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STRUCTURED VERSUS MINIMAL TRELLISES COMPLEXITY COMPARISON FOR SOME BEST $[n, k, d]$ CODES

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This paper presents new linear binary block codes with codeword length n, number of information symbols in codewords k and code distance d_m, which reach the upper or lower bounds on maximal code-distance. They were found using the Kasahara *et al* construction. This method supports constructions of structured trellises, which has practical importance for wireless communications and software defined radio (SDR). The knowledge of the structured trellis can simplify decoding of the corresponding code using different soft (turbo) algorithms. In this way the set of useful codes for practical applications is broadened to some 'best' codes that would otherwise be difficult to decode using soft algorithms.

K e y w o r d s: linear block codes, Hamming distance, Kasahara construction, minimal trellis, structured trellis, turbo-decoding

1 INTRODUCTION

Turbo-codes based on convolutional codes and LDPC codes found applications in wireless systems [3, 4]. The decoding complexity of such codes is still an important factor to take into account especially when SDR realization is necessary. It is dependent on the possibility of using the structure of the codes. One way to exploit structure is to use product codes. There are integrated circuits in which Hamming codes and extended Hamming codes are used as the constituent codes [5]. It is known, however, that most product codes fail to reach the upper bounds on code distance d_m for a given codeword length n and dimension k [6]. It would be useful to increase the number of codes that could be used for different soft decoding algorithms and Turbo-encoding/decoding in future wireless systems and SDR. For example the Sandbridge Sandblaster® SDR platform SB3011 includes four DSPs. Each DSP is 8-way multithreaded allowing to execute up to 32 independent instruction streams. One way how to adapt the decoding procedure to such SDR architecture can be by finding a structured trellis for the code with identical parallel subtrellises.

The goal of this paper is to find new codes that reach the upper or lower bounds on $d_{\max}(n, k)$ with structure, which enables structured trellises to be found. Such trellises allow the application of different soft decoding algorithms and turbo algorithms with more acceptable complexity. An interactive interface to the upper and lower bounds on $d_{\max}(n, k)$, the maximum possible Code-distance (Hamming distance) of some linear binary, ternary and quadruple block codes, can be found at [9]. In the tables of [9] the lower bounds correspond to existing codes and in some cases the lower bounds are identical to the upper bounds. The tables contain also information on how the codes were constructed. Many different methods were used, including, not only algebraic, geometric and other mathematical branches, but also a computerized search [7, 8]. Therefore, the structures of the trellises of such codes (if known) are in most cases irregular and quite complicated.

From the practical point of view, the decoding algorithm complexity of different soft turbo-decoding methods can restrict or enable their usage to be considered for the underlying code of a wireless application or other system. In the following section, it will be shown that some new codes could be obtained using the Kasahara *et al* construction, with identical n, k, d_m parameters, as the best codes from [2]. In Section 3 the method for constructing structured trellises for the codes obtained in Section 2 will be presented. Finally, Section 4 contains some concluding remarks.

2 BASIC THEORY OF LINEAR BLOCK CODES

Let $C = [n, k, d_m]$ be a linear block code defined over finite field $GF(q)$. n denotes the number of codeword symbols, k denotes the number of information symbols and d_m denotes the code distance. Important terms in coding theory are Hamming distance and Hamming weight. Hamming distance of two vectors $v = (v_1, v_2, \ldots, v_n)$ and $u = (u_1, u_2, \ldots, u_n)$ is the number of symbols in which these two vectors differ. It is denoted as $d(v, u)$. By minimal or code distance is denoted the minimal Hamming distance between any two codewords of a given code. The code distance is denoted as d_m. A linear block code (LBC) is capable of correcting t errors if $d_m \geq 2t + 1$. The Hamming weight of vector $v = (v_1, v_2, \ldots, v_n)$ is the number of its non-zero elements. It is denoted as $w(v)$. The linear block code is completely defined by its generator matrix G with dimensions $k \times n$.

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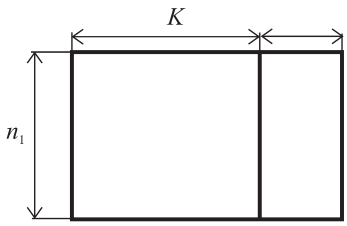

Fig. 1. Principle of Kasahara *et al* construction

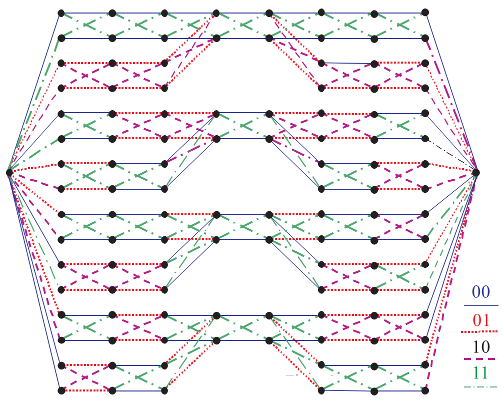

Fig. 2. The trellis of the code C1 $[18, 12, 4]$.

