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DISCRETE–TIME VARIABLE STRUCTURE CONTROLLER FOR AIRCRAFT ELEVATOR CONTROL

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A discrete-time variable structure controller for aircraft elevator control using the method for control of plants with finite zeros in canonical subspace is proposed in this paper. First, a discrete mathematical model of the system over canonical space, using the delta transform, is given. Then, decomposition of the canonical space to subspaces with and without control is carried out by introducing the output variable delta transform. Finally, a relation providing the quasi-sliding mode over the canonical subspace with control is derived. The realized system is stable and robust against parameter and external disturbances.

K e y w o r d s: discrete-time variable structure system, canonical space, aircraft, elevator

1 INTRODUCTION

The real-life plants that have the practical importance and their mathematical models have finite zeros are: aircrafts, helicopters, rockets, nuclear reactors, DC-to-DC power converters and others. In the past literature special attention is dedicated to the aircraft dynamic stability. The concept of dynamic stability studies what happens to the aircraft in one time period, when it took out the balanced position. The aircraft longitudinal motion is the aircraft response to the disturbances.

The aircraft as a plant is a dynamic system on that the control and external disturbances act in the course of the flight. The aircraft answers to outputs that are the function of control and external disturbances. The goal of aircraft control is that the control system (pilot-autopilot) controls the aircraft position and path. As it is known, the aircraft can have the control system for the control around three axes (Fig. 1):

- x axis — rolling axis (longitudinal axis),
- y axis — pitching axis (transverse axis),
- z axis — yawing axis (normal axis in downward direction),

that they enable the aircraft motion in the desired direction.

In classical approaches, the flight control systems can be realized by: neuronic net [1], adaptive systems [2–5], conventional PID regulator [6] and variable structure system [7–13]. All these control systems are obtained for a continual time domain, while the attempts of this realization were not in the discrete domain [14]. In this paper we will make an attempt for a discrete-time variable structure system (DTVSS) for the aircraft elevator control, on the basis of a method obtained for the control of plants with finite zeros in the canonical subspace [15–17].

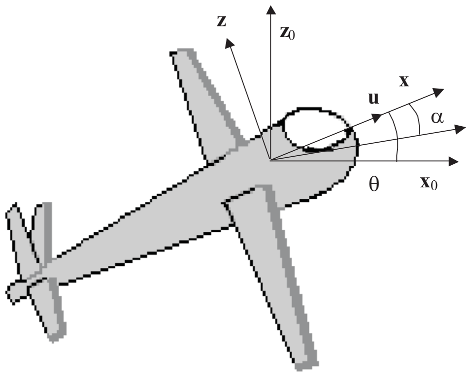

Fig. 1. Aircraft longitudinal motion.

2 AIRCRAFT LONGITUDINAL MOTION

The equations of motion for the aircraft are derived by applying Newton's laws of motion which relate the summation of all external forces and moments to the linear and angular accelerations of the body. Newton's laws are related to the coordinate system (x, y, z), which is fixed to the aircraft and rotates, while the axis system (x_0, y_0, z_0) is the inertial coordinate system of the earth. After that linearization, the equations of motion for the aircraft are obtained in the following form [18]:

$$\begin{aligned} \sum F_x &= m(\dot{U} + WQ)\,, & \sum M_x &= \dot{P} I_x - \dot{R} J_{xz}\,, \\ \sum F_y &= m(\dot{V} + UR - WP)\,, & \sum M_y &= \dot{Q} I_y\,, \\ \sum F_z &= m(\dot{W} - UQ)\,, & \sum M_z &= \dot{R} I_z - \dot{P} J_{xz}\,. \end{aligned} \tag{1}$$

where: m is the mass, F_x, F_y, F_z are the external forces in the directions of x, y and z axes, M_x is the rolling moment, M_y is the pitching moment, M_z is the yawing moment, u, v, w are the components of the aircraft

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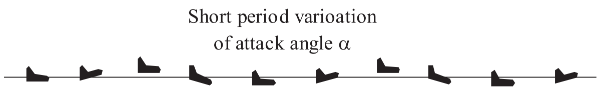

Fig. 2. The short-period oscillations.

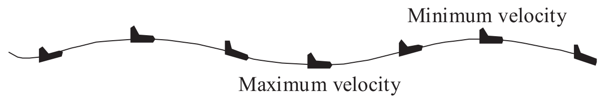

Fig. 3. The phugoid oscillations.

linear velocity v_T in the directions of x, y and z axes, P, Q, R are the components of the aircraft angular velocity ω in the directions of x, y and z axes, I_x, I_y, I_z are the moments of inertia in the directions of x, y and z axes and J_{xz} is the product of inertia I_x and I_z. These six equations (1) can be broken up into two sets of three simultaneous equations and the aircraft motion can be broken up to:

- aircraft longitudinal motion,
- aircraft lateral motion.

