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*COMMUNICATIONS*

# AN ITERATIVE METHOD FOR WIENER–HAMMERSTEIN SYSTEMS PARAMETER IDENTIFICATION

**Jozef Vörös** *

The paper deals with parameter identification of nonlinear dynamic systems using the Wiener-Hammerstein model. Application of an operator decomposition technique provides a special output equation that is linear in the parameters of all the model blocks. As two unmeasurable internal variables are included into the model description, an iterative parameter estimation algorithm is proposed.

K e y w o r d s: parameter identification, Wiener-Hammerstein model

## 1 INTRODUCTION

The class of nonlinear dynamic systems which can be represented by the block-oriented models, *ie*, by interconnection of linear dynamic and nonlinear static subsystems, has been studied by many authors. In the simplest case the models consist of a combination of two blocks giving the so-called Hammerstein (nonlinear-linear) and Wiener (linear-nonlinear) models and there are many methods for nonlinear system identification using these models (see [8] and [9] for a complete bibliography).

The Wiener-Hammerstein model (also the general or sandwich model, see Fig. 1), defined as a linear system in cascade with a static nonlinear element followed by another linear system, represents a more complex case of block-oriented models. Nonlinear system identification using this model has been studied for many years. In the classical studies [2, 3, 6, 7], identification algorithms for the Wiener-Hammerstein model based on correlation analysis have been proposed. The identification problem has been decoupled into two distinct steps; identification of the linear dynamic subsystems and characterization of the static nonlinearity. Since then numerous contributions to the Wiener-Hammerstein systems identification have been published, *eg*, [1, 4, 5, 11, 12, 18], and this is still an active research area because many real systems are of this type (*eg*, sensor systems, electromechanical systems in robotics, mechatronics, biological and chemical systems).

In this paper a new approach to the Wiener-Hammerstein model parameter estimation is presented. It is based on a new form of model description resulting from a decomposition technique [13] that is sequentially applied to the general model description. The model is linear in parameters and nonlinear in variables. As two unmeasurable internal variables are included into the model description, the estimation problem is solved iteratively as a pseudo-linear one, where the parameters of the nonlinear and two linear blocks are estimated simultaneously using the input and output variables and the estimates of two internal variables.

## 2 DECOMPOSITION TECHNIQUE

In the previous work [13], a simple decomposition technique has been proposed for composite mappings based on the so-called key term separation principle. This consists in separating a key term (variable) in the outer mapping and consequent half-substituting of the inner mapping for the key term only. Then the inner mapping appears both explicitly and implicitly in the outer one. Application of this approach to the Hammerstein and Wiener models has led to useful simplifications because the original composite mappings were replaced by simpler ones more appropriate for the identification and control purposes [14–17].

In the case of more complex composite mappings the additive form of this decomposition technique can be also applied sequentially more times. If there are three mappings

$$v = \alpha(u)\,, \tag{1}$$

$$x = \beta(v) = \beta\big(\alpha(u)\big)\,, \tag{2}$$

$$y = \gamma(x) = \gamma\big(\beta(v)\big) = \gamma\big(\beta(\alpha(u))\big)\,, \tag{3}$$

defined on proper sets $U$, $V$, $X$, and $Y$, let us assume the following decompositions exist:

$$x = v + \beta'(v)\,, \tag{4}$$

$$y = x + \gamma'(x)\,, \tag{5}$$

where $\beta'(\cdot)$ and $\gamma'(\cdot)$ are the 'remainders' of $\beta(\cdot)$ and $\gamma(\cdot)$, after separations of variables $v$ and $x$, respectively.

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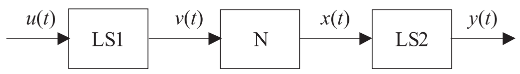


**Fig. 1.** Wiener-Hammerstein model.

Then the sequential half-substitution of the corresponding inner mappings only for the separated (key terms) $v$ and $x$ will lead to

$$x = \alpha(u) + \beta'(v)\,, \tag{6}$$

$$y = \alpha(u) + \beta'(v) + \gamma'(x)\,. \tag{7}$$

In this case, the composite mapping $\gamma \circ \beta \circ \alpha$ given by (3) can be replaced by (7), that may be simpler, *eg*, may be linear in parameters. In the following, the above decomposition will be used to simplify the description of the Wiener-Hammerstein system and the estimation of the corresponding model parameters.

