COMMUNICATIONS

ROBUST CONTROL OF ELECTRICAL MACHINES WITH LOAD UNCERTAINTY

Ziqian Liu * — Qunjing Wang **

This paper presents a new approach toward the design of a robust control for different types of electrical machines, such as, DC motor, induction motor, synchronous motor, etc, with load uncertainty. In order to develop the main theoretical result, the concepts of Lyapunov stability and linear optimal state feedback control are used. The proposed design is easy to implement in industrial applications.

K e y w o r d s: robust control, Lyapunov stability, linear optimal control, electrical motors

1 INTRODUCTION

Utkin indicates that the control of electrical machines is one of the most challenging applications because of the use of electrical servomechanisms in control systems, the advances of high-speed power electronics, and nonlinear high-order characteristics of control plants [1]. Therefore, it is observed that much effort has been conducted to study the robust control for different types of electrical machines [2], [3] and [4]. In this paper, we provide a solution to the problem of analytically investigating the robust control of position loop for a wide range of electrical machines, such as, DC motor, induction motor, synchronous motor, etc, without the knowledge of a load torque. The control algorithm and the selection of corresponding parameters are analyzed. With an advanced microprocessor control system, it is easy to implement the proposed approach in many industrial applications.

2 PROBLEM FORMULATION

It is well known that the basic model for a variety of electrical motor drives can be described by an electrical part (from u to T_e) plus a mechanical part (from T_e to Θ), both are shown in Fig.1, which is presented in paper [5]. It is the most popular and the simplest model used in industry. This model is based on the knowledge that the electrical dynamics is much faster than the mechanical dynamics in an electrical motor control system. In this model, the authors consider the electrical part as a first order transfer function plus a time delay, in which d takes a tiny value. It is obvious that the time constant of the first order transfer function should be sufficiently small to reflect the faster electrical dynamics. In their paper, a typical value $\tau = 0.16$ ms, which is corresponding to a bandwidth of 1 KHz in a torque loop, is given as an example.

In order to analyze the system more effectively, the model of Fig. 1 is approximated to the model shown in Fig. 2 for our further discussion. Now let us describe the model of Fig. 2 by state space form. The position error can be defined as

$$X_1 = \Theta_{\text{ref}} - \Theta$$

with Θ_{ref} the position command, the derivative of the position error as

$$X_2 = \dot{X}_1 = -\dot{\Theta} = -\omega$$

and

$$X_3 = \dot{X}_2 = -\dot{\omega}$$

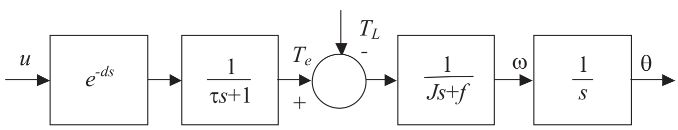

Fig. 1. Basic model of electrical motor drives

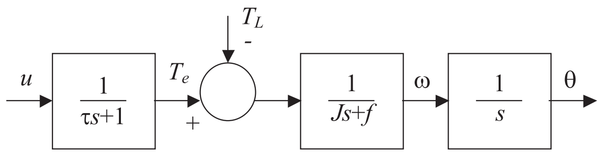

Fig. 2. Improved model of electrical motor drives

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Then, the model of Fig.2 can be illustrated as

$$\dot{X} = AX + Bu + W \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{f}{J\tau} & -(\frac{f}{J} + \frac{1}{\tau}) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -b \end{bmatrix}, \quad b = \frac{1}{J\tau}$$

$$\begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix}, \quad w = \frac{T_L}{J\tau}$$

and T_L is an uncertain constant, *ie*, uncertain load, with a range of $[0, T_L^{\max}]$.

From the equation above, we notice that

$$\frac{w}{b} = T_L \tag{2}$$

and

$$W = -\frac{w}{b}B = -T_L B \tag{3}$$

3 DESIGN PROCEDURE

Now we use the principles of sliding-mode control, Lyapunov stability, and linear-quadratic-regulator (LQR) design approach to develop a robust control. Similar to the result of sliding-mode control [1], we choose our control law as follows

$$u = -M \ \mathrm{sign}(s) \tag{4}$$

where $M > 0$ is the appropriate amplitude of the control signal, and s is the switching surface.