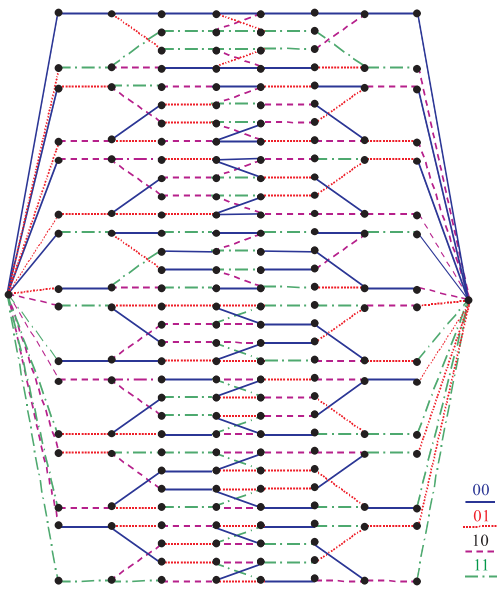

Fig. 3. The trellises of the code C3 $[18, 6, 8]$.

If the Hamming distance in LBC of two codewords v and v' equals d, then $w(v - v') = d$. It means that the code distance of LBC d_m equals to the minimal weight of its non-zero codeword. It implies that the exact knowledge of weight distribution of all codewords by their weights called weight spectrum allows to determine exactly d_m. Several methods exist for the calculation of the weight spectrum for some codes:

- Analytical methods — based on exact formulas for the weight spectra calculation dependent on n and k which are known only for certain types of codes. One example is the group of well-known Reed-Solomon codes.
- Exhaustive methods — based on direct calculation of weights of all exhaustively generated codewords.
- Statistical methods — based on different approximations methods, which applies only for certain types of codes.

For the new codes obtained in this paper the analytical method for obtaining the weight spectrum is not known and statistical methods give only approximate results. It was decided to use exhaustive calculations of exact weight spectra. It is well known that the codes constructed by the Kasahara method belong to LBC [11].

3 NEW CODES WITH PARAMETERS IDENTICAL TO BEST KNOWN CODES

Kasahara *et al* published in [2] the following well known construction. Let C_1 be a $[n_1, k_1, d_1]$ code over $GF(2^k)$ and let the codewords of the code C_1 be written as column vectors. If each symbol of the codeword is written in the form of a binary vector with K coordinates, we get a binary $n_1 \times K$ matrix. Adding one parity symbol to each row, we get a binary $[n_1(K+1), Kk_1, 2d_1]$ code C' (see Fig. 1). This is a concatenated code. The code distance is greater than or equal to $2d_1$, because in each nonzero codeword there are at least d_1 non-zero rows and the weight of each non-zero row is at least two. In [2] it was shown that the dimension of the code can be increased by k_3 by adding, modulo two, another C_3 code over $GF(2)$, with parameters $[n_1, k_3, 2d_1]$, to the last column of C' (Fig. 1). The new resulting code C is then an $[n_1(K+1), Kk_1 + k_3, 2d_1]$ code. Table 1 shows the different combinations of basic parameters of the codes and the resulting parameters of the new codes that could be obtained by the aforementioned construction. The step in which addition of the parity symbol is done, is equivalent to forming a cascade code or product code composed of C_1 and C_2. The code C_2 is a $[2, 1, 2]$ code.

4 STRUCTURED VS. MINIMAL TRELLISES OF THE NEW CODES

Complexity of a decoding process based on trellises generally depends on number of vertexes and edges of trellis. For all linear block codes the minimal trellis representation can be found. In Table 2 are the trellis complexities for same new codes found by decimal permutation algorithm [12].

Another way how decoding complexity can be reduce is the use of the structured trellis representation for linear block codes. The structure of the new codes, presented in the previous section, does allow structured trellises to be obtained. The construction is described in [1] in more detail. For each sub-code a separate trellis is constructed and the resulting trellis is obtained as a product of all these trellises.

Table 1. Basic parameters of some codes for Kasahara *et al* construction

C	$C1$	$C3$
$[24,9,8]$	$[12,7,4]$	$[12,2,8]$
$[30,14,8]$	$[15,10,4]$	$[15,4,8]$
$[32,16,8]$	$[16,11,4]$	$[16,5,8]$
$[36,18,8]$	$[18,12,4]$	$[18,6,8]$
$[38,20,8]$	$[19,13,4]$	$[19,7,8]$
$[40,22,8]$	$[20,14,4]$	$[20,8,8]$
$[42,24,8]$	$[21,15,4]$	$[21,9,8]$
$[44,26,8]$	$[22,16,4]$	$[22,10,8]$

Table 2. Minimal trellis complexity

	Binary		Quarter		Octet	
C	V	E	V	E	V	E
$[18,6,8]$	238	284	106	152	–	–
$[18,12,4]$	220	374	100	270	–	–
$[36,18,8]$	9470	13052	3754	7336	1826	5664
$[22,10,8]$	1534	1980	618	1064	–	–
$[22,16,4]$	262	444	114	328	–	–
$[44,26,8]$	58622	89596	–	–	–	–