In aircraft longitudinal motion (Fig. 1) the control magnitudes are:

- α – the angle of attack,
- θ – the pitch angle,
- u – the variation of flight velocity along the longitudinal axis x,

and the control input is:

- δ_e – the elevator deflection.

The aircraft longitudinal motion can be represented as a system of equations in which the forces and moments are obtained by the control magnitudes: α, θ and u [18]:

$$\begin{aligned} &(A_1\dot{u}_n - C_{x_u}u_n) + (A_2C_{x_\alpha}\dot{\alpha}_n - C_{x_\alpha}\alpha_n) + \\ &\qquad (A_2C_{x_q}\dot{\theta} - C_w(\cos\Theta)\theta) = C_{F_{x_\alpha}}\,, \\ &-C_{z_u}\dot{u}_n + (A_4\dot{\alpha}_n - C_{z_\epsilon}\alpha_n) + \\ &\qquad (A_5\dot{\theta} - C_w(\sin\Theta)\theta) = C_{F_{z_\alpha}}\,, \\ &-C_{m_u}\dot{u}_n + (A_2\dot{\alpha}_n - C_{m_\alpha}) + \\ &\qquad \Big(\frac{I_y}{Sqc}\ddot{\theta} + A_2C_{m_q}\dot{\theta}\Big) = C_{m_\alpha}\,, \end{aligned} \tag{2}$$

where: $A_1 = \frac{mU}{Sq}$, $A_2 = -\frac{c}{2U}$, $A_3 = A_1 + A_2C_{z_\alpha}$, $A_4 = -A_1 + A_2C_{z_q}$, C_{**} is the aerodynamic constant of the aircraft, S is the wing area, q is the dynamic pressure, $u_n = \frac{u}{U}$, $\alpha_n = \frac{w}{U}$, $\dot{\alpha}_n = \frac{\dot{w}}{U}$ are nondimensional coefficients, u, w are the changes of the linear velocity components and Θ is the angle between the longitudinal axes of the movable (x) and inertial (x_0) coordinate system. Taking the Laplace transform of equations (2) with the initial conditions zero, the characteristic equation is obtained in the form:

$$\big(s^2 + 2\xi_p\omega_{np}s + \omega_{np}^2\big)\big(s^2 + 2\xi_s\omega_{ns}s + \omega_{ns}^2\big) = 0\,,$$

where: ω_{np}, ω_{ns} are the natural frequencies, ξ_p, ξ_s are the damping factors. We can observe the existence of two oscillation types:

- the short-period oscillations (ω_{ns}, ξ_s) — the oscillations of short period with relatively heavy damping (Fig. 2),
- the phugoid oscillations (ω_{np}, ξ_p) — the oscillations of long period with relatively light damping (Fig. 3).

The periods and the damping of these oscillations vary from aircraft to aircraft with the flight conditions. The short-period oscillations challenge the variations of α_n and θ with a very little change of u_n, and the phugoid oscillations challenge the variations of θ and u_n with a very little change of α_n. The phugoid can be thought of as a change of potential and kinetic energies.

3 TRANSFER FUNCTION OF AIRCRAFT ELEVATOR

We consider the aircraft flying in a straight and level flight at with a velocity of . For this aircraft the aerodynamic constants values are [18]:

$$\begin{gathered} \Theta = 0\,,\ m = 5800\,\text{slugs},\ U = 600\ \text{ft/sec}, \\ S = 2400\,\text{sq ft},\ q = 2.62\times10^{-6}\text{slug ft}^2,\ c_w = -0.74\,, \\ C_{x_u} = -0.088\,,\ C_{x_\alpha} = 0.392\,,\ C_w = -0.74\,, \\ C_{z_u} = -1.48\,,\ C_{z_\alpha} = -4.46\,,\ C_{z_\alpha} = -4.46\,, \\ C_{m_\alpha} = -0.619\,,\ C_{m_\alpha} = -3.27\,,\ C_{m_q} = -11.4\,. \end{gathered} \tag{3}$$

