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DESIGN OF ROBUST GUARANTEED COST PID CONTROLLER FOR NETWORKED CONTROL SYSTEMS

Quang Thuan Nguyen — Vojtech Veselý *

The paper addresses the problem of an output feedback guaranteed cost controller design for Networked Control Systems (NCSs) with time-delay and polytopic uncertainties. By constructing a new parameter-dependent Lyapunov functional and applying the free-weighting matrices technique, the parameter-dependent, delay-dependent design method will be obtained to synthesize PID controllers achieving a guaranteed cost such that the NCSs can be stabilized for all admissible uncertainties and time-delays. Finally, numerical examples are given to illustrate the effectiveness of the proposed method.

K e y w o r d s: PID controller, output feedback, Networked Control Systems (NCSs), polytopic system, parameter-dependent quadratic stability, time-delay system

1 INTRODUCTION

Feedback control systems, wherein the loops are closed through real-time networks, are called Networked Control Systems (NCSs) [11, 12 14 16]. Advantages of using NCSs in the control area include simplicity, cost-effectiveness, ease of system diagnosis and maintenance, increased system agility and testability. However, integration of communication real-time networks into feedback control loops inevitably leads to some problems. As a result, it leads to a network-induced delay in the networked control closed-loop system. The existence of such a kind of delay in a network-based control loop can induce instability or poor performance of control systems [8].

In the recent years, the stability analysis and controller synthesis for systems with time-delay are important in theory and practice [1 3]. In the time domain, there are two approaches to controller design and studying of stability of closed-loop systems: the Razumikhin theorem and the Lyapunov-Krasovskii functional (LKF) approach. It is well know that the LKF approach can provide less conservative results than the Razumikhin theorem [4 6 13] and references therein. The existing criteria for asymptotic stability of the time-delay system can be classified into categories: delay-independent criteria and delay-dependent ones. It is also know that the delay-dependent criteria make use of information on the length of delays, they are less conservative than the delay-independent ones, even if the time delays are very small. On the other hand, a wide class of uncertainty types studied in the system and control literature fall into the polytopic perturbations. For the time-delay system with polytopic-type uncertainties, the parameter-dependent stability condition is of lower conservativeness than the quadratic stability condition. Recently, free-weighting matrices method or slack-variable method and cross term bounding method were developed to obtain a less conservative condition [7 10] and reference therein.

The guaranteed cost control approach has been extended to the uncertain time-delay systems, for the state feedback case, see [9, 15, 17] and for output feedback [5]. In the paper the authors consider the full order strictly proper dynamic output feedback controller. However, it seems that there is no previous result on the delay-dependent guaranteed cost control via PID output feedback.

Motivated by the above observation, in this article the parameter-dependent, delay-dependent design method will be studied to design a robust output feedback PID controller achieving a guaranteed cost such that the NCSs can be stabilized for all admissible polytopic-type uncertainties and time-delays. The sufficient condition for the existence of a guaranteed cost output feedback controller is established in terms of matrix inequalities.

This paper is organized as follows. Section 2 gives the problem formulation. Section 3 explains the main results of the paper. In section 4, numerical examples are presented to show the effectiveness of the proposed method. Notation: Throughout this paper, for real matrix M, the notation $M \geq 0$ (or $M > 0$) means that matrix M is symmetric and positive semi-definite (or positive definite); "$*$ denotes a block that is readily inferred by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 PRELIMINARIES AND PROBLEM FORMULATION

Consider the following linear time-delay system described

$$\begin{aligned} \dot{x}(t) &= A(\xi)x(t) + A_d(\xi)x(t-\tau) + B(\xi)u(t)\,, \\ yt) &= Cx(t)\,, \\ x(t) &= \varphi(t)\,,\ t \in [-\tau_M, 0] \end{aligned} \tag{1}$$

* Slovak University of Technology, Faculty of Electrical Engineering and Information Technology, Ilkovičova 3, 812 19 Bratislava, Slovakia; (thuan.quang,vojtech.vesely)@stuba.sk