### 3 WIENER–HAMMERSTEIN SYSTEMS

The Wiener-Hammerstein system is given by the cascade connection of a linear dynamic system LS1 followed by a static nonlinearity block N, which is followed by a linear dynamic system LS2 (Fig. 1). The first linear block can be described by the difference equation

$$v(t) = q^{-m} B\big(q^{-1}\big)u(t) + \big[1 - A\big(q^{-1}\big)\big]\, v(t) \tag{8}$$

where $u(t)$ and $v(t)$ are the inputs and outputs, respectively, $A\big(q^{-1}\big)$ and $B\big(q^{-1}\big)$ are scalar polynomials in the unit delay operator $q^{-1}$

$$A\big(q^{-1}\big) = 1 + a_1 q^{-1} + \cdots + a_{na} q^{-na}, \tag{9}$$

$$B\big(q^{-1}\big) + b_0 + b_1 q^{-1} + \cdots + b_{nb} q^{-nb}, \tag{10}$$

and $q^m$ represents the pure delay of LS1. The characteristic of nonlinear block N is assumed to be approximable by a polynomial given as follows:

$$x(t) = \sum_{k=1}^{r} f_k v^k(t)\,, \tag{11}$$

where $v(t)$ and $x(t)$ are the inputs and outputs, respectively. The second linear dynamic block is given by

$$y(t) = q^{-n} D\big(q^{-1}\big)x(t) + \big[1 - C\big(q^{-1}\big)\big]\, y(t)\,, \tag{12}$$

where $x(t)$ and $y(t)$ are the inputs and outputs, respectively, $C\big(q^{-1}\big)$ and $D\big(q^{-1}\big)$ are scalar polynomials in the unit delay operator $q^{-1}$

$$C\big(q^{-1}\big) = 1 + c_1 q^{-1} + \cdots + c_{nc} q^{-nc}, \tag{13}$$

$$D\big(q^{-1}\big) = d_0 + d_1 q^{-1} + \cdots + d_{nd} q^{-nd}, \tag{14}$$

and $q^{-n}$ represents the pure delay of LS2. The model inputs $u(t)$ and outputs $y(t)$ are measurable, while the internal variables $v(t)$ and $x(t)$ are not.

The input-output description of a Wiener-Hammerstein system resulting from direct substitutions of the corresponding variables from (8) into (11) and then into (12) would be strongly nonlinear both in the variables and in the parameters, hence not very appropriate for the parameter estimation. Therefore, the above mentioned decomposition will be applied with the aim to derive a simpler form of the model description.

The Wiener-Hammerstein system is a cascade connection of three blocks, which can be modelled by (8), (11), and (12). It means that many combinations of parameters can be found to fulfil the model equation. To obtain a unique parametrization, one parameter in at least two blocks has to be (and can be) fixed in the mathematical model. Moreover, this will simplify the separation of the key terms in the Wiener-Hammerstein model description.

First, let us assume $d_0 = 1$, *ie*, the second linear dynamic system can be described as follows:

$$y(t) = x(t-n) + \big[D\big(q^{-1}\big) - 1\big]\, x(t-n) + \big[1 - C\big(q^{-1}\big)\big]\, y(t)\,, \tag{15}$$

where the internal variable $x(t-n)$ is separated. Then, let us assume that $f_1 = 1$ and the nonlinear static block is characterized by the polynomial

$$x(t) = v(t) + \sum_{k=2}^{r} f_k v^k(t)\,, \tag{16}$$

where again the internal variable $v(t)$ is separated. Now, to complete the sequential decomposition, the corresponding half-substitutions can be performed, *ie*, (i) from (8) into (16) only for the first term, and (ii) from (16) into (15) again only for the first term. The resulting output equation of the Wiener-Hammerstein model will be

$$y(t) = B\big(q^{-1}\big)u(t-m-n) + \big[1 - A\big(q^{-1}\big)\big]\, v(t-n) + \sum_{k=2}^{r} f_k v^k(t-n) + \big[D\big(q^{-1}\big) - 1\big]\, x(t-n) + \big[1 - C\big(q^{-1}\big)\big]\, y(t)\,. \tag{17}$$

