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# MODELING AND IDENTIFICATION OF HYSTERESIS USING SPECIAL FORMS OF THE COLEMAN–HODGDON MODEL

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A simple approach to modeling and identification of hysteretic systems using the Coleman-Hodgdon model is presented. Two forms of model are introduced for symmetric and asymmetric hysteresis based on special piecewise linear 'material functions'. For the model parameter identification an iterative approach with internal variable estimation is proposed. Illustrative examples are included.

K e y w o r d s: hysteresis, modeling, identification, Coleman-Hodgdon model

## 1 INTRODUCTION

Hysteresis phenomena are encountered in many different areas of science and technology. The hysteresis is a special type of dynamic nonlinearity, because it is a multi-branching nonlinearity that occurs when the output of a system "lags behind" the input. The best known examples of hysteresis affected phenomena can be found in ferromagnetism, piezoelectricity, plasticity, friction, but also in superconductivity, spin glasses, semiconductors, economics, and physiology.

The hysteresis can be positive and advantageous (*eg*, in magnetic recording for the storage of information), however, in most applications, should be appropriately controlled to minimize or neutralize certain undesirable effects, because it is harmful to the accuracy and performance of those devices. For example, actuators and sensors play an important role in the design of control systems, but they display hysteretic behavior in some regime of operation. This dynamic nonlinearity can lead to instability in closed-loop operations, and complicates the task of controller design and analysis [16]. Therefore, it is of great importance to know the hysteresis description, more precisely, to find the best model approximating the hysteretic nonlinearity.

System identification means the determination of the model of a system from given input/output data, *ie*, selecting the 'best' model from a class of models measured by a chosen criterion or cost function. Perhaps the most crucial element of nonlinear systems identification is the specification of the class of models. If a class of models can be represented by models which are completely determined by a number of parameters, we speak about parameterized model class and the system identification turns to the model parameter estimation.

The fact that hysteresis occurs in a wide variety of areas has probably been the main reason that a range of model classes is available and each has been developed within the context of its own application. To describe the behavior of hysteretic processes several mathematical models have been proposed and a survey may be found in [18]. For a class of hysteresis systems a first order scalar time-domain differential equation can be used to describe the system behavior [5]. A relatively simple differential model of hysteresis, which is appropriate for the representation of rate independent hysteretic systems, is the so-called Coleman-Hodgdon model proposed in [6], [10], [11]. This model (a first-order nonlinear differential equation) is able to capture, in an analytical form, a range of shapes of hysteretic loops, which match the behavior of a wide class of hysteretic systems.

Although more approaches have been proposed for the identification and control of different types of hysteretic systems [2], [4], [7–9], [12–15], [17], [19], there are only two papers dealing with the identification of hysteretic systems using the Coleman-Hodgdon model [1], [3]. The parameters of Coleman-Hodgdon model are calculated by solving complicated transcendental and integral equations in the approach presented in [1]. In the work [3] a series of experiments are executed to determine the parameters characterizing the Coleman-Hodgdon hysteresis, included into the cascade system, on the basis of more sets of input/output data.

In this paper a new and simple approach to modeling and identification of systems with a hysteresis using the Coleman-Hodgdon model is presented. Two special forms of parametric models are introduced for symmetric and more complicated asymmetric hysteresis based on simple piecewise linear 'material functions'. These models enable to perform the identification of hysteretic systems on the basis of available input/output data. For the hysteresis model parameter estimation two iterative approaches with internal variable estimation are proposed and tested by simulation.

## 2 COLEMAN–HODGDON HYSTERESIS MODEL

The differential model of hysteresis according to [6], [10], [11] is a representation of a rate-independent dynamic effect in the form of the first order nonlinear differential equation in the time domain

$$\dot{y} = f(u,y)|\dot{u}| + g(u,y)\dot{u} \tag{1}$$

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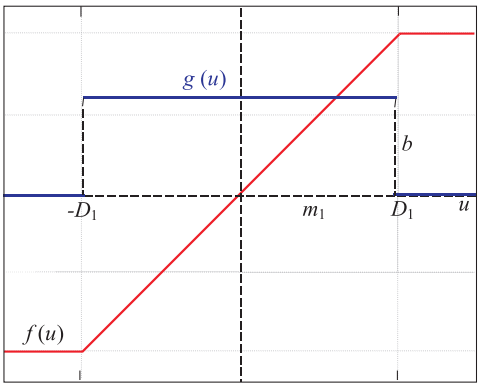