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where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^l$ is the controlled output (measured output). Matrices $A(\xi), A_d(\xi), B(\xi) \in S$ belong to the convex hull, and S is a polytope with N vertices $S_1, S_2, \ldots, S_N$ which can formally be defined as

$$S := \left\{ \begin{array}{c} A(\xi), A_d(\xi) \in R^{n\times n}, B(\xi) \in R^{n\times m}, \\ A(\xi) = \sum_{i=1}^N \xi_i A_i\,,\ A_d(\xi) = \sum_{i=1}^N \xi_i A_{di}\,, \\ B(\xi) = \sum_{i=1}^N \xi_i B_i\,,\ \sum_{i=1}^N \xi_i = 1\,,\ \xi_i \geq 0 \end{array} \right\}, \tag{2}$$

where A_i, A_{di}, B_i are constant matrices with appropriate dimensions and ξ_i is time-invariant uncertainty, τ_M is the upper bound of time delay and $\varphi(t)$ is a continuously differentiable initial function. Note that S is a convex and bounded domain. We assume that a real-time communication network is integrated into feedback control loops of system (1), and the network induced delay in NCS is given by $0 < \tau \leq \tau_M$ and $\dot{\tau} \leq \mu \leq 1$.

For system (1), we consider the following PID control algorithm

$$u(t) = K_p y(t-\tau) + K_I \int_0^t y(t-\tau)\mathrm{d}t + K_D \frac{\mathrm{d}}{\mathrm{d}t} y(t-\tau)\,. \tag{3}$$

Consider $z(t) = \int y(t-\tau)\mathrm{d}t$, $\frac{\mathrm{d}}{\mathrm{d}t} y(t-\tau) = C_d \dot{x}(t-\tau)$, where C_d is an output matrix for derivative output feedback, and then by using the Newton-Leibniz formulas

$$x(t-\tau) = x(t) - \int_{t-\tau}^t \dot{x}(s)\mathrm{d}s\,,$$

and

$$\dot{x}(t-\tau) = \dot{x}(t) - \int_{t-\tau}^t \ddot{x}(s)\mathrm{d}s\,,$$

the PID control algorithm (3) can be written as

$$u(t) = FC_n X(t) + F_D C_D \dot{X}(t) - F_p C_p \int_{t-\tau}^t \dot{X}(s)\mathrm{d}s - F_d C_d \int_{t-\tau}^t \ddot{X}(s)\mathrm{d}s\,, \tag{4}$$

where

$$X(t) = \left[x^\top(t), z^\top(t)\right]^\top\,,$$
$$F = [\,K_P \quad K_I\,]\,,\ F_P = [\,K_P \quad 0\,]\,,\ F_D = [\,K_D \quad 0\,]\,,$$
$$C_n = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix},\ C_P = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix},\ C_D = \begin{bmatrix} C_d & 0 \\ 0 & 0 \end{bmatrix}.$$

Consider $\dot{z}(t) = C_i x(t-\tau) = C_i x(t) - C_i \int_{t-\tau}^t \dot{x}(s)\mathrm{d}s$, where C_i is an output matrix for integral output feedback , the system (1) can be expanded in the following form

$$\dot{X}(\xi) = A_n(\xi)X(t) + B_n(\xi)u(t) - A_{dn}(\xi)\int_{t-\tau}^t \dot{X}(s)\mathrm{d}s\,, \tag{5}$$

where

$$A_n(\xi) = \sum_{i=1}^N \xi_i A_{ni}\,, \qquad A_{ni} = \begin{bmatrix} A_i + A_{di} & 0 \\ C_i & 0 \end{bmatrix},$$
$$B_n(\xi) = \sum_{i=1}^N \xi_i B_{ni}\,, \qquad B_{ni} = \begin{bmatrix} B_i \\ 0 \end{bmatrix},$$
$$A_{dn}(\xi) = \sum_{i=1}^N \xi_i A_{dni}\,, \qquad A_{dni} = \begin{bmatrix} A_{di} & 0 \\ C_i & 0 \end{bmatrix}.$$

