

$$G(x) = A(x)\sigma(x) = A_i \sigma_i \eta_A(x) \eta_\sigma(x)$$
$$= G_i \eta_G(x) = G_i \Big(1 + \sum_{k=1}^{s+r} \eta_{Gk} x^k\Big) \tag{3}$$

where $G_i = A_i \sigma_i$ is the electrical conductance at node i and the polynomial $\eta_G(x)$ expresses the variation along the element. The order of electrical conductance polynomial is $s + r$.

2.4 Electric conductance matrix

The potential at point x is determined by

$$V(x) = [N_i(x) \ \ N_j(x)] \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{4}$$

where $N_i(x)$ and $N_j(x)$ are the shape functions and V_i and V_j are nodal potentials — see Fig. 1.

The new shape functions for the polynomial variation of link conductance (3) have been derived by solving the differential equations of the link with non-constant coefficients, [3, 4].

The new shape functions have the form

$$N_i(x) = 1 - \frac{b'_{G2}(x)}{b'_{G2}}\,, \quad N_j(x) = \frac{b'_{G2}(x)}{b'_{G2}} \tag{5}$$

where $b_{G2}(x)$ is the transfer function, $b'_{G2}(x)$ is its first derivative and b'_{G2} is $b'_{G2}(L)$ and is called the transfer constant. The transfer function is defined in differential form [4, 5]

$$b''_{G2}(x) = \frac{1}{\eta_G(x)} \tag{6}$$

where $\eta_G(x)$ is defined by (3) — for the sake of consistency, the process of transfer constant solution is shown in the Appendix.

The potential gradient can be expressed as

$$\operatorname{grad} V(x) = \frac{\mathrm{d}V(x)}{\mathrm{d}x} = \left[\frac{\mathrm{d}N_i(x)}{\mathrm{d}x}, \frac{\mathrm{d}N_j(x)}{\mathrm{d}x}\right] \begin{bmatrix} V_i \\ V_j \end{bmatrix} = \mathbf{B}\mathbf{V}^{\mathrm{e}}. \tag{7}$$

Matrix $\mathbf{B}$ is a row matrix of shape function derivatives, and $\mathbf{V}^{\mathrm{e}}$ is the vector of nodal potentials. The derivatives of the shape functions (5) are

$$\frac{\mathrm{d}N_i}{\mathrm{d}x} = -\frac{b''_{G2}(x)}{b'_{G2}}\,, \quad \frac{\mathrm{d}N_j}{\mathrm{d}x} = \frac{b''_{G2}(x)}{b'_{G2}}\,. \tag{8}$$

The electrical conductance matrix of this finite element has been derived by minimization of the potential energy functional [7], and its form is

$$\mathbf{K}_G^{\mathrm{e}} = \int_{V^{\mathrm{e}}} \mathbf{B}^\top \mathbf{D} \mathbf{B} \mathrm{d}V \tag{9}$$

where the matrix of material properties $\mathbf{D}$ is equal to electrical conductivity $\sigma(x)$ in our case.

Substituting Eqs. (2), (7) and (8) into Eq. (9) and using Eqs. (3), (6) and relationship $\mathrm{d}V = A(x)\mathrm{d}x$, we get the electrical conductance matrix of 1D link element with variation of the cross-section and/or electrical conductivity

$$\mathbf{K}_G^{\mathrm{e}} = \int_L \Big(\frac{b''_{G2}(x)}{b'_{G2}}\Big)^2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \sigma(x) A(x) \mathrm{d}x =$$
$$\frac{G_i}{\big(b'_{G2}\big)^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \int_L b''_{G2}(x) \mathrm{d}x = \frac{G_i}{b'_{G2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \tag{10}$$

In the case, when the cross-section and electrical conductivity are constant along the element length, the element conductance matrix coincides with the classical conductance matrix because in this case $b''_{G2} = L$.

The system of equilibrium equations for one element has the classical FEM form

$$\mathbf{K}_G^{\mathrm{e}} \mathbf{V}^{\mathrm{e}} = \mathbf{I}^{\mathrm{e}} \tag{11}$$

where $\mathbf{I}^{\mathrm{e}}$ is the vector of nodal currents, $\mathbf{I}^{\mathrm{e}} = [I_i,\, I_j]^\top$, a positive value of electric current represents the current flowing into the node from outside of the element — see Fig. 1.