**Fig. 1.** Functions $f(u)$ and $g(u)$

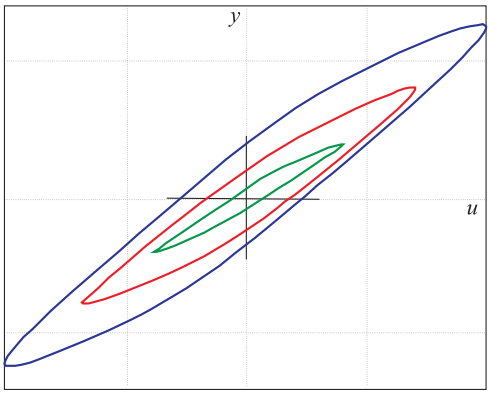


**Fig. 2.** Hysteresis

where $y$ is the output and $u$ is the input. Both $y$ and $u$ are real-valued functions of time with piecewise continuous derivatives, $\dot{y}$ and $\dot{u}$. In the following, the differential models of hysteresis based on a function $f(u,y)$ that is affine in $y$, and a function $g(u,y)$ that is constant in $y$, will be considered. Then the resulting system function of the hysteresis model can be written as:

$$\dot{y} = \alpha[f(u) - y]|\dot{u}| + g(u)\dot{u} \tag{2}$$

or in the discrete form as

$$y(t+1) - y(t) = \alpha\big\{f[u(t)] - y(t)\big\}|u(t+1) - u(t)| + g[u(t)][u(t+1) - u(t)] \tag{3}$$

where $0 < \alpha$ is a real number. It is assumed that [3]:

(i) real-valued function $f(\cdot)$ is odd, monotone increasing and piece-wise continuously differentiable with a finite limit for its first order derivative at positive infinity;

(ii) real-valued function $g(\cdot)$ is even, piece-wise continuous and at infinity of such a finite value that

$$\lim_{s\to\infty} \frac{\mathrm{d}f(s)}{\mathrm{d}s} = \lim_{s\to\infty} g(s)\,; \tag{4}$$

(iii) real-valued functions $f\cdot)$ and $g(\cdot)$ are such that

$$\frac{\mathrm{d}f(s)}{\mathrm{d}s} \geq g(s) \quad \forall s < \infty \tag{5}$$

and

$$\alpha e^{\alpha s} \int_s^\infty \Big[\frac{\mathrm{d}f(\zeta)}{\mathrm{d}\zeta} - g(\zeta)\Big] e^{-\alpha\zeta} \mathrm{d}\zeta \leq g(s) \quad \forall s < \infty\,. \tag{6}$$

## 3 SYMMETRIC HYSTERESIS

### Model

The above described Coleman-Hodgdon hysteresis model can be used for a broad class of hysteretic systems by an appropriate choice of the functions $f(\cdot)$ and $g(\cdot)$. In the simplest case [3] the following functions can be chosen:

$$f[u(t)] = \begin{cases} m_1 D_1\,, & \text{if } D_1 < u(t) \\ m_1 u(t)\,, & \text{if } -D_1 \leq u(t) \leq D_1 \\ -m_1 D_1\,, & \text{if } u(t) < -D_1 \end{cases} \tag{7}$$

$$g[u(t)] = \begin{cases} 0\,, & \text{if } D_1 < u(t) \\ b\,, & \text{if } -D_1 \leq u(t) \leq D_1 \\ 0\,, & \text{if } u(t) < -D_1 \end{cases} \tag{8}$$

where $m_1$ is the slope of the central segment of the function $f(\cdot)$, the constant $D_1 > 0$ determines the range of the central segment as well as the range of $g(\cdot)$ with the constant value $b$ — see Fig. 1. Evidently, the functions $f(\cdot)$ and $g(\cdot)$ agree with the conditions (i)–(iii) and an example of the major hysteresis loop and some minor loops generated by the hysteresis model (3) based on the functions (7) and (8) using triangular inputs u(t) with different amplitudes is shown in Fig. 2. Note that the parts of the graph connecting the loops are not plotted to improve the visibility of loops.

In this case, the hysteresis model is characterized by 4 parameters: $\alpha$, $m_1$, $D_1$, and $b$. However, to estimate the model parameters on the basis of inputs $u(t)$ and outputs $y(t)$ is not easy because of the complicated mathematical model description. One way how to solve this problem is to consider the so-called loading and unloading forms of model equation by substituting the corresponding segments of $f(\cdot)$ and $g(\cdot)$ from (7) and (8) into (3).