Applying the PID control algorithm (4) to system (5) will result in the closed-loop system

$$M_d(\xi)\dot{X}(t) + A_c(\xi)X(t) + A_{dc}(\xi)\int_{t-\tau}^t \dot{X}(s)\mathrm{d}s + A_{dd}(\xi)\int_{t-\tau}^t \ddot{X}(s)\mathrm{d}s = 0\,, \tag{6}$$

where

$$M_d(\xi) = \sum_{i=1}^N \xi_i M_{di}\,, \qquad M_{di} = I - B_{ni}F_D C_D\,,$$
$$A_c(\xi) = \sum_{i=1}^N \xi_i A_{ci}\,, \qquad A_{ci} = -(A_{ni} + B_{ni}FC_n)\,,$$
$$A_{dc}(\xi) = \sum_{i=1}^N \xi_i A_{dci}\,, \qquad A_{dci} = A_{dni} + B_{ni}F_P C_P\,,$$
$$A_{dd}(\xi) = \sum_{i=1}^N \xi_i A_{ddi}\,, \qquad A_{ddi} = B_{ni}F_D C_D\,,$$

Given positive definite symmetric matrices Q, R and S, we will consider the cost function

$$J = \int_0^\infty J(t)\mathrm{d}t\,, \tag{7}$$

where
$J(t) = X^\top(t)QX(t) + u^\top(t)Ru(t) + \dot{X}^\top(t-\tau)S\dot{X}(t-\tau)$.
Consider $\eta(t) =$
$\left[\dot{X}^\top(t) \quad X^\top(t) \quad \int\limits_{t-\tau}^{t} \dot{X}^\top(s)\mathrm{d}s \quad \int\limits_{t-\tau_M}^{t-\tau} \dot{X}^\top(s)\mathrm{d}s \quad \int\limits_{t-\tau}^{t} \ddot{X}^\top(s)\mathrm{d}s\right]^\top$
and by substituting $u(t)$ from (4) to $u^\top(t)Ru(t)$ we obtain

$$u^\top(t)Ru(t) = \eta^\top K^\top RK\eta(t)\,,$$

where $K = [\,F_D C_D \quad FC_n \quad -F_p C_p \quad 0 \quad -F_D C_D\,]$. Using $\eta(t)$, $J(t)$ can be rewritten as follows

$$J(t) = \eta^\top(t) M_Q(\xi)\eta(t)\,, \tag{8}$$

where

$$M_Q(\xi) = \begin{bmatrix} C_D^\top F_D^\top RF_D C_D + S & C_D^\top F_D^\top RFC_n & -C_D^\top F_P^\top RF_P C_P & 0 & -C_D^\top F_D^\top RF_D C_D - S \\ * & C_n^\top F^\top RFC_n + Q & -C_n^\top F^\top RF_P C_P & 0 & -C^\top F^\top RF_D C_D \\ * & * & C_P^\top F_P^\top RF_P C_P & 0 & C_P^\top F_P^\top RF_D C_D \\ * & * & * & 0 & 0 \\ * & * & * & * & C_D^\top F_D^\top RF_D C_D + S \end{bmatrix}$$

Associated with the cost, the guaranteed cost controller is defined as follows:

DEFINITION 1. Consider the uncertain system (1). If there exists a controller of form (3) and a positive scalar J_0 such that for all uncertainties (2) the closed-loop system (6) is asymptotically stable and closed-loop value of the cost function (7) satisfies $J \leq J_0$ then J_0 is said to be a guaranteed cost and the controller (2) is said to be a guaranteed cost controller.

Finally we introduce the well known results from LQ theory.

LEMMA 1. *Consider the continuous-time delay system* (5) *with control algorithm* (3). *The control algorithm* (3) *is the guaranteed cost control for system* (5) *if and only if there exists LKF* $V(\xi, t)$ *such that the following condition holds*

$$\frac{d}{dt}V(\xi,t) + J(t) \leq 0\,. \tag{9}$$

The objective of this paper is to develop a procedure to design a robust PID controller of form (4) which ensures parameter-dependent the closed-loop system stability and guaranteed cost.

3 MAIN RESULTS

The following theorem provides robust parameter-dependent quadratic stability and robust performance results for the closed-loop system (6).