Now the output equation (17) and those of (8) and (16) defining the internal variables $v(t)$ and $x(t)$, respectively, represent the Wiener-Hammerstein model.

The derived Wiener-Hammerstein model is linear-in-parameters and can be written in the vector form

$$y(t) = \Phi^{\top}(t,\theta)\theta\,, \tag{18}$$

where the vector of parameters and the vector of data are

$$\theta = [b_0, \ldots, b_{nb}, a_1, \ldots, a_{na}, f_2, \ldots, f_r, d_1, \ldots, d_{nd}, c_1, \ldots, c_{nc}]^{\top}, \tag{19}$$

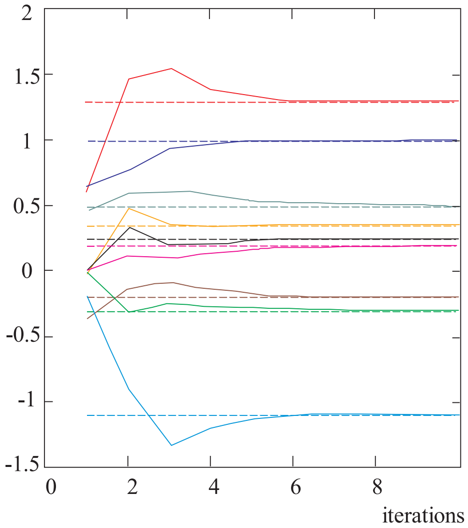


**Fig. 2.** The process of parameter estimation.

$$\Phi(t,\theta) = [u(t-m-n), \dots, u(t-m-n-nb), -v(t-n-1), \dots, -v(t-n-na), v^2(t-n), \dots, v^r(t-n), x(t-n-1), \dots, x(t-n-nd), -y(t-1), \dots, y(t-nc)]^\top . \quad (20)$$

As all the model parameters in (17) are separated, the proposed form of Wiener-Hammerstein model consists of the least possible number of parameters to be estimated. Note, that the constants m and n are considered to be known.

## 4 PARAMETER ESTIMATION

The above Wiener-Hammerstein model description contains internal variables, which are generally unmeasurable; hence the parameter estimation cannot be performed directly on the basis of (18). It will be appropriate to apply the iterative approach with internal variable estimation proposed for Hammerstein and Wiener models in [13], as this can be simply extended to the estimation of two internal variables.

The iterative algorithm is based on the use of the previous estimates of the model parameters for the estimation of the internal variables, *ie*, for the computation of their estimates. Assigning the estimated variables in the $s$-th step as

$${}^s v(t) = \sum_{i=1}^{nb} {}^s b_i u(t-m-i) - \sum_{j=1}^{na} {}^s a_j {}^s v(t-j) , \quad (21)$$

$${}^s x(t) = {}^s v(t) + \sum_{k=2}^{r} {}^s f_k {}^s v^k(t) , \quad (22)$$

the error to be minimized in the $s$-th step is gained from (18) as

$${}^s e(t) = y(t) - {}^s\Phi^\top(t, {}^s\theta)^{s+1}\theta , \quad (23)$$

where ${}^s\Phi$ is the data vector with the corresponding estimates of internal variables according to (21)-(22) and ${}^{s+1}\theta$ is the $(s+1)$-th estimate of the parameter vector. The steps in the iterative procedure may be now stated as follows:

a) Minimizing a proper criterion based on (23) the estimates of parameters ${}^{s+1}\theta$ are yielded using ${}^s\Phi$ with the $s$-th estimates of the internal variables.

b) Using (21) the estimates of ${}^{s+1}v(t)$ are evaluated by means of the estimates of corresponding model parameters.

c) Using (22) with the estimates of ${}^{s+1}v(t)$, the estimates of ${}^{s+1}x(t)$ are evaluated by means of the estimates of parameters $f_k$.

d) If the estimation criterion is met the procedure ends, else it continues by repeating steps a)–c).