Another way is to consider appropriate analytical forms of (7) and (8) enabling to solve the problem of hysteresis model parameter estimation as a pseudo-linear one. Using the switching function defined as

$$h(s) = \begin{cases} 0\,, & \text{if } s > 0 \\ 1 & \text{if } s \leq 0 \end{cases} \tag{9}$$

the functions (7) and (8) can be rewritten as follows:

$$f(u) = m_1 u\big[1 - h(D_1 - |u|)\big] + D_1 m_1 h(D_1 - |u|)\,\mathrm{sign}(u), \tag{10}$$

$$g(u) = b\big[1 - h(D_1 - |u|)\big]\,. \tag{11}$$

After substituting (10) and (11) into (3) the hysteresis model will be linear in the parameters $\alpha$ and $b$, and in the products of parameters $\alpha m_1$ and $\alpha m_1 D_1$, but the parameter $D_1$ appears also nonlinearly in the switching function. However, defining the following internal variable

$$\xi(t) = h\big[1 - h(D_1 - |u(t)|)\big] \tag{12}$$

and assigning

$$m = \alpha m_1\,, \quad D = \alpha m_1 D_1 \tag{13}$$

the model equation can be written as

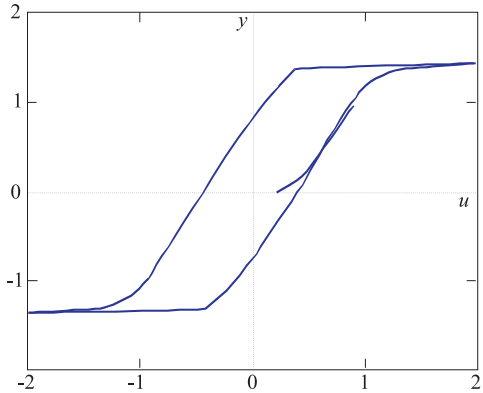


**Fig. 3.** Hysteresis – example 1

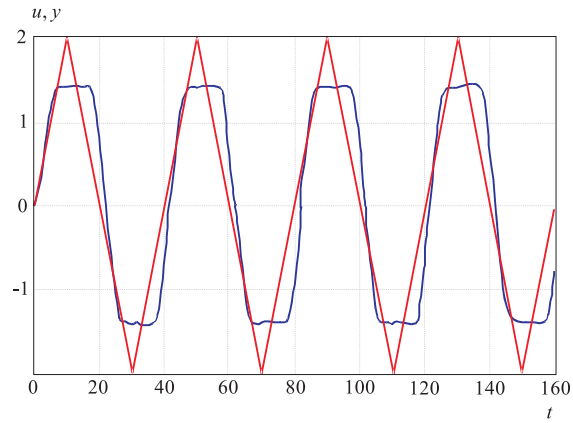


**Fig. 4.** Input $u(t)$ (thin line) and noisy output $y(t)$ (thick line) – example 1

$$\begin{aligned} y(t+1)-y(t) = &-\alpha y(t)|u(t+1)-u(t)| \\ &+ mu(t)[1-\xi(t)]|u(t+1)-u(t)| \\ &+ D\xi(t)\,\mathrm{sign}[u(t)]|u(t+1)-u(t)| \\ &+ b[1-\xi(t)][u(t+1)-u(t)]\,. \end{aligned} \tag{14}$$

Now the model equation is in the form enabling the estimation of the parameters characterizing the hysteresis by the iterative method with internal variable estimation proposed in [20]. It means that in each iteration, the model parameters are estimated using (14) with the estimates of the internal variable $\xi(\cdot)$ based on the previous estimate of parameter $D_1$.