Therefore, there are two design tasks. One is to decide the value or the range of M. the other is to determine the switch surface. Next we use the LQR design approach to determine the switching surface and decide the range of M by a Lyapunov function. The LQR design principle is popular in many industrial applications. Its design approach can be found in many control software packages. The design principle is well known as follows.

Given a linear system as

$$\dot{X} = AX + Bu \tag{5}$$

and a performance index as

$$J = \int_0^\infty \left(X^T QX + u^T Ru\right) \mathrm{d}t \tag{6}$$

where Q is a positive-semidefinite matrix, R is a positive-definite matrix, X is a n-dimensional state vector, and u is a m-dimensional input vector. Therefore, the optimal state-feedback control law is given

$$u^* = -KX, \quad K = R^{-1}B^T P \tag{7}$$

where P is the solution to the following matrix Riccati equation

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{8}$$

and

$$P = P^T > 0 \tag{9}$$

Let us choose the switching surface

$$s = -u^* \tag{10}$$

Therefore, we have

$$s = KX, \quad K = R^{-1}BP \tag{11}$$

Then we study a Lyapunov function

$$V = X^T PX \tag{12}$$

Taking a differentiation of (12) along the system of (1), we have

$$\dot{V} = \dot{X}^T PX + x^T P\dot{X} = \\ = (X^T A^T + u^T B^T + W^T)PX + x^T P(AX + Bu + W) \tag{13}$$

In our case

$$u^T = u, \quad B^T PX = X^T PB,$$

and

$$W^T PX = X^T PW$$

Therefore

$$\dot{V} = X^T \left(A^T P + PA\right) X + 2X^T PBu + 2X^T PW \tag{14}$$

Using (3), (4) and (11), we have

$$\dot{V} = X^T \left(-Q + PBR^{-1}B^T P\right) X \\ -2X^T PBM \,\mathrm{sign}(KX) - 2X^T P(T_L B) \tag{15}$$

Since R and T_L are scalars, we obtain

$$\dot{V} = -X^T QX + RR^{-1}X^T PBR^{-1}B^T PX \\ -2RR^{-1} X^T PB(M \,\mathrm{sign}(KX) + T_L) = \\ = -X^T QX + R(KX)(KX - 2(M \,\mathrm{sign}(KX) + T_L)) \tag{16}$$

From (16), we can see if

$$(KX)(KX - 2(M \,\mathrm{sign}(KX) + T_L)) < 0 \tag{17}$$

then $\dot{V} < 0$ and the system achieves the asymptotic stability.

Case 1: $KX > 0$

Then

$$KX - 2(M + T_L) < 0$$

Therefore we have

$$2(M + T_L) > KX > 0 \tag{18}$$

From equation (18), we consider that when the initial state value is large, the selection of the feedback K should be small in order to hold (18).

Case 2: $KX < 0$

Then

$$KX - 2(-M + T_L) > 0$$

Therefore we have

$$2(M - T_L) > -KX > 0 \tag{19}$$

It is obvious that equation (19) holds if

$$M > T_L \tag{20}$$

In summary, from the analysis above, we know that when the control law is defined as

$$u = -M\,\mathrm{sign}(s)$$

with (i) $M > T_L^{\max}$, and (ii) $s = -u^*$ (u^* is the solution of a LQR problem). Thus, the asymptotical stability of the system (1) is guaranteed.

4 CONCLUSION

Based on a general model for different types of electrical motor drives, a design method of robust control to achieve the asymptotical stability was presented. The contribution of this study is to solve this problem without the knowledge of the load torque. In the process of developing the main theoretical result, the concepts of sliding-mode control, Lyapunov stability, and linear optimal state feedback control are used. The developed control design is able to be employed in a wide variety of industrial applications.

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