The system (11) with the electrical conductance matrix (10) produces exact results because they are equal to the solution results of differential equation of the problem with appropriate boundary conditions.

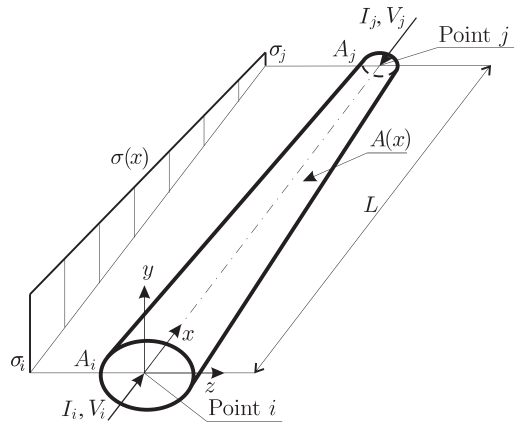

Fig. 2. Example 1 – electrical conductor with variation of cross-section and electrical conductivity.

2.5 Computation of the Joule heat

The differential of electrical performance is defined by the scalar product

$$\mathrm{d}P_J(x) = \mathbf{E}(x) \cdot \mathbf{J}(x)\mathrm{d}V = \frac{1}{\sigma(x)} J(x)^2 \mathrm{d}V \tag{12}$$

where $\mathbf{E}(x)$ is the electric intensity vector and $\mathbf{J}(x)$ is the current density vector. For 1D problem, the current density vector is

$$\mathbf{J}(x) = \frac{I(x)}{A(x)} \boldsymbol{i} = J(x)\boldsymbol{i} \tag{13}$$

where $\boldsymbol{i}$ is a unit vector of x direction, $A(x)$ is the cross-sectional area where the current is applied on and $J(x)$ is the current density value. The cross-section and electrical conductivity along the element have been defined by Eqs. (1) and (2), respectively. In our case, current $I(x)$ is a constant value along the element, because there is no distributed load in the electric analysis, and has value $I(x) = I = I_i = -I_j$.

The product of the cross-sectional area $A(x)$ and electrical performance per volume $\mathbf{E}(x) \cdot \mathbf{J}(x)$ gives the length electrical performance

$$p(x) = q(x)A(x) = E(x)J(x)A(x) = \frac{I^2}{A_i\sigma_i} \, \frac{1}{\eta_A(x)\eta_\sigma(x)}. \tag{14}$$

In the thermal analysis, $p(x)$ represents the internal heat source distributed along the element and $q(x)$ represents the internal heat source per volume.

By integration of Eq. (14) along the element, length L, the electrical losses (Joule heat) in element volume are given by

$$P_J(L) = P_J = \int_L p(x)\mathrm{d}x = \frac{I^2}{A_i\sigma_i} \int_L \frac{1}{\eta_A(x)\eta_\sigma(x)} \mathrm{d}x\,. \tag{15}$$

Expression (15) can be rewritten using the transfer constant b'_{G2} to the form

$$P_J = \frac{I^2}{A_i\sigma_i} b'_{G2}\,. \tag{16}$$

3 COMPARISON OF RESULTS OBTAINED BY NUMERICAL AND ANALYTICAL APPROACHES

Numerical experiments contain two examples. The first example shows the effectiveness and accuracy of our new link element. In the second example, the numerical analysis of the electrical conductor has been performed using our new link element.

3.1 Conductor of varying electric conductivity and varying cross-section

The conductor shown in Fig. 2 has a variation of the cross-sectional area and electrical conductivity

$$A(x) = A_i(1 - 0.9x)$$
$$\sigma(x) = \sigma_i(1 - 0.9x)$$

where $A_i = 0.001\,\mathrm{m}^2$, and $\sigma_i = 40$ S/m. The boundary conditions of the problem are the electric potential at point i, $V_i = 20$ V, and the electric current at point j, $I_j = 2$ A. The length of electrical conductor is $L = 1$ m. The conductor is ideally electrical insulated around the external surface. We assume 1D electrical conduction.

The main goal is to find the electric potential and current density distribution along the conductor length. The problem was solved analytically, numerically using our new link element and numerically using the link element LINK68 of Ansys code.

Analytical solution

Variation of the cross-section $A(x)$ and electrical conductivity $\sigma(x)$ was chosen in a such way that the analytical solution exists.