## Parameter estimation

As the internal variable in (14) is not available and must be estimated, an iterative parameter estimation process has to be considered. Defining the following vector of data

$$\begin{aligned} \varphi(t) = \big[&-y(t)|u(t+1)-u(t)|\,,\, u(t)[1-\xi(t)]|u(t+1)-u(t)|\,, \\ &\xi(t)\,\mathrm{sign}[u(t)][u(t+1)-u(t)], [1-\xi(t)]|u(t+1)-u(t)|\big]^{\top} \end{aligned} \tag{15}$$

and the vector of parameters

$$\theta = [\alpha\,,\, m\,,\, D\,,\, b]^{\top} \tag{16}$$

the hysteresis model can be written in the form

$$y(t+1)-y(t) = \varphi^{\top}(t)\theta\,. \tag{17}$$

Now assigning the estimated variable in the $s$-th step as

$${}^{s}\xi(t) = h\big[{}^{s}D_1 - |u(t)|\big] \tag{18}$$

the error to be minimized for the estimation procedure is

$${}^{s+1}e(t) = y(t+1)-y(t) - {}^{s}\varphi^{\top}\,{}^{s+1}\theta \tag{19}$$

where ${}^{s}\varphi(t)$ is the data vector with the corresponding estimates of internal variable according to (15) and ${}^{s+1}\theta$ is the $(s+1)$-th estimate of the parameter vector. If the mean squares errors criterion is used, the following functional will be minimized with respect to the parameter vector:

$${}^{s+1}J = \frac{1}{N}\sum_{t=1}^{N} {}^{s+1}e^2(t) \tag{20}$$

where $N$ is the number of input/output samples.

The steps in the iterative procedure may be now stated as follows:

a) Minimizing ${}^{s+1}J$ based on (19) the estimates of parameters ${}^{s+1}\theta$ are computed using ${}^{s}\varphi(t)$ with the $s$-th estimates of internal variable.

b) Using (18) the estimates of ${}^{s+1}\varphi(t)$ are evaluated by means of the recent estimate of parameter $D_1$.

c) If the estimation criterion is met the procedure ends, else it continues by repeating steps a) and b).

In the first iteration a nonzero initial value of $D_1$ has to be used for evaluation of ${}^{1}\varphi(t)$ to start up the iterative algorithm.

## Simulation study

The first step of the identification procedure is the choice of an appropriate periodic input signal. A good choice of the input signal is a triangle-wave one, where the frequency of the input signal is irrelevant in our case. To identify the parameter $D_1$, the amplitude of the input signal should be chosen large enough, *ie*, larger then this constant. If such inputs cannot be selected (because of physical limitations), then the limit cycle is not observed and we are not able to identify the parameters of the model.

The method for the identification of hysteresis was implemented and tested in MATLAB. Several cases were simulated and the estimations of parameters were carried out on the basis of input and output records. The performance of the proposed method is illustrated on the following example.

Example 1. The symmetric hysteretic system was simulated with the following real parameters: $\alpha = 3.0$, $b = -0.5$, $m_1 = 2.0$, $D_1 = 0.7$. The hysteresis loop

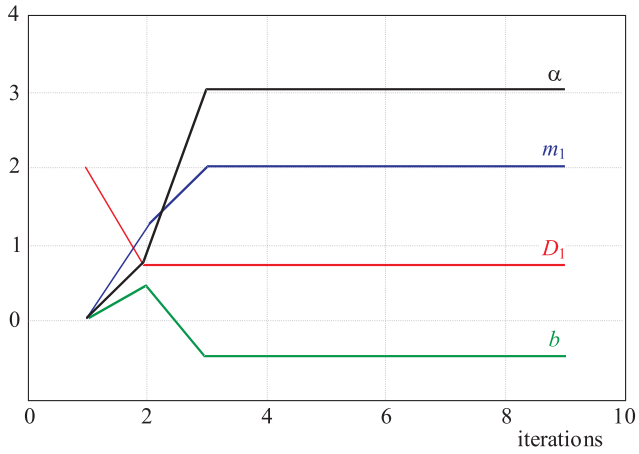


**Fig. 5.** Parameter estimation – example 1

is shown in Fig. 3. The identification was performed on the basis of triangle-wave inputs (thin line) and simulated outputs (thick line) as shown in Fig. 4. To simulate the eventual output measurement errors, normally distributed random noise with zero mean and signal-to-noise ratio – SNR= 50 (the square root of the ratio of output and noise variances) was added to the outputs. The iterative algorithm with estimation of the internal variable $\xi(t)$ was applied choosing ${}^1D_1 = 2.0$ for the first estimate of $\xi(t)$, while the initial values of other parameters were zero. The process of parameter estimation is shown in Fig. 5. The estimates converge to the values of real parameters after 4 iterations.

## 4 ASYMMETRIC HYSTERESIS

### Model

More general forms of the functions $f(\cdot)$ and $g(\cdot)$ were considered in [6], [10], [11] for modeling ferromagnetic hysteresis. These can be generalized by using the asymmetric piecewise linear functions shown in Fig. 6.