THEOREM 1. *Consider the uncertain linear time-delay system* (1) *with network-induced delay* τ *satisfying* $0 \leq \tau \leq \tau_M$, $\dot{\tau} \leq \mu \leq 1$ *and the cost function* (7). *If there exists a PID controller of form* (3), *scalar* J_0, *and matrices* $P_i > 0$, $G_i > 0$, $G_{1i} > 0$, $G_{2i} > 0$, $G_{3i} > 0$ $(i = 1, \ldots, N)$, N_1, N_2, N_3, N_4, *and* N_5 *that satisfy the following matrix inequality*

$$W_i = \begin{bmatrix} w_i^{11} & w_i^{12} & w_i^{13} & w_i^{14} & w_i^{15} \\ * & w_i^{22} & w_i^{23} & w_i^{24} & w_i^{15} \\ * & * & w_i^{33} & w_i^{34} & w_i^{35} \\ * & * & * & w_i^{44} & w_i^{45} \\ * & * & * & * & w_i^{55} \end{bmatrix} \leq 0\,, \tag{10}$$

where

$$\begin{aligned}
w_i^{11} &= N_1 M_{di} + M_{di}^\top N_1^\top + \tau_M G_{1i} + \mu G_{3i} + \\
&\quad C_D^\top F_D^\top R F_D C_D + S\,, \\
w_i^{12} &= N_1 A_{ci} + M_{di}^\top N_2^\top + P_i + C_D^\top F_D^\top R F C_n\,, \\
w_i^{13} &= N_1 A_{dci} + M_{di}^\top N_3^\top - C_D^\top F_D^\top R F_P C_P\,, \\
w_i^{14} &= M_{di}^\top N_4^\top\,, \\
w_i^{15} &= N_1 A_{ddi} + M_{di}^\top N_5^\top + (1-\mu)\mu G_{3i} \\
&\quad - C_D^\top F_D^\top R F_D C_D - S\,, \\
w_i^{22} &= N_2 A_{ci} + A_{ci}^\top N_2^\top + \mu G_i + C_n^\top F^\top R F + Q\,, \\
w_i^{23} &= N_2 A_{dci} + A_{ci}^\top N_3^\top + (1-\mu) G_i + G_{2i} \\
&\quad - C_n^\top F^\top R F_P C_P\,, \\
w_i^{24} &= A_{ci}^\top N_4^\top + G_{2i}\,, \\
w_i^{25} &= N_2 A_{ddi} + A_{ci}^\top N_5^\top - C_n^\top N^\top R F_D C_D\,, \\
w_i^{33} &= N_3 A_{dci} + A_{dci}^\top N_3^\top - (1-\mu) G_i - \frac{1}{\tau_M} G_{1i} - G_{2i} + \\
&\quad C_P^\top F_P^\top R F_P C_P\,, \\
w_i^{34} &= A_{dci}^\top N_4^\top - G_{2i}\,, \\
w_i^{35} &= N_3 A_{ddi} + A_{dci}^\top N_5^\top + C_P^\top F_P^\top R F_D C_D\,, \\
w_i^{44} &= -G_{2i} - \frac{1}{\tau_M} G_{1i}\,, \\
w_i^{45} &= N_4 A_{ddi}\,, \\
w_i^{55} &= N_5 A_{ddi} + A_{ddi}^\top N_5^\top - (1-\mu) G_{3i} + \\
&\quad C_D^\top F_D^\top R F_D C_D + S\,.
\end{aligned}$$

then the uncertain system (1) *with controller* (3) *is parameter-dependent quadratically stable and the cost function* (7) *satisfies the following bound*

$$J \leq J_0 = \sqrt{\lambda_{MP}^2 + \lambda_{MG}^2 + \lambda_{MG1}^2 + \lambda_{MG2}^2 + \lambda_{MG3}^2} * J_M \tag{11}$$

