DESIGN OF ROBUST GUARANTEED COST PID CONTROLLER FOR NETWORKED CONTROL SYSTEMS

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The paper addresses the problem of an output feedback guaranteed cost controller design for Networked Control Systems (NCSs) with time-delay and polytopic uncertainties. By constructing a new parameter-dependent Lyapunov functional and applying the free-weighting matrices technique, the parameter-dependent, delay-dependent design method will be obtained to synthesize PID controllers achieving a guaranteed cost such that the NCSs can be stabilized for all admissible uncertainties and time-delays. Finally, numerical examples are given to illustrate the effectiveness of the proposed method.

Key words: PID controller, output feedback, Networked Control Systems (NCSs), polytopic system, parameter-dependent quadratic stability, time-delay system

1 INTRODUCTION

Feedback control systems, wherein the loops are closed through real-time networks, are called Networked Control Systems (NCSs) [11, 12, 14, 16]. Advantages of using NCSs in the control area include simplicity, cost-effectiveness, ease of system diagnosis and maintenance, increased system agility and testability. However, integration of communication real-time networks into feedback control loops inevitably leads to some problems. As a result, it leads to a network-induced delay in the networked control closed-loop system. The existence of such a kind of delay in a network-based control loop can induce instability or poor performance of control systems [8].

In the recent years, the stability analysis and controller synthesis for systems with time-delay are important in theory and practice [1, 3]. In the time domain, there are two approaches to controller design and studying of stability of closed-loop systems: the Razumikhin theorem and the Lyapunov-Krasovskii functional (LKF) approach. It is well know that the LKF approach can provide less conservative results than the Razumikhin theorem [4, 6, 13] and references therein. The existing criteria for asymptotic stability of the time-delay system can be classified into categories: delay-independent criteria and delay-dependent ones. It is also know that the delay-dependent criteria make use of information on the length of delays, they are less conservative than the delay-independent ones, even if the time delays are very small. On the other hand, a wide class of uncertainty types studied in the system and control literature fall into the polytopic perturbations. For the time-delay system with polytopic-type uncertainties, the parameter-dependent stability condition is of lower conservativeness than the quadratic stability condition. Recently, free-weighting matrices method or slack-variable method and cross term bounding method were developed to obtain a less conservative condition [7, 10] and reference therein.

The guaranteed cost control approach has been extended to the uncertain time-delay systems, for the state feedback case, see [9, 15, 17] and for output feedback [5]. In the paper the authors consider the full order strictly proper dynamic output feedback controller. However, it seems that there is no previous result on the delay-dependent guaranteed cost control via PID output feedback.

Motivated by the above observation, in this article the parameter-dependent, delay-dependent design method will be studied to design a robust output feedback PID controller achieving a guaranteed cost such that the NCSs can be stabilized for all admissible polytopic-type uncertainties and time-delays. The sufficient condition for the existence of a guaranteed cost output feedback controller is established in terms of matrix inequalities.

This paper is organized as follows. Section 2 gives the problem formulation. Section 3 explains the main results of the paper. In section 4, numerical examples are presented to show the effectiveness of the proposed method. Notation: Throughout this paper, for real matrix $M$, the notation $M \geq 0$ (or $M > 0$) means that matrix $M$ is symmetric and positive semi-definite (or positive definite); $\ast$ denotes a block that is readily inferred by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 PRELIMINARIES AND PROBLEM FORMULATION

Consider the following linear time-delay system described

$$\dot{x}(t) = A(\xi)x(t) + A_d(\xi)x(t - \tau) + B(\xi)u(t),$$

$$y(t) = Cx(t),$$

$$x(t) = \varphi(t), \quad t \in [-\tau_M, 0]$$

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where \( x(t) \in R^n \) is the state vector, \( u(t) \in R^n \) is the control input, \( y(t) \in R^l \) is the controlled output (measured output). Matrices \( A(\xi), A_d(\xi), B(\xi) \in S \) belong to the convex hull, and \( S \) is a polytope with \( N \) vertices \( S_1, S_2, \ldots, S_N \) which can formally be defined as