The differential equation of the problem is a Laplace equation for steady state current field as follows (1D problem), [7]

$$\frac{\partial}{\partial x}\Big(\sigma(x)A(x)\frac{\partial V(x)}{\partial x}\Big) = 0\,.$$

Applying boundary conditions $V_i = 20$ V and $I_j = 2$ A, we get the electric potential

$$V(x) = V_i + I_j \frac{1}{0.9A_i\sigma_i}\Big(\frac{1}{1-0.9x} - 1\Big)$$

and the gradient of potential is

$$\operatorname{grad} V(x) = I_j \frac{1}{0.9A_i\sigma_i} \, \frac{0.9}{(1-0.9x)^2}\,.$$

The current density along the conductor length is

$$J(x) = -\frac{I_j}{A(x)} = -\frac{I_j}{A_i(1-0.9x)}$$

Table 1. Example 1 — numerical values of the transfer constant, potential, its gradient and current density for specific points obtained by our new element.

Location x (m)	Transfer constant $b'_{G2}(x)$ (–)	Potential $V(x)$ (V)	Grad $V(x)$ (V/m)	Current density $J(x)$ (A/m^2)
0	0	20	50	-2000
0.2	0.244	32.19	74.36	-2439
0.4	0.625	51.25	122.07	-3125
0.6	1.305	85.22	236.29	-4347
0.8	2.857	162.86	637.75	-7142
1	10	520	5000	-20000

Numerical solution using our new link element

We used only one new link element for the conductor — points i and j represent nodes i and j, respectively (see Fig. 2). The system of FEM equations for the unknown potential V_j and the electric current I_i has the form (11).

The electrical conductance at node i is

$$G_i = A_i\sigma_i = 0.04 \text{ Sm}.$$

The variation of electrical conductance is given by polynomial (3)

$$\eta_G(x) = \eta_A(x)\eta_\sigma(x) = (1-0.9x)^2 .$$

The constant b'_{G2}, that was evaluated according to the numerical algorithm [3, 4] has value 10. FEM equations have the form

$$\frac{0.04}{10}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 20 \\ V_j \end{bmatrix} = \begin{bmatrix} I_i \\ 2 \end{bmatrix} .$$

Solving the FEM equations we get

$$V_j = 520\,\text{V}, \;\; I_i = -2\,\text{A}.$$

The potential along the conductor is expressed by the shape functions (4)

$$V(x) = \Big(1-\frac{b'_{G2}(x)}{b'_{G2}}\Big)V_i + \Big(\frac{b'_{G2}(x)}{b'_{G2}}\Big)V_j$$

and gradient of potential (7)

$$\operatorname{grad} V(x) = \Big(-\frac{b''_{G2}(x)}{b'_{G2}}\Big)V_i + \Big(\frac{b''_{G2}(x)}{b'_{G2}}\Big)V_j = \frac{V_j - V_i}{b'_{G2}\eta_G(x)} .$$

The current density can be written as

$$J(x) = -\sigma(x)\operatorname{grad} V(x) = -\sigma_i\frac{\eta_\sigma(x)}{b'_{G2}\eta_G(x)}(V_j - V_i)$$

because $b''_{G2}(x) = \dfrac{1}{\eta_G(x)}$.

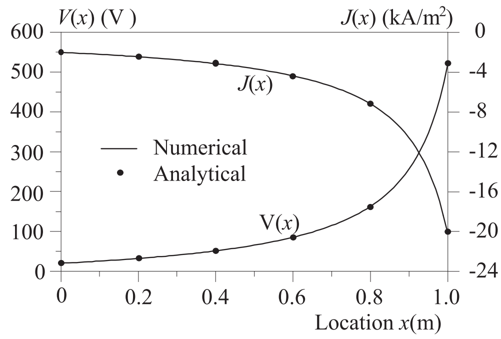

Fig. 3. Example 1 — comparison of analytical and our new element solution — potential $V(x)$ and current density $J(x)$.