where

$$\begin{aligned}
\lambda_{MP} &= \operatorname*{Max}_{i=1\ldots N}\big(\text{MaxEigenvalue}(P_i)\big)\,, \\
\lambda_{G_i} &= \operatorname*{Max}_{i=1\ldots N}\big(\text{MaxEigenvalue}(G_i)\big)\,, \\
\lambda_{MG1} &= \operatorname*{Max}_{i=1\ldots N}\big(\text{MaxEigenvalue}(G_{1i})\big)\,, \\
\lambda_{MG2} &= \operatorname*{Max}_{i=1\ldots N}\big(\text{MaxEigenvalue}(G_{2i})\big)\,, \\
\lambda_{MG3} &= \operatorname*{Max}_{i=1\ldots N}\big(\text{MaxEigenvalue}(G_{3i})\big)\,,
\end{aligned}$$

$$\begin{aligned}
J_M = \Bigg(&\|x_0\|^4 + \Big(\int_{-\tau}^{0} \|\varphi(s)\|^2 ds\Big)^2 + \Big(\int_{-\tau}^{0} d\theta \int_{\theta}^{0} \|\dot{\varphi}(s)\|^2 ds\Big)^2 \\
&+ \Big(\int_{-\tau_M}^{-\tau} \|\varphi(s)\|^2 ds\Big)^2 + \Big(\int_{-\tau}^{0} \|\dot{\varphi}(s)\|^2 ds\Big)^2\Bigg)^{1/2}.
\end{aligned}$$

P r o o f . Consider the Lyapunov-Krasovskii functional as follows

$$\begin{aligned}
V(\xi,t) &= \sum_{i=1}^{5} V_i(\xi,t)\,, \qquad (12) \\
V_1(\xi,t) &= X^\top(t) P(\xi) X(t)\,, \\
V_2(\xi,t) &= \int_{t-\tau}^{t} X^\top(s) G(\xi) X(s) \mathrm{d}s\,, \\
V_3(\xi,t) &= \int_{-\tau}^{0} \mathrm{d}\theta \int_{t+\theta}^{t} \dot{X}^\top(s) G_1(\xi) \dot{X}(s) \mathrm{d}s\,, \\
V_4(\xi,t) &= \int_{t-\tau_M}^{t} X^\top(s) G_2(\xi) X(s) \mathrm{d}s\,,
\end{aligned}$$

$$V_5(\xi,t)=\int_{t-\tau}^{t}\dot{X}^\top(s)G_3(\xi)\dot{X}(s)\mathrm{d}s\,.$$

Differentiating $V(\xi,t)$ with respect to t and using the Newton-Leibniz formula $x(t-\tau)=x(t)-\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s$ we obtain

$$\begin{aligned}
\dot{V}_1(\xi,t)&=zX^\top(t)P(\xi)\dot{X}(t)\,,\\
\dot{V}_2(\xi,t)&\le\eta_1^\top(t)\begin{bmatrix}\mu G(\xi) & (1-\mu)G(\xi)\\ * & -(1-\mu)G(\xi)\end{bmatrix}\eta_1(t)\,,\\
\eta_1^\top(t)&=\begin{bmatrix}X^\top(t) & \int_{t-\tau}^{t}\dot{X}^\top(s)\mathrm{d}s\end{bmatrix}\,,\\
\dot{V}_3&\le\tau_M\dot{X}^\top(t)G_1(\xi)\dot{X}(t)\\
&\quad-\frac{1}{\tau_M}\int_{t-\tau}^{t}\dot{X}^\top(s)\mathrm{d}s\,G_1(\xi)\int_{t-\tau}^{t}\dot{X}(s)\mathrm{d}s\,,\\
\dot{V}_4(\xi,t)&\le\eta_2^\top(t)\begin{bmatrix}0 & G_2(\xi) & G_2(xi)\\ * & -G_2(xi) & -G_2(xi)\\ * & * & -G_2(xi)\end{bmatrix}\eta_2(t)\,,\\
\eta_2^\top(t)&=\begin{bmatrix}X^\top(t) & \int_{t-\tau}^{t}\dot{X}^\top(s)\mathrm{d}s & \int_{t-\tau_M}^{t-\tau}\dot{X}^\top(s)\mathrm{d}s\end{bmatrix}\,,\\
\dot{V}_5(\xi,t)&\le\eta_3^\top(t)\begin{bmatrix}\mu G_3(\xi) & (1-\mu)G_3(\xi)\\ * & -(1-\mu)G_3(\xi)\end{bmatrix}\eta_3(t)\,,\\
\eta_3^\top&=\begin{bmatrix}X^\top & \int_{t-\tau}\ddot{X}^\top(s)\mathrm{d}s\end{bmatrix}\,.
\end{aligned}$$