To obtain the transfer function of the aircraft elevator, it is necessary to define the positive deflection of the elevator. Down elevator is defined as a positive elevator by NASA convention. Thus a positive elevator deflection produces a negative $-\dot{\theta}$. Taking the Laplace transform of relations (2) with the initial conditions zero and after the substitution of the appropriate values (3) the transform function of the aircraft elevator is obtained:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-1.31(s+0.016)(s+0.3)}{(s^2+0.00466s+0.0053)(s^2+0.806s+1.311)}\,. \tag{4}$$

The transfer function (4) shows that a considerable variation in θ occurs at both short-period ($\omega_{ns} = 1.14$ rad/sec, $\xi_s = 0.352$) and phugoid ($\omega_{np} = 0.073$ rad/sec, $\xi_p = 0.032$) oscillations. These observations lead to the following approximations of the phugoid and short-period oscillations:

- Short-period approximation

The short-period oscillations occur at an almost constant

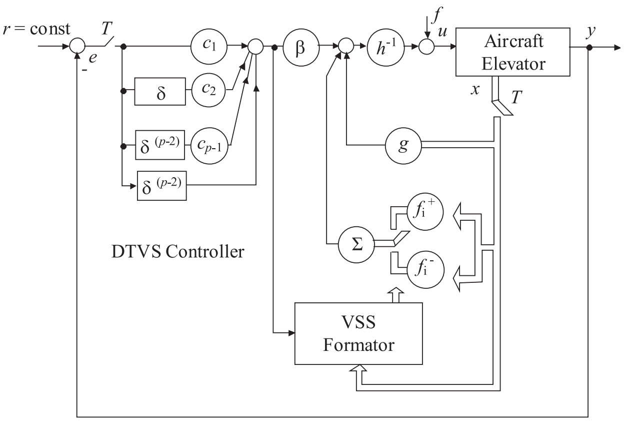

Fig. 4. DTVSS for aircraft elevator control.

flight speed u, because forces in the x direction contribute mostly to changes in the flight speed. The transfer function (4) can be approximated by function:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-1.39(s+0.306)}{s(s^2+0.805s+1.325)}\,. \tag{5}$$

The short-period approximation (5) shows very good agreement in the vicinity of the natural frequency of the short-period oscillations. It is wing for the simulation of the aircraft elevator transfer function.

• Phugoid approximation

The phugoid oscillations take place at an almost constant angle of attack α. As the phugoid oscillations are of long period, θ is varying quite slowly, therefore, the inertia forces can be neglected. The transfer function (4) can be approximated by function:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{0.018(s+0.00637)}{s^2+0.00645s+0.00582}\,. \tag{6}$$

Comparison of (4) and (6), shows good agreement for the natural frequencies and damping ratios. There is a 180° phase difference. Thus, the phugoid approximation is not satisfactory for simulation purposes.

4 DTVSS FOR AIRCRAFT ELEVATOR CONTROL

For aircraft elevator control a DTVSS is proposed that is shown in Fig. 4. The discrete-time variable structure controller synthesis is performed on the basis of the elevator transfer function short-period approximation (5) that can be introduced in the state space as:

$$\begin{aligned}\dot{\overline{\mathbf{x}}}(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}\overline{\mathbf{x}}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\boldsymbol{u}(t)\,,\\ \boldsymbol{y}(t) &= [\,d_0 \quad d_1 \quad 0\,]\,\overline{\mathbf{x}}(t)\,,\\ &a_0 = 0\,,\ 1.06 \le a_1 \le 1.59\,,\ 0.744 \le a_2 \le 0.966\,,\\ &-0.5 \le d_0 \le -0.34\,,\ -1.668 \le d_1 \le -1.112\,.\end{aligned} \tag{7}$$

The DTVS controller synthesis is performed using the method for control of plants with finite zeros in canonical subspace [15–17] because the transfer function (5) has finite zero. The elevator the under the action of a disturbance:

$$f(t) = 0.1\big(h(t-7) - h(t-9)\big)\,.$$

First, we must determine the system mathematical model (7) in canonical space. Applying delta transformation for the discretization period $T = 0.1$ ms, the DTVSS model takes the form:

$$\begin{aligned}\delta\overline{\mathbf{x}}(kT) &= \begin{bmatrix} 0 & 0.99999999 & 0.00004999 \\ 0 & -0.0006625 & 0.99995975 \\ 0 & -1.32494667 & -0.80493385 \end{bmatrix}\overline{\mathbf{x}}(kT) + \\ &\qquad \begin{bmatrix} 0 \\ 0.00004999 \\ 0.99995975 \end{bmatrix} u(kT)\,,\\ y(kT) &= [\,-0.42534 \quad -1.39 \quad 0\,]\overline{\mathbf{x}}(kT)\,.\end{aligned} \tag{8}$$