\[
S := \left\{ A(\xi), A_d(\xi) \in R^{m \times n}, B(\xi) \in R^{n \times m}, \right. \\
\left. A(\xi) = \sum_{i=1}^{N} \xi_i A_i, \quad A_d(\xi) = \sum_{i=1}^{N} \xi_i A_{di}, \quad B(\xi) = \sum_{i=1}^{N} \xi_i B_i, \quad \sum_{i=1}^{N} \xi_i = 1, \quad \xi_i \geq 0 \right\}
\] (2)

where \( A_i, A_{di}, B_i \) are constant matrices with appropriate dimensions and \( \xi_i \) is time-invariant uncertainty. \( \tau_M \) is the upper bound of time delay and \( \varphi(t) \) is a continuously differentiable initial function. Note that \( S \) is a convex and bounded domain. We assume that a real-time communication network is integrated into feedback control loops of system (1), and the network induced delay in NCS is given by \( 0 < \tau \leq \tau_M \) and \( \tau \leq \mu \leq 1 \).

For system (1), we consider the following PID control algorithm

\[
u(t) = K_p y(t-\tau) + K_I \int_0^t y(t-\tau) dt + K_D \frac{d}{dt} y(t-\tau)
\] (3)

Consider \( z(t) = \int_0^t y(t-\tau) dt, \frac{d}{dt} y(t-\tau) = C_d \dot{x}(t-\tau) \), where \( C_d \) is an output matrix for derivative output feedback, and then by using the Newton-Leibniz formulas

\[
x(t-\tau) = x(t) - \int_{t-\tau}^{t} \dot{x}(s) ds,
\]

and

\[
\dot{x}(t-\tau) = \dot{x}(t) - \int_{t-\tau}^{t} \ddot{x}(s) ds,
\]

the PID control algorithm (3) can be written as

\[
u(t) = FC_n x(t) + F_D C_D \dot{x}(t) - F_P C_P \int_{t-\tau}^{t} \dot{x}(s) ds
- F_D C_d \int_{t-\tau}^{t} \ddot{x}(s) ds,
\] (4)

where

\[
X(t) = [x^T(t), \dot{x}^T(t)]^T,
F = [K_P \quad K_I], \quad F_P = [K_P \quad 0], \quad F_D = [K_D \quad 0],
C_n = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}, \quad C_P = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}, \quad C_D = \begin{bmatrix} C_0 & 0 \\ 0 & 0 \end{bmatrix}.
\]

Consider \( \dot{x}(t) = C_i x(t-\tau) - C_i \int_{t-\tau}^{t} \dot{x}(s) ds \), where \( C_i \) is an output matrix for integral output feedback, the system (1) can be expanded in the following form

\[
\dot{X}(\xi) = A_n(\xi) X(t) + B_n(\xi) u(t) - A_{dn}(\xi) \int_{t-\tau}^{t} \dot{X}(s) ds,
\] (5)

where

\[
A_n(\xi) = \sum_{i=1}^{N} \xi_i A_n, \quad A_{ni} = \begin{bmatrix} A_i + A_{di} & 0 \\ C_i & 0 \end{bmatrix},
B_n(\xi) = \sum_{i=1}^{N} \xi_i B_n, \quad B_{ni} = \begin{bmatrix} B_i & 0 \end{bmatrix},
A_{dn}(\xi) = \sum_{i=1}^{N} \xi_i A_{dni}, \quad A_{dn} = \begin{bmatrix} A_{di} & 0 \\ C_i & 0 \end{bmatrix}.
\]

Applying the PID control algorithm (4) to system (5) will result in the closed-loop system

\[
M_d(\xi) \dot{X}(t) + A_c(\xi) X(t) + A_{dc}(\xi) \int_{t-\tau}^{t} \dot{X}(s) ds
+ A_{dd}(\xi) \int_{t-\tau}^{t} \ddot{X}(s) ds = 0,
\] (6)

where

\[
M_d(\xi) = \sum_{i=1}^{N} \xi_i M_{di}, \quad M_{di} = I - B_{ni} F_P C_D,
A_c(\xi) = \sum_{i=1}^{N} \xi_i A_{ci}, \quad A_{ci} = -(A_{ni} + B_{ni} F_C),
A_{dc}(\xi) = \sum_{i=1}^{N} \xi_i A_{dni}, \quad A_{dni} = A_{di} + B_{ni} F_P C_P,
A_{dd}(\xi) = \sum_{i=1}^{N} \xi_i A_{ddi}, \quad A_{ddi} = B_{ni} F_D C_D.
\]