Applying the free-weighting matrices technique, equation (8) is represented in the following equivalent form

$$\begin{aligned}\alpha(t)=2\eta^\top(t)\,[\,N_1^\top\quad N_2^\top\quad N_3^\top\quad N_4^\top\quad N_5^\top\,]^\top\\ [\,M_d(\xi)\quad A_c(\xi)\quad A_{dc}(\xi)\quad 0\quad A_{dd}(\xi)\,]\,\eta(t)=0\,.\end{aligned}$$

After manipulation of the above equation, we obtain

$$\alpha(t)=\eta^\top(t)M_\alpha(\xi)\eta(t)=0 \tag{13}$$

where

$$M_\alpha(\xi)=\begin{bmatrix}
N_1M_d(\xi)+M_d^\top(\xi)N_1^\top & N_1A_c(\xi)+M_d^\top(\xi)N_2^\top & N_1A_{dc}(\xi)+M_d^\top(\xi)N_3^\top & M_d^\top(\xi)N_4^\top & N_1A_{dd}(\xi)+M_d^\top(\xi)N_5^\top\\
* & N_2A_c(\xi)+A_c^\top(\xi)N_2^\top & N_2A_{dc}(\xi)+A_c^\top(\xi)N_3^\top & A_c^\top(\xi)N_4^\top & N_2A_{dd}(\xi)+A_c^\top(\xi)N_5^\top\\
* & * & N_3A_{dc}(\xi)+A_{dc}^\top(\xi)N_3^\top & A_{dc}(\xi)N_4^\top & N_3A_{dc}(\xi)+A_{dc}^\top(\xi)N_5^\top\\
* & * & * & 0 & N_4A_{dd}(\xi)\\
* & * & * & * & N_5A_{dd}(\xi)+A_{dd}^\top(\xi)N_5^\top
\end{bmatrix}.$$

Because of $\alpha(t)=0$, thus

$$\dot{V}(\xi,t)=\sum_{i=1}^{5}\dot{V}_i(\xi,t)+\alpha(t)\le\eta^\top(t)[M_\alpha(\xi)+M_V(\xi)]\eta^\top(t)\,, \tag{14}$$

where

$$M_V(\xi)=\begin{bmatrix}
\tau_MG_1(\xi)+\mu G_3(\xi) & P(\xi) & 0 & 0 & (1-\mu)G_3(\xi)\\
* & \mu G(\xi) & (1-\mu)G(\xi) & G_2(\xi) & 0\\
* & * & -(1-mu)G(\xi)-G_2(\xi) & -G_2(\xi) & 0\\
* & * & -\frac{1}{\tau_M}G_1(\xi) & -G_2(\xi)-\frac{1}{\tau_M}G_1(\xi) & 0\\
* & * & * & * & -(1-\mu)G_3(\xi)
\end{bmatrix}$$

Due to Lemma 1, the closed-loop system (6) is robustly asymptotically stable and give an upper bound (a guaranteed cost) for the cost function (7) if

$$\dot{V}(\xi,t)+J(t)\le\eta^\top(t)W(\xi)\eta^\top(t)\le0\iff W(\xi)\le0\,, \tag{15}$$

where $W(\xi)=\sum_{i=1}^{N}\xi_iW_i=M_\alpha(\xi)+M_V(\xi)+M_Q(\xi)$. If for each $W_i\le0$, $I=1,\dots,N$, then $W(\xi)=\sum_{i-1}^{N}\xi_iW_i\le0$. Therefore, $\dot{V}(\xi,t)\le-J(t)\le0$ ($J(t)>0$), or $J(t)\le-\dot{V}(\xi,t)$. By integrating $J(t)\le-\dot{V}(\xi,t)$ we obtain