The functions $f(\cdot)$ and $g(\cdot)$ can be written as multi-segment piecewise-linear functions in the form proposed in [20] as follows:

$$f(u) = m_{R1}h(-u)u + (m_{R2} - m_{R1}h(D_R - u)u - D_R\xi_1 + m_{L1}h(u)u + (m_{L2} - m_{L1})h(u - D_L)u - D_L\xi_2, \quad (21)$$

$$\xi_1 = \xi_1(u) = (m_{R2} - m_{R1})h(D_R - u)\,, \quad (22)$$

$$\xi_2 = \xi_2(u) = (m_{L2} - m_{L1})h(u - D_L)\,, \quad (24)$$

$$g(u) = b_1 + (b_2 - b_1)h(u - D_R)h(D_L - u) \quad (24)$$

where $0 < m_{R1} < \infty$, $0 \le m_{R2} < \infty$ are the corresponding segment slopes and $0 \le D_R < \infty$ is the constant for the positive inputs, $0 < m_{L1} < \infty$, $0 \le m_{L2} < \infty$ are the corresponding segment slopes and $-\infty < D_L \le 0$ is the constant for the negative inputs; $\xi_1$ and $\xi_2$ are internal variables introduced for the same reason as in the previous case.

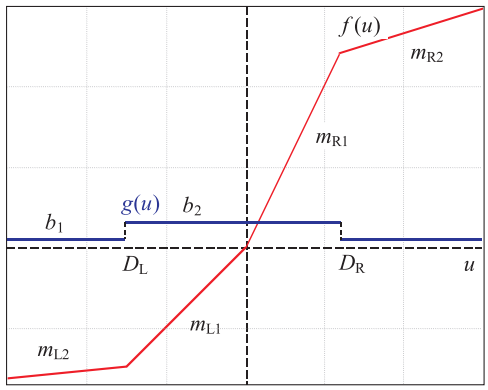


**Fig. 6.** Asymmetric functions $f(u)$ and $g(u)$

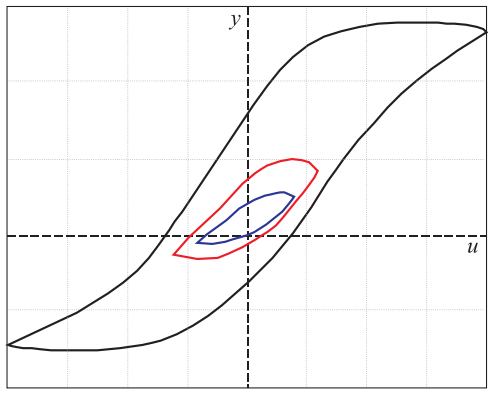


**Fig. 7.** Asymmetric hysteresis

After substituting (21) and (24) into (3) the hysteresis model equation can be written in the form,

$$\begin{aligned} y(t+1) - y(t) = {} & -p_1 y(t)|u(t+1) - u(t)| \\ & + p_2 h[-u(t)]u(t)|u(t+1) - u(t)| \\ & + p_3 h[u(t)]u(t)|u(t+1) - u(t)| \\ & + p_4 h[D_R - u(t)]u(t)|u(t+1) - u(t)| \\ & + p_5 h[u(t) - D_L]u(t)|u(t+1) - u(t)| - p_6\xi_1(t)|u(t+1) - u(t)| \\ & - p_7\xi_2(t)|u(t+1) - u(t)| + p_8[u(t+1) - u(t)] \\ & + p_9 h[u(t) - D_R]h[D_L - u(t)][u(t+1) - u(t)] \end{aligned} \quad (25)$$

where

$$\begin{aligned} & p_1 = \alpha\,, \quad p_2 = \alpha m_{R1}\,, \quad p_3 = \alpha m_{L1}\,, \\ & p_4 = \alpha(m_{R2} - m_{R1})\,, \quad p_5 = \alpha(m_{L2} - m_{L1})\,, \\ & p_6 = \alpha D_R\,,\ p_7 = \alpha D_L\,,\ p_8 = b_1\,,\ p_9 = (b_2 - b_1)\,. \end{aligned} \quad (26)$$

The Coleman-Hodgdon hysteresis model given by (25) is pseudo-linear in parameters and the model parameters can be estimated iteratively as in the previous case, *ie*, using the iterative method with internal variable estimation. Evidently, all the parameters characterizing the hysteresis can be determined from (26), using the estimates of the parameters $p_1, p_2, \ldots, p_9$. An example of asymmetric hysteresis (the major hysteresis loop and some minor loops) generated by (21)–(25) is shown in Fig. 7. Again the parts of the graph connecting the loops are not plotted.