Given positive definite symmetric matrices \( Q, R \) and \( S \), we will consider the cost function

\[
J = \int_0^{\infty} J(t) dt,
\] (7)

where

\[
J(t) = X^T(t) Q X(t) + u^T(t) R u(t) + \dot{X}^T(t-\tau) S \dot{X}(t-\tau).
\]

Consider \( \eta(t) = \int_{t-\tau}^{t} \dot{X}(s) ds \) and by substituting \( u(t) \) from (4) to \( u^T(t) R u(t) \) we obtain

\[
u^T(t) R u(t) = \eta^T K \dot{K} \eta(t),
\]

where \( K = [F_D C_D \quad F_C, -F_P C_P \quad 0 \quad -F_D C_D] \). Using \( \eta(t), J(t) \) can be rewritten as follows

\[
J(t) = \eta^T(t) M Q(\xi) \eta(t),
\] (8)

where

\[
\end{bmatrix}.
\]
Associated with the cost, the guaranteed cost controller is defined as follows:

**Definition 1.** Consider the uncertain system (1). If there exists a controller of form (3) and a positive scalar $J_0$ such that for all uncertainties (2) the closed-loop system (6) is asymptotically stable and closed-loop value of the cost function (7) satisfies $J \leq J_0$ then $J_0$ is said to be a guaranteed cost and the controller (2) is said to be a guaranteed cost controller.

Finally we introduce the well known results from LQ theory.

**Lemma 1.** Consider the continuous-time delay system (5) with control algorithm (3). The control algorithm (3) is the guaranteed cost control for system (5) if and only if there exists LKF $V(\xi, t)$ such that the following condition holds

$$
\frac{d}{dt} V(\xi, t) + J(t) \leq 0. \quad (9)
$$

The objective of this paper is to develop a procedure to design a robust PID controller of form (4) which ensures parameter-dependent the closed-loop system stability and guaranteed cost.

### 3 MAIN RESULTS

The following theorem provides robust parameter-dependent quadratic stability and robust performance results for the closed-loop system (6).

**Theorem 1.** Consider the uncertain linear time-delay system (1) with network-induced delay $\tau$ satisfying $0 \leq \tau \leq \tau_M$, $\hat{\tau} \mu \leq 1$ and the cost function (7). If there exists a PID controller of form (3), scalar $J_0$, and matrices $P_i > 0$, $G_i > 0$, $G_{ii} > 0$, $G_{2i} > 0$, $G_{3i} > 0$ ($i = 1, \ldots, N$), $N_1, N_2, N_3, N_4$, and $N_5$ that satisfy the following matrix inequality

$$
W_i = \begin{bmatrix} w_{i1}^{11} & w_{i1}^{12} & w_{i1}^{13} & w_{i1}^{14} & w_{i1}^{15} \\
* & w_{i2}^{22} & w_{i2}^{23} & w_{i2}^{24} & w_{i2}^{25} \\
* & * & w_{i3}^{33} & w_{i3}^{34} & w_{i3}^{35} \\
* & * & * & w_{i4}^{44} & w_{i4}^{45} \\
* & * & * & * & w_{i5}^{55} \end{bmatrix} \leq 0, \quad (10)
$$

where

$$
w_{i1}^{11} = N_1 M_{ai} + M_{d1i} N_{i2}^T + \tau_M G_{1i} + \mu G_{3i} + C_{d1}^T F_{D} R F_{C} C_0 + S,
$$

$$
w_{i1}^{12} = N_1 M_{ai} + M_{d1i} N_{i2}^T + P_i + C_{d1}^T F_{D} R F_{C} C_0,
$$

$$
w_{i1}^{13} = N_1 M_{ai} + M_{d1i} N_{i3}^T - C_{d1}^T F_{D} R F_{C} C_0 + S,
$$

$$
w_{i1}^{14} = M_{d1i} N_{i4}^T,
$$

$$
w_{i1}^{15} = N_1 M_{ai} + M_{d1i} N_{i5}^T + (1 - \mu) \mu G_{3i} - C_{d1}^T F_{D} R F_{C} C_0 + S,
$$