For simplicity of the relations in the future explanation $\bullet(kT)$ will be $\bullet(k)$. Using the matrix coordinate transformation [15–17]:

$$\mathbf{x}(k) = \mathbf{P}\overline{\mathbf{x}}(k)\,,$$

where:

$$\mathbf{P} = \begin{bmatrix} 0.999959776882 & 0.000099996646 & 0.000000001667 \\ 0 & 0.999959748451 & 0.000049998658 \\ 0 & 0 & 0.999959748871 \end{bmatrix},$$

the system (8) is transformed into controllable canonical form:

$$\begin{aligned}\delta\mathbf{x}(k) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1.324947 & -0.804934 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u(k)\,,\\ y(k) &= [\,-0.42534 \quad -1.39 \quad 0\,]\mathbf{x}(k)\,.\end{aligned} \tag{9}$$

Then, we do the decomposing of the canonical space (9) to subspace with and without control. Successively finding the delta transform of the output variable $y(k)$, the system model over the two-dimensional subspace in the following form:

$$\begin{aligned}\delta y(k) &= [\,0 \ \ -0.425323 \ \ -1.389986\,]\mathbf{x}(k)\,,\\ \delta^2 y(k) &= [\,0 \ \ 1.841658 \ \ 0.693524\,]\mathbf{x}(k) - 1.389986u(k)\,.\end{aligned} \tag{10}$$

will be composed.

The aim of the DTVS controller is to select control $u(k)$ so that for any arbitrary initial conditions the stable discrete-time sliding mode on the switching line:

$$g(k) = cy(k) + \delta y(k)\,, \quad c = \text{const}, \tag{11}$$

will occur. To choose the switching line parameter c, let us determine the equivalent control $u_{eq}(k)$ from the condition [19] for the system to remain on the switching

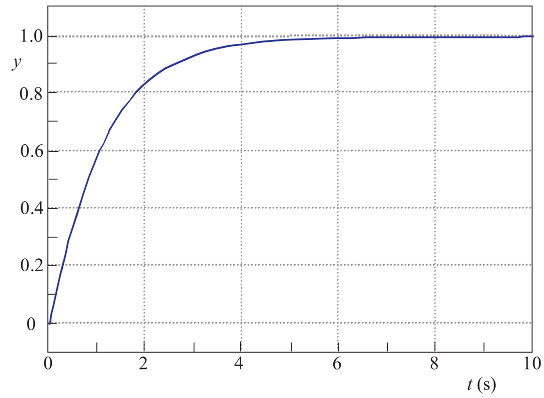

Fig. 5. Step response of the nominal plant with load disturbance $f(t) = 0.1\big(h(t-7) - h(t-9)\big)$.

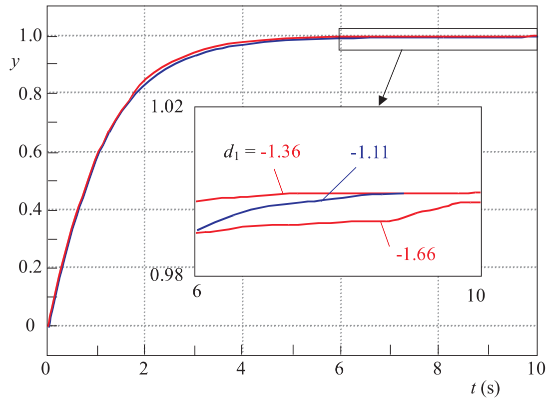

Fig. 6. Step responses of the plant with parameter d_1 variation.

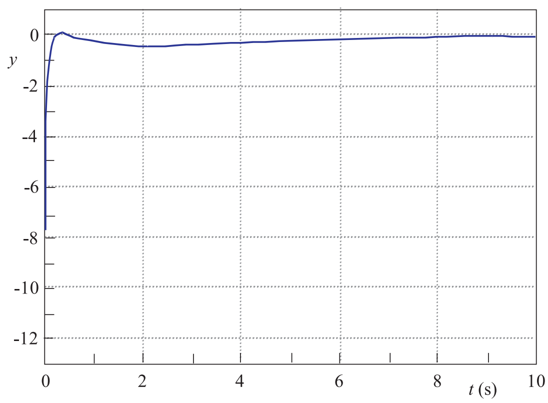

Fig. 7. Control signal of the nominal plant with load disturbance $f(t) = 0.1\big(h(t-7) - h(t-9)\big)$.