Table 3. Structured trellis complexity

	Trellis		Parallel part		Type
C	V	E	V	E	
$[18,6,8]$	194	256	24	32	quarter
$[18,12,4]$	114	240	28	60	quarter
$[36,18,8]$	2562	6656	320	832	octet
$[22,10,8]$	738	1120	92	140	quarter
$[22,16,4]$	250	608	62	152	quarter

In Table 3 are structured trellis complexities for some new codes. As we can see the number of vertices and the edges are higher compared to minimal trellises (Table 2). However significant reduction is in the particular parallel parts of the trellises.

For example the code $[36,18,8]$ is composed of the $[18,12,4]$ code, another simple parity check code $[2,1,2]$ and the $[18,6,8]$ code. The structured trellises for these codes are presented in Figs. 2-3. The trellis of the resulting code could be constructed as a product trellis of these two trellises. In this case the resulting structured trellis contains eight structures with the same shape as depicted in Fig. 2. This could allow parallel processing during Turbo-decoding and especially for SDR platforms which support more processors or multithread processing [10]. For example if we use the Viterbi algorithm in the decoder, we can look at each parallel subtrellis as at a single trellis and process them independently. Same way can be used also with different turbo algorithms such as BCJR *etc.*

A similar method is used in the trellis construction for another code $[44,26,8]$ from Table 1. This code is composed of the $[22,16,4]$ code, another simple parity check code $[2,1,2]$, and an $[22,10,8]$ code.

The trellises have complexity resulting also from the number of codewords. Therefore the trellises for final codes $[36,18,8]$ and $[44,26,8]$ are so complex that it is not possible to show them here. But they could be found on the web page
http://www.ktl.elf.stuba.sk/∼obona/trellises/.

5 CONCLUSION

In Section 2 the tables of new codes obtained by the Kasahara construction [2] were presented. These codes have the same basic parameters n, k, d_m, as the best binary linear block codes. In Section 3 an example was presented of how to exploit the regularities in the construction to obtain a structured trellis for the new code. Such trellises allow the new codes to be decoded with much lower computational complexity than without knowledge of the trellises. The reduced complexity could allow the application of more efficient decoding algorithms, such as turbo-decoding or other soft algorithms.

References

[1] FARKAŠ, P.—HOC, R.—HEBBES, L.: On Structured Trellises of some Best $[n,k,d]$ Codes, In: 5th International Symposium on Wireless Personal Multimedia Communications: Honolulu, USA, 27–30 October 2002, pp. 1262–1264.

[2] KASAHARA, M.—SUGIYAMA, Y.—HIRASAWA, S.—NAMEKAWA, T.: New Classes of Binary Codes Constructed on the Basis of Concatenated Codes and Product Codes, IEEE Trans. on Inform. Theory **IT-22** (1976), 462–468.

[3] HANZO, L.—CHERRIMAN, P. J.—STREIT, J.: Wireless Video Communications — Second to Third Generation and Beyond, IEEE Press, New York, 2000..

[4] http://www.qualcomm.com/technology/flash-ofdm/products/index.html, Qualcomm FLASH-OFDM, accessed 1.6.2007.

[5] RISLEY, A.—PICKAVANCE, K.: Turbo Product Codes in Wireless Applications, In: Proc. of 6th International Symposium on Communications Theory and Applications, ISCTA'2001, Ambleside, 15-16 July 2001, p. 322.

[6] CHIARALUCE, F.—GARELLO, R.: On Asymptotic Performance of Hamming Product Codes, In: Proc. of, ISCTA'2001, Ambleside, 15-16 July 2001, p. 322.

[7] BROUWER, A. E.—VERHOEFF, T.: An Updated Table of Minimum-Distance Bounds for Binary Linear Code, IEEE **IT-39** No. 2 (1993), 662–677.

[8] FARKAŠ, P.—BRUEHL, K.: Three Best Binary Linear Block Codes of Minimum Distance Fifteen, IEEE Trans. on Inform. Theory **IT-40** No. 3 (1994), 949–951.

[9] http://www.win.tue.nl/∼aeb/voorlincod.html.

[10] GLOSSNER, J.—IANCU, D.—MOUDGRILL, M.—SCHULTE, M. J.: The Sandbridge Sandblaster® SDR platform, In: Proc. of Symposium on Trend in Communications, SympoTIC'2006, Slovakia, 24–27 June 2006, pp. II..

[11] McWILLIAMS, F. J.—SLOANE, N. J. A.: The Theory of Error-Correcting Codes, 10.imp. NorthHolland Mathematical Library, vol. 16, 1998.

[12] MANOUKIAN, H. H.—HONARY, B.: BCJR Trellis Construction for Binaty Linear Block Codes, IEE Processings-Communication **144** (1997), 367–371.

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