The first estimation can be performed for the quasi-Wiener part only, *ie*, considering ${}^0v(t) = y(t)$ in the first linear dynamic block, ${}^0v(t) = u(t)$ in the static block, while the second linear dynamic block is not considered.

To illustrate the feasibility of the proposed identification method, the following example shows the parameter estimation process for the Wiener-Hammerstein system, where the first linear dynamic system was given by the difference equation

$$v(t) = u(t-1) + 0.5u(t-2) + 0.2v(t-1) - 0.35v(t-2) ,$$

the static nonlinearity was characterized by the polynomial

$$x(t) = v(t) + 1.3v^2(t) - 1.1v^3(t) ,$$

and the second linear dynamic system was given by the difference equation

$$y(t) = x(t-1) + 0.25x(t-2) - 0.2y(t-1) + 0.3y(t-2) .$$

The identification was carried out with 2000 samples of normally distributed random inputs and the generated outputs. The outputs were corrupted with Gaussian white noise with zero mean and the signal to noise ratio SNR = 20 (SNR – the square root of the ratio of output and noise variances). The least squares method was used for the repeated estimations of all the model parameters. The initial values of the parameters were chosen zero. The process of parameter estimation is shown in Fig. 2 (the top-down order of parameters is $f_2$, $b_1$, $b_2$, $a_2$, $d_2$, $c_1$, $a_1$, $c_2$, $f_3$). The parameter estimates are almost equal to the true values after 9 iterations and the iterative process of parameter estimation shows good convergence.

**Table 1.** The error values

| It. | MSE-$v$ | MSE-$x$ | MSE-$y$ |
|---|---|---|---|
| 1 | 0.03966695700059 | 0.12382447022760 | 0.12382447022760 |
| 2 | 0.00549091226696 | 0.00784181024804 | 0.01151075681834 |
| 3 | 0.00120247348792 | 0.00277729972956 | 0.00325753546815 |
| 4 | 0.00014067547056 | 0.00049502349351 | 0.00083331672128 |
| 5 | 0.00004343085291 | 0.00014901483207 | 0.00040619150597 |
| 6 | 0.00003194832355 | 0.00004358887604 | 0.00030707763229 |
| 7 | 0.00001084824299 | 0.00001534858099 | 0.00029357596750 |
| 8 | 0.00000336637924 | 0.00000474567614 | 0.00028886837999 |
| 9 | 0.00000194158632 | 0.00000204202544 | 0.00028561689467 |

The corresponding values of the mean square error for the internal variables $v(t)$, $x(t)$ and the model output $y(t)$ are in Table 1. This shows good convergence of the internal variables estimates.

## 5 CONCLUSION

The presented approach to the Wiener-Hammerstein systems parameter identification is based on a new form of system description resulting from two consecutive decompositions of the corresponding compound mappings. An iterative parameter estimation algorithm with internal variable estimations has been proposed and illustrated on an example of a simulated Wiener-Hammerstein system.

As the requirement to fix one parameter in the description of the nonlinear static block is not too severe, other types of nonlinearities [10] can also be considered in the Wiener-Hammerstein model, *eg*, discontinuous characteristics or two-segment nonlinearities.

Note that a general convergence proof for this type of model parameter estimation is not available (neither for the simpler Hammerstein and Wiener models). The presented algorithm is a modified relaxation one, similar to [11], which is not convergent in all cases, but is in most practical applications. The proposed form of the Wiener-Hammerstein model can be superior in applications where simple descriptions of block-oriented systems are required.

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