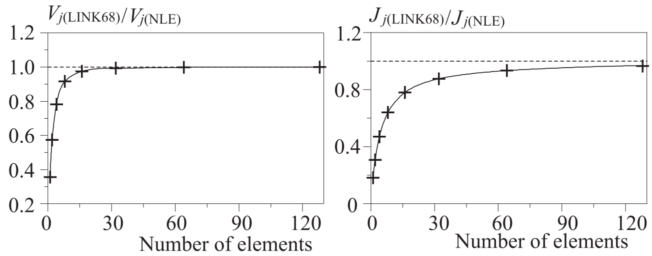

Fig. 4. Example 1 – potential and current density convergence of LINK68 to new link element results (NLE).

Table 1 shows numerical values of transfer constant $b'_{G2}(x)$, potential $V(x)$, gradient of $V(x)$ and current density $J(x)$ for specific locations x.

Comparison of the analytical solution of $V(x)$ and $J(x)$ with a numerical solution obtained by only one new link element is shown in Fig. 3. As we can see from this figure, the numerical results — potential $V(x)$ and current density $J(x)$ coincide with analytical results.

Numerical solution using the LINK68 element of Ansys code

The solution of the same problem has been made using the classical 1D-electric link element LINK68 of Ansys code, [1]. In order to show the convergence of Ansys solution to our new element solution, the number of elements was gradually increased. Individual LINK68 elements contain average values of the cross-section and electrical conductivity in the element range. Table 2 contains

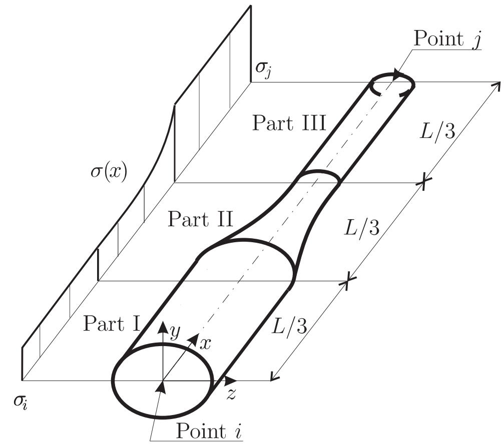

Fig. 5. Example 2 — the current path with varying conductance.

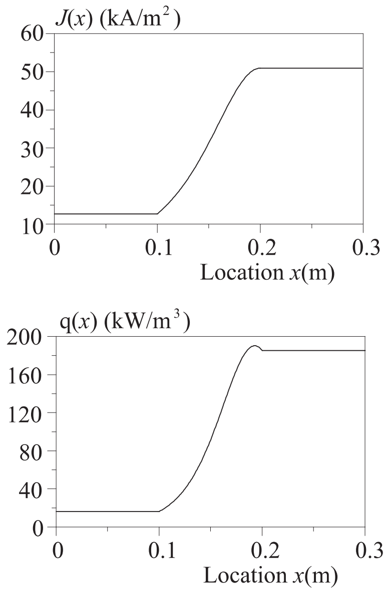

Fig. 6. Example 2 – distribution of current density $J(x)$ and Joule heat $q(x)$ along the conductor length.

the potential $V_{j(\text{LINK68})}$ and current density $J_{j(\text{LINK68})}$ at the end of the conductor and also new link element results (NLE).

Figure 4 shows the convergence of LINK68 results to new link element results (NLE).

As we can see from Table 2 and mainly from Fig. 4, potential $V_{j(\text{LINK68})}$ has a better convergence to NLE results than current density $J_{j(\text{LINK68})}$. For example, the current density error at point j, when the conductor is divided into 64 elements, is more than 6.5 % while the potential error at point j in the same mesh density is less than 1 %.

3.2 Electric analysis of the current path of varying electric conductance

The conductor of length $L = 1$ m shown in Fig. 5 is built from parts of constant and varying conductances.

Conductor parameters:

- Part I
  - diameter $d_{\text{I}} = 0.02$ m,
  - electric conductivity $\sigma_{\text{I}} = 10000$ S/m.
- Part II
  - diameter $d_{\text{II}}(x) = d_{\text{I}}(1 - 9x + 30x^2 + 100x^3)$ m,
  - electric conductivity $\sigma_{\text{II}}(x) = \sigma_{\text{I}}(1+20x^2+200x^3)$ S/m, x is local axis of part II $x \in \langle 0, L/3\rangle$.
- Part III
  - diameter $d_{\text{III}} = 0.01$ m,
  - electric conductivity $\sigma_{\text{III}} = 14000$ S/m.