$$
w_{i2}^{22} = N_2 M_{ci} + M_{c1i} N_{i2}^T + \mu G_{i} + C_{n}^T F_{C} R F + Q,
$$

$$
w_{i3}^{33} = N_3 M_{dci} + M_{dci} N_{i3}^T - (1 - \mu) G_{i} - \frac{1}{\tau_M} G_{1i} + C_{d1}^T F_{D} R F + S,
$$

$$
w_{i3}^{34} = N_3 M_{dci} + M_{dci} N_{i3}^T - C_{d1}^T F_{D} R F_{C} C_0 + S,
$$

$$
w_{i3}^{35} = N_3 M_{dci} + M_{dci} N_{i5}^T + (1 - \mu) G_{3i} + C_{d1}^T F_{D} R F + S,
$$

$$
w_{i4}^{44} = \frac{1}{\tau_M} G_{1i} + C_{d1}^T F_{D} R F_{C} C_0 + S,
$$

$$
w_{i4}^{45} = N_4 M_{dci} + M_{dci} N_{i4}^T + (1 - \mu) G_{3i} + C_{d1}^T F_{D} R F + S,
$$

$$
w_{i5}^{55} = N_5 M_{dci} + M_{dci} N_{i5}^T + (1 - \mu) G_{3i} + C_{d1}^T F_{D} R F + S,
$$

$$
J_M = \left( \left\| x_0 \right\| ^4 + \int_{-\tau}^{0} \left( \left\| \varphi(s) \right\| ^2 ds \right) ^2 + \int_{-\tau}^{0} \left( \int_{-\tau}^{0} \left\| \varphi(s) \right\| ^2 ds \right) ^2 \right) ^{1/2}.
$$

**Proof.** Consider the Lyapunov-Krasovskii functional as follows

$$
V(\xi, t) = \sum_{i=1}^{N} V_i(\xi, t),
$$

$$
V_i(\xi, t) = X^T(t) P_i X(t),
$$

$$
V_2(\xi, t) = \int_{t-\tau}^{t} X^T(s) G_i X(s) ds,
$$

$$
V_3(\xi, t) = \int_{-\tau}^{0} \left( \int_{t-\tau}^{0} X^T(s) G_i X(s) ds \right) ds,
$$

$$
V_4(\xi, t) = \int_{-\tau}^{0} \left( \int_{t-\tau}^{0} X^T(s) G_2(\xi) X(s) ds \right) ds,
$$

$$
V_5(\xi, t) = \int_{-\tau}^{0} \left( \int_{t-\tau}^{0} X^T(s) G_2(\xi) X(s) ds \right) ds,
$$

where

$$
\lambda_{MP} = \max_{i=1, \ldots, N} \left( \text{MaxEigenvalue}(P_i) \right),
$$

$$
\lambda_{G_i} = \max_{i=1, \ldots, N} \left( \text{MaxEigenvalue}(G_i) \right),
$$

$$
\lambda_{MG1} = \max_{i=1, \ldots, N} \left( \text{MaxEigenvalue}(G_{1i}) \right),
$$

$$
\lambda_{MG2} = \max_{i=1, \ldots, N} \left( \text{MaxEigenvalue}(G_{2i}) \right),
$$

$$
\lambda_{MG3} = \max_{i=1, \ldots, N} \left( \text{MaxEigenvalue}(G_{3i}) \right),
$$

$$
J_M = \left( \left\| x_0 \right\| ^4 + \int_{-\tau}^{0} \left( \left\| \varphi(s) \right\| ^2 ds \right) ^2 + \int_{-\tau}^{0} \left( \int_{-\tau}^{0} \left\| \varphi(s) \right\| ^2 ds \right) ^2 \right) ^{1/2}.
$$
where

\[ M_V(\xi) = \begin{bmatrix} \tau_M G_1(\xi) + \mu G_3(\xi) & P(\xi) \\ \mu G(\xi) & \mu G(\xi) \end{bmatrix} \]

Due to Lemma 1, the closed-loop system (6) is robustly asymptotically stable and give an upper bound (a guaranteed cost) for the cost function (7) if

\[
\dot{V}(\xi, t) + J(t) \leq \eta^T(t) W(\xi, t) \eta(t) = 0 \iff W(\xi) \leq 0,
\]

where \( W(\xi) = \sum_{i=1}^N \xi_i W_i = M_a(\xi) + M_V(\xi) + M_G(\xi) \).