$$\begin{aligned}J\le-\int_0^\infty\dot{V}(\xi,t)=V_0=X_0^\top P(\xi)X_0+\\
\int_{-\tau}^{0}X^\top(s)G(\xi)X(s)\mathrm{d}s+\int_{-\tau}^{0}\mathrm{d}\theta\int_{\theta}^{0}\dot{X}^\top(s)G_1(\xi)\dot{X}(\xi)\mathrm{d}s\\
+\int_{-\tau_M}^{0}X^\top G_2(\xi)X(s)\mathrm{d}s+\int_{-\tau}^{0}\dot{X}^\top(s)G_3(\xi)\dot{X}(s)\mathrm{d}s\end{aligned}$$

Because of $X(t)=[\,\varphi^\top(t)\quad 0\,]$, $\forall t\in[-\tau_M,0]$

$$\begin{aligned}V_0\le\lambda_{MP}\|x_0\|^2+\lambda_{MG}\int_{-\tau}^{0}\|\varphi(s)\|^2\mathrm{d}s\\
+\lambda_{MG1}\int_{-\tau}^{0}\mathrm{d}\theta\int_{\theta}^{0}\|\dot{\varphi}(s)\|^2\mathrm{d}s+\lambda_{MG2}\int_{-\tau_M}^{0}\|\varphi(s)\|^2\mathrm{d}s\\
+\lambda_{MG3}\int_{-\tau}^{0}\|\dot{\varphi}(s)\|^2\mathrm{d}s\,.\end{aligned}$$

It is known that for two arbitrary vectors $\boldsymbol{X},\boldsymbol{Y}$, the following inequality hold

$$|\boldsymbol{XY}|\le\|\boldsymbol{X}\|\|\boldsymbol{Y}\|\,. \tag{16}$$

Consider $\vec{X}=[\,\lambda_{MP}\quad\lambda_{MG}\quad\lambda_{MG1}\quad\lambda_{MG2}\quad\lambda_{MG3}\,]^\top$,

$$\boldsymbol{Y}=\Big[\|x_0\|^2\quad\int_{-\tau}^{0}\|\varphi(s)\|^2\mathrm{d}s\quad\int_{-\tau}^{0}\mathrm{d}\theta\int_{\theta}^{0}\|\dot{\varphi}(s)\|^2\mathrm{d}s\quad\int_{-\tau_M}^{0}\|\varphi(s)\|^2\mathrm{d}s\quad\int_{-\tau}^{0}\|\dot{\varphi}(s)\|^2\mathrm{d}s\Big]^\top.$$

Applying inequality (16) to the above equation the upper bound cost function (7) J_0 is obtained as (11). The theorem 1 is proved.

4 EXAMPLES

In this section we present the results of numerical calculations of two examples to design a robust output feedback PID controller with guaranteed cost for NCSs with time-delay. The design procedure is based on BMI inequalities (10).

Example 1 has been borrowed from [2] to demonstrate the use of algorithm (10) for the problem of robustly stabilizing, with a guaranteed cost, vertical take-off and landing of a helicopter. The system is controlled through NCS with a time-varying time- delay $0 < \tau \leq \tau_M = 200$ ms, $\dot{\tau} \leq \mu \leq 0.99$. Let uncertain matrices A, B, C, A_d be defined as

$$A = \begin{bmatrix} -0.036 & 0.0270 & 0.0188 & -0.4555 \\ 0.0482 & -1.010 & 0.0024 & -4.0208 \\ 0.1002 & q_1(t) & -0.707 & q_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 4.422 & 0.1761 \\ q_3(t) & -7,59222 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}, \; C^\top = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \; A_d = 0$$

with parameters bounds for all time $-0.6319 \leq q_1(t) \leq 1.3691$, $1.22 \leq q_2(t) \leq 1.420$, $2.7446 \leq q_3(t) \leq 4.3446$. The above model has been recalculated to the form (1). The respective eight vertices are calculated. Note that matrix A is unstable with $\max \operatorname{real} \operatorname{eigenvalue}(A) = 1.2675$. The results of calculation for the case $r = 1$, $q = 0.1$, $s = 0.001$, $r_0 = 10$ are as follows

$$F = [\, K_P \quad K_I \,] = \begin{bmatrix} -0.2788 & 0.0927 \\ 0.5857 & 0.4086 \end{bmatrix}, K_D \cong 0\,.$$