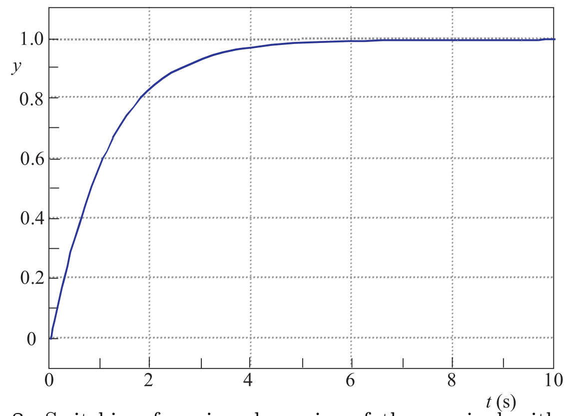

Fig. 8. Switching funcyion dynamics of the nominal with load disturbance $f(t) = 0.1\big(h(t-7) - h(t-9)\big)$.

line $g(k+1) = g(k)$ for each k, that is, from the condition, that:

$$g(k) = 0\,, \quad \delta g(k) = 0\,. \tag{12}$$

From the condition (12) the equivalent control $u_{eq}(k)$ is obtained as:

$$u_{eq}(k) = [\,0 \quad 0.305991c + 1.324934 \quad -c + 0.498943\,]\mathbf{x}(k)\,. \tag{13}$$

The system characteristics equation with the equivalent control $u_{eq}(k)$ (13) is:

$$z^3 + (c + 0.305991)z^2 + (0.305991c + 0.000013)z = 0\,.$$

Applying the Jury's stability test [20], the parameter c of the switching line is chosen as $c = 1$, in such way that the system is stable over the whole canonical space (over the subspaces both with and without control). Then the switching line (11) has the following form:

$$g(k) = y(k) + \delta y(k)\,,$$

and the equivalent control $u_{eq}(k)$ (13) is:

$$u_{eq}(k) = [\,0 \quad 1.018943 \quad -0.501057\,]\mathbf{x}(k)\,.$$

At the end the quasi-sliding mode in canonical subspace (10) with control is realized by theorem [15–17]:

Theorem. *The system is stable in canonical subspace, if the control:*

$$u(k) = h^{-1}\mathbf{g}\mathbf{x}(k) + h^{-1}\mathbf{f}\mathbf{x}(k) + h^{-1}\beta g(k)\,,$$

where:

β – *constant* $|\beta| < 1$, $\beta \neq 0$,

$\mathbf{f} = [\,f_1 \quad f_2 \quad \dots \quad f_n\,]$ – *commutation coefficients, such that:*

$$|f_i^+| = f_i^-| = f_i = \begin{cases} 0 & |g(k) \le \gamma(k)\,, \\ -f\,\mathrm{sgn}\big(\beta g(k) x_i(k)\big) & |g(k)| > \gamma(k)\,, \end{cases}$$

$i = 1, \dots, n$,

$\gamma(k) = \frac{f}{2|\beta|} \sum_{i=1}^{n} |x_i(k)|$,

$0 < f < 2|\beta| \max_{1 \le i \le n} \Big(\mathbf{c}_p \left[\, \boldsymbol{d}_\delta^\top \quad (\boldsymbol{d}_\delta \mathbf{A}_\delta^\top \quad \dots \quad (\boldsymbol{d}_\delta (\mathbf{A}_\delta^{p-1})^\top \,\right]^\top \Big)$.

The following parameter values according to theorem

$$\mathbf{g} = [\,0 \quad 1.018943 \quad -0.501057\,] \times 10^{-4},$$
$$h = -1.89986 \times 10^{-4},$$
$$f = 9 \times 10^{-6}, \quad \beta = 0.0015\,.$$

are determined. The simulation results are shown in the form of step responses (Figs. 5 and Fig. 6), for plant parameters variation in the above given boundaries and disturbance action, control signal (Fig. 7) and switching function dynamics (Fig. 8). Based on the diagram (Fig. 6), it is concluded that the system is stable, invariant and robust against plant parameters variation in the given boundaries. It is robust for disturbation action (Fig. 7).

6 CONCLUSION

Discrete-time variable structure controller for aircraft elevator control, using the method for control of plants with finite zeros in canonical subspace is proposed in this paper. First, a discrete mathematical model of the system over canonical space, using the delta transform, is given. Then, decomposition of the canonical space to subspaces with and without control is carried out by introducing the output variable delta transform. Finally, a relation providing the quasi-sliding mode over the canonical subspace with control is derived. The realized system is stable and robust against parameter and external disturbances.

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