If for each \( W_i \leq 0, \ i = 1, \ldots, N, \) then \( W(\xi) = \sum_{i=1}^N \xi_i W_i \leq 0. \) Therefore, \( \dot{V}(\xi, t) \leq -J(t) \leq 0 \) (\( J(t) > 0 \)), or \( J(t) \leq -\dot{V}(\xi, t) \). By integrating \( J(t) \leq -\dot{V}(\xi, t) \) we obtain

\[
J \leq -\int_0^\infty \dot{V}(\xi, t) dt = V_0 = X_0^T P(\xi) X_0 + \int_{-\tau}^0 X^T(s) G(\xi) X(s) ds + \int_{-\tau}^0 d\theta \int_{-\tau}^0 X^T(s) G_1(\xi) \dot{X}(s) ds
\]

\[ + \int_{-\tau}^0 X^T G_2(\xi) X(s) ds + \int_{-\tau}^0 \dot{X}^T(s) G_3(\xi) \dot{X}(s) ds \]

Because of \( X(t) = [\varphi^T(0) \ 0] \), \( \forall t \in [-\tau, 0] \)

\[
V_0 \leq \lambda_{MP} \|x_0\|^2 + \lambda_{MG} \int_{-\tau}^0 \|\varphi(s)\|^2 ds + \lambda_{MG1} \int_{-\tau}^0 \|\varphi(s)\|^2 ds + \lambda_{MG2} \int_{-\tau}^0 \|\varphi(s)\|^2 ds + \lambda_{MG3} \int_{-\tau}^0 \|\varphi(s)\|^2 ds.
\]

It is known that for two arbitrary vectors \( X, Y \), the following inequality hold

\[
\|X Y\| \leq \|X\|\|Y\|.
\]

Consider \( \ddot{X} = [\lambda_{MP} \lambda_{MG} \lambda_{MG1} \lambda_{MG2} \lambda_{MG3}]^T \),

\[
Y = \left[ \|x_0\|^2 \int_{-\tau}^0 \|\varphi(s)\|^2 ds \right] \left[ \int_{-\tau}^0 \|\varphi(s)\|^2 ds \right] \left[ \int_{-\tau}^0 \|\varphi(s)\|^2 ds \right].
\]

Applying inequality (16) to the above equation the upper bound cost function (7) \( J_0 \) is obtained as (11). The theorem 1 is proved.
4 EXAMPLES

In this section we present the results of numerical calculations of two examples to design a robust output feedback PID controller with guaranteed cost for NCSs with time-delay. The design procedure is based on BMI inequalities (10).

Example 1 has been borrowed from [2] to demonstrate the use of algorithm (10) for the problem of robustly stabilizing, with a guaranteed cost, vertical take-off and landing of a helicopter. The system is controlled through the use of algorithm (10) for the problem of robustly stabilizing, with a guaranteed cost, vertical take-off and landing of a helicopter.

\[ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

Example 2. We consider the linear model of two cooperating DC motors. The system is to design two PI controllers for a laboratory MIMO system which guarantees robust stability with a guaranteed cost. The system is controlled through a NCS with time-varying time-delay. The respective four vertices are calculated. Note that matrix \( A \) is unstable with max real eigenvalue \( \lambda(A) = 1.2675 \). The results of calculation for the case \( r = 1 \), \( q = 0.1 \), \( s = 0.001 \), \( r_0 = 10 \) are as follows

\[ F = [K_P \ K_I] = \begin{bmatrix} -0.2788 & 0.0927 \\ 0.5857 & 0.4086 \end{bmatrix}, \quad K_D \equiv 0. \]

The results of calculation for the case \( r = 1 \), \( q = 0.1 \), \( s = 0.001 \), \( r_0 = 20 \) are as follows

\[ F = [K_P \ K_I] = \begin{bmatrix} -0.8196 & 0.1128 & 0.2026 & -0.6964 \\ -0.4310 & -1.8916 & 0.8782 & -1.4985 \end{bmatrix}. \]

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The example shows the effectivity of the proposed method.

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