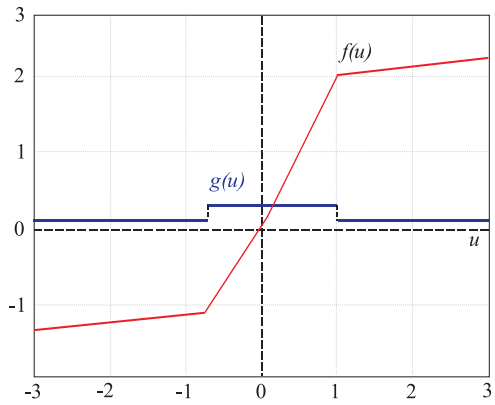


**Fig. 8.** Asymmetric functions $f(u)$ and $g(u)$ – example 2

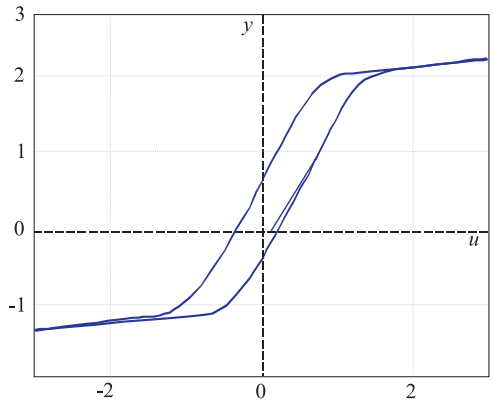


**Fig. 9.** Hysteresis example 2

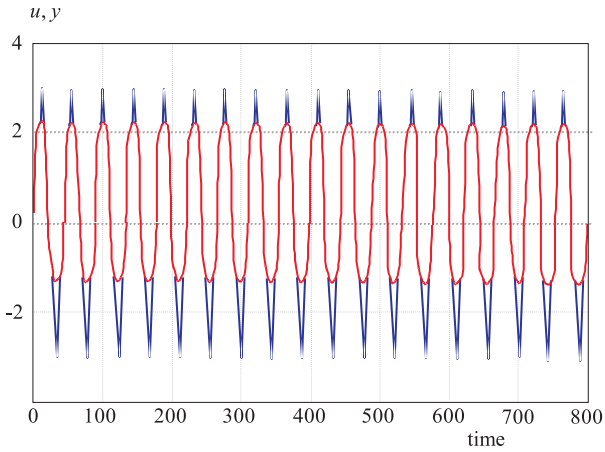


**Fig. 10.** Input $u(t)$ (thin line) and noisy output $y(t)$ (thick line) – example 2

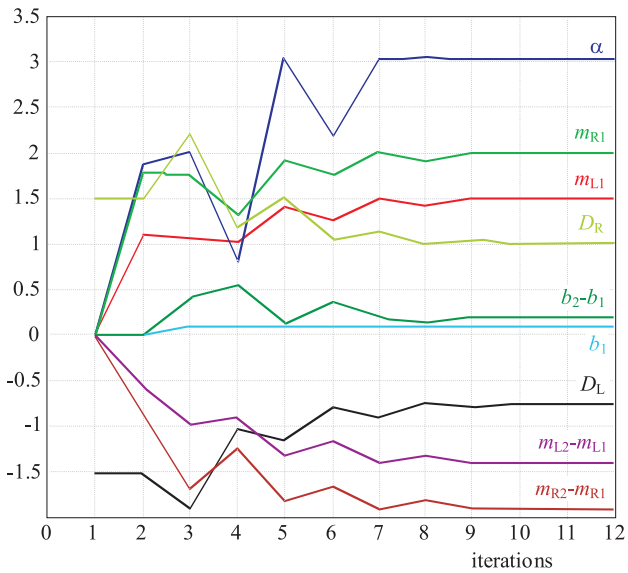


**Fig. 11.** Parameter estimation – example 2

## Parameter estimation

Again an iterative parameter estimation process is considered as the internal variables $\xi_1$ and $\xi_2$ in (25) are not available and must be estimated. Defining the following vector of data