The boundary conditions of the problem are the electric potential at point i, $V_i = 20$ V and the electric current at point j, $I_j = 4$ A.

The main goal is to find the electric potential, current density and Joule's heat distribution along the current path. The current path has been divided to 3 new finite elements. The solution results have been obtained as follows:

- nodal potential $V(L/3) = 20.1273$ V, $V(2L/3) = 20.4063$ V, $V(L) = V_j = 20.7701$ V;
- the current density in single parts $J_{\text{I}} = 12732.4\,\text{A/m}^2$, $J_{\text{II}}(x) = \frac{4}{\pi(d_{\text{II}}(x)/2)^2}\,\text{A/m}^2$, $J_{\text{III}} = 50929.6\,\text{A/m}^2$;
- Joule heat per volume in single parts $q_{\text{I}} = 16211.4\,\text{Wm}^{-3}$, $q_{\text{II}}(x) = \frac{16}{A(x)^2\sigma(x)}\text{Wm}^{-3}$, $q_{\text{III}} = 185273\,\text{Wm}^{-3}$.

Figure 6 shows the current density and Joule heat per volume distribution along the current path. The conductance variation causes the variations of the current density and Joule heat per volume along part II. This example has been also formulated in such a way that analytical solution is possible. It can be verified that our very effective numerical solution coincides with the analytical one. Of course, if a more complicated variation of electric conductance is given (the polynomial of higher order), analytical solution is impossible.

Table 2. Example 1 — numerical values of potential and current density at the end of conductor by increasing number of elements LINK68

Number of elements (–)	Potential $V_{j(\text{LINK68})}$ (V)	Current density $J_{j(\text{LINK68})}$ (A/m^2)
1	185.289	-3636
2	298.310	-6154
4	406.472	-9412
8	476.335	-12800
16	506.697	-15610
32	516.437	-17534
64	519.092	-18686
128	519.772	-19321
1 NLE	520	-20000

4 CONCLUSION

A new link element with variations of the cross-section and/or with electrical conductivity is presented in this paper. The electric conductance matrix contains transfer constants which depend on the cross-section and electrical conductivity variation along the element. The comparison between analytical and numerical solutions proves the accuracy of the new element. The comparison of results between the classical link element and the element with varying conductance proves the effectiveness of this new element. In opposite to the classical link elements, the accuracy of our link element does not depend on element mesh sdensity. The new link element with varying electric conductance can be used for analysis of conductor paths in composite or functionally graded materials. The rise of temperature and thermo-elastic analysis of this current path will be the matter of our future contribution. After that, the electric-thermal-structural analysis of construction parts with varying material properties can be performed using our new link element.

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Appendix

Determination of the transfer functions and transfer constants occurring in the stiffness matrix and shape functions is based on the following expression

$$b''_{Gj+2}(x) = \frac{a_j(x)}{\eta_G(x)}$$

where the function $a_j(x) = \frac{x^j}{j!}$ for $j \geq 0$, and for $j \leq 0$, $a_0 = 1$, $a_j = 0$. Closed solutions for the 1st and 2nd integrals of the function $b''_{Gj+2}(x)$ are known only for lower degree polynomials $\eta_G(x)$. For their numerical solution, which is more general, a recurrence rule was derived

$$b^{(n)}_{Gj}(x) = a_{j-n}(x) - \sum_{k=1}^{m} \eta_{Gk} \frac{(j-2+k)!}{(j-2)!} b^{(n)}_{Gj+k}(x) \qquad \text{for} \quad j \geq 2\,,\ n = 0 \text{ and } 1\,.$$

After some manipulation we get

$$b^{(n)}_{Gj}(x) = a_{j-n}(x) \sum_{t=0}^{\infty} \beta_{t,0}(x)$$

where $\beta_{t,0}(x)$ is expressed by

$$\beta_{t,0}(x) = -\sum_{k=1}^{m} \Big[\eta_{Gk} \beta_{t,k}(x) \prod_{r=-k}^{-1} (s-1+r) \Big]$$

with parameters

$$s = 1 + t,\ e = \frac{x}{s-n},\ \ \beta_{t,k} = e\beta_{t-1,k-1} \ \text{for } k = 1, \dots, m,$$

and initial values are

$$\beta_{0,0} = 1, \quad \beta_{0,k} = 0 \qquad \text{for } k = 1, \dots, m.$$

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