The $\max \operatorname{real} \operatorname{eigenvalue}(\textit{Close-loop}) = -0.072209$ and guaranteed cost $J_0 = 17.9128 J_M$, where $\lambda_{MP} = \lambda_{MG} = \lambda_{MG2} = 8.9794$, $\lambda_{MG1} = 8.8841$, $\lambda_{MG3} = 0.2319$.

Example 2. We consider the linear model of two cooperating DC motors. The problem is to design two PI controllers for a laboratory MIMO system which guarantee robust stability with a guaranteed cost. The system is controlled through a NCS with time-varying time-delay $0 < \tau \leq \tau_M = 100$ ms, $\dot{\tau} \leq \mu = 0.2$. The system model is given with a time invariant matrix affine type uncertain structure, where

$$A_0 = \begin{bmatrix} 0 & -.2148 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1.014 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2605 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -.9107 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.1639 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -.8137 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.2279 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -.8251 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} .3148 & 0 \\ .0478 & 0 \\ 0 & -.1028 \\ 0 & -.0091 \\ -.0841 & 0 \\ -.0287 & 0 \\ 0 & .3676 \\ 0 & .2448 \end{bmatrix}, \; B_1 = \begin{bmatrix} .0625 & 0 \\ .0798 & 0 \\ 0 & -.0462 \\ 0 & -.0449 \\ .0016 & 0 \\ .0072 & 0 \\ 0 & .077 \\ 0 & .003 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & -.025 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1395 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.0938 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2911 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0188 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0208 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.1173 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & .0125 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .0594 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0116 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0308 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0188 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0156 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0208 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} .0094 & 0 \\ .0151 & 0 \\ 0 & .0019 \\ 0 & -.003 \\ -.0121 & 0 \\ -.03 & 0 \\ 0 & -.064 \\ 0 & .0189 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}, \; A_d = 0\,.$$

The above model has been recalculated to the form (1). The respective four vertices are calculated. The results of calculation for the case $r = 1$, $q = 0.1$, $s = 0.001$, $r_0 = 20$ are as follows

$$F = [\, K_P \;\; K_I \,] = \begin{bmatrix} -1.8916 & 0.1128 & 0.2026 & -0.6964 \\ -0.4310 & -1.8916 & 0.8782 & -1.4985 \end{bmatrix}.$$

The $\max \operatorname{real} \operatorname{eigenvalue}(\textit{Close-loop}) = -0.17726$. And guaranteed cost $J_0 = 37.7659\, J_M$ where $\lambda_{MP} = 18.916$, $\lambda_{MG} = 15.0011$, $\lambda_{MG2} = 18.916$, $\lambda_{MG1} = 15.5723$, $\lambda_{MG3} = 15.5918$.

5 CONCLUSION

The guaranteed cost control problem is studied in this paper for a class of linear time-delay uncertain polytopic systems and a given quadratic cost function with three terms (QRS). On the basis of the Lyapunov-Krasovskii functional, new sufficient parameter-dependent quadratic stability conditions are given for the output feedback PID controller proposed design procedure in terms of bilinear matrix inequality.

The examples show the effectiveness of the proposed method.

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Nguyen Quang Thuan born in 1982 in Hanoi-VietNam. Has been studying from 2003 to August 2007 for Bachelor of Science at Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovakia and from 2007 to 2009 for Master of Science at the same place, where he is from 2009 until now doing further studies to get his PhD title.

Vojtech Veselý (Prof, Ing, DrSc) was born in 1940. Since 1964 he has worked at the Department of Automatic Control Systems at the Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, where he has supervised 19 PhD students. Since 1986 he has been Full Professor. His research interests include the areas of power system control, decentralized control of large-scale systems, process control and optimization. He is author and coauthor of more than 250 scientific papers.