$$\Phi(t) = \big[-y(t)|u(t{+}1){-}u(t)|\,,\, h[-u(t)]u(t)|u(t{+}1){-}u(t)|\,,$$
$$h[u(t)]u(t)|u(t{+}1){-}u(t)|\,,\, h[D_R{-}u(t)]u(t)|u(t{+}1){-}u(t)|\,,$$
$$h[u(t)-D_L]u(t)|u(t+1)-u(t)|\,,\, -\xi_1(t)|u(t+1)-u(t)|\,,$$
$$-\,\xi_2(t)|u(t+1)-u(t)|\,,\, [u(t+1)-u(t)]\,,$$
$$h[u(t)-D_R]h[D_L-u(t)][u(t+1)-u(t)]\big]^\top \qquad (27)$$

and the vector of parameters

$$\Theta = [p_1, p_2, \ldots, p_9]^\top \qquad (28)$$

the hysteresis model can be written in the form

$$y(t+1) - y(t) = \Phi^\top(t)\Theta\,. \qquad (29)$$

Assigning the estimated variables in the $s$-th step as

$$^s\xi_1(t) = \big({}^s m_{R2} - {}^s m_{R1}\big)h\big[{}^s D_R - u(t)\big] \qquad (30)$$
$$^s\xi_2(t) = \big({}^s m_{L2} - {}^s m_{L1}\big)h\big[u(t) - {}^s D_L\big] \qquad (31)$$

the error to be minimized is

$$^{s+1}e(t) = y(t+1) - y(t) - {}^s\Phi^\top(t)\,{}^{s+1}\Theta \qquad (32)$$

where $^s\Phi(t)$ is the data vector with the corresponding estimates of internal variables according to (26), (27) and $^{s+1}\Theta$ is the $(s+1)$-th estimate of the parameter vector.

If the mean squares errors criterion is used, the functional (20) will be minimized and the steps in the iterative procedure are as follows:

a) Minimizing (20) based on (32) the estimates of parameters $^{s+1}\Theta$ are computed using $^s\Phi(t)$ with the $s$-th estimates of internal variables.

b) Using (30) and (31) the estimates of internal variables in $^{s+1}\Phi(t)$ are evaluated by means of the recent estimates of model parameters.

c) If the estimation criterion is met the procedure ends, else it continues by repeating steps a) and b).

In the first iteration the internal variables are considered to be zero, hence only the parameters $p_1, p_2, \ldots, p_5$ are estimated. However, nonzero initial values of $D_L$ and $D_R$ have to be used for evaluation of the switching functions in the data vector.

## Simulation study

The conditions on the identification of asymmetric hysteresis are the same as in the previous case. The following example shows the identification of a simulated hysteresis system.

Example 2. The asymmetric hysteretic system shown in Fig. 9 was simulated with the following parameters: $\alpha = 3.0$, $m_{R1} = 2.0$, $m_{R2} = 0.1$, $m_{L1} = 1.5$, $m_{L2} = 0.1$, $D_R = 1.0$, $D_L = -0.75$, $b_1 = 0.1$, $b_2 = 0.3$. The corresponding functions $f$ and $g$ are shown in Fig. 8. The identification was performed on the basis of inputs (thin line) and noisy outputs (thick line) with SNR = 100 as shown in Fig. 10. The iterative algorithm with estimation of the internal variables $\xi_1(t)$ and $\xi_2(t)$ was used with the initial values ${}^0D_R = 1.5$, ${}^0D_L = -1.5$ for the first estimates of $\xi_1(t)$ and $\xi_2(t)$, while the initial values of other parameters were zero. The process of parameter estimation is shown in Fig. 11. The estimates converge after about 10 iterations.

## 5 CONCLUSIONS

A new approach to modeling and identification of hysteretic systems using the Coleman-Hodgdon model has been presented. The proposed new models for symmetric and asymmetric hysteresis are based on the special form of $f(\cdot)$ and $g(\cdot)$ representations. Note that these functions are specific to a material (material functions) and their knowledge is fundamental for the hysteresis systems. For both models, iterative identification algorithms were proposed with internal variable estimation based on input/output data.

The proposed identification method is of an easy use. Furthermore, no a priori knowledge on the parameters is assumed except very rough initial values of the parameters required for the first estimates of internal variables. This approach may be a novel contribution to the solution of problems arising in many application areas dealing with the hysteresis characteristics and requiring their analytical descriptions.

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