

HYBRID CHAOS SYNCHRONIZATION OF FOUR-SCROLL SYSTEMS VIA ACTIVE CONTROL

Rajagopal Karthikeyan* — Vaidyanathan Sundarapandian**

This paper investigates the hybrid chaos synchronization of identical Wang four-scroll systems (Wang, 2009), identical Liu-Chen four-scroll systems (Liu and Chen, 2004) and non-identical Wang and Liu-Chen four-scroll systems. Active control method is the method adopted to achieve the hybrid chaos synchronization of the four-scroll chaotic systems addressed in this paper and our synchronization results are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active control method is effective and convenient to hybrid synchronize identical and different Wang and Liu-Chen four-scroll chaotic systems. Numerical simulations are also shown to illustrate and validate the hybrid synchronization results derived in this paper.

Key words: active control, chaos, chaotic systems, four-scroll systems, Wang system, Liu-Chen system

1 INTRODUCTION

Chaotic systems are nonlinear dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the *butterfly effect* [1]. The chaos phenomenon was first observed in weather models by Lorenz ([2], 1963). Chaos is an interesting nonlinear phenomenon and it has been intensively and extensively studied in the last three decades. Chaos theory has wide applications in several fields such as physical systems [3], chemical systems [4], ecological systems [5], secure communications [6–8], *etc.*

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging research problem.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the slave system so that the states of the slave system track the states of the master system asymptotically.

Since the seminal work by Pecora and Carroll ([9], 1990) on complete synchronization of chaotic systems, several approaches have been developed for chaos synchronization such as the OGY method [10], sampled-data feedback synchronization method [11], the time-delay feedback method [12, 13], the active control method [14–18], the adaptive control method [19–23], the backstep-

ping method [24–26], the sliding mode control method [27–30] and others.

So far, various synchronization methods have been developed such as the complete synchronization [9], the phase synchronization [31], the generalized synchronization [32], the anti-synchronization [33–36], the projective synchronization [37] and the generalized projective synchronization [38–40].

Complete synchronization (CS) is characterized by the equality of state variables evolving in time, while the anti-synchronization (AS) is characterized by the disappearance of the sum of relevant state variables evolving in time. Projective synchronization (PS) is characterized by the fact the master and slave systems could be synchronized up to a scaling factor, whereas in generalized projective synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α . It is easy to see that the complete synchronization and the anti-synchronization are the special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

In hybrid synchronization of chaotic systems [41], one part of the systems (for instance, the odd-numbered states) is completely synchronized, while the other part (for instance, the even numbered states) is anti-synchronized so that complete synchronization (CS) and anti-synchronization (AS) co-exist in the master-slave chaotic systems. The co-existence of CS and AS is very useful in secure communication and chaotic encryption schemes.

In this paper, we apply the active control method to derive new results for the hybrid synchronization of identical Wang four-scroll chaotic systems ([42], 2009), identical Liu-Chen four-scroll chaotic systems ([43], 2004) and non-identical Wang and Liu-Chen four-scroll chaotic systems. Our hybrid synchronization results for the four-

* Department of ECE, Velammal Institute of Technology, Panchetti, Tiruvalluvar District, India, rkarthikeyan@gmail.com ** Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University, Avadi, Chennai-600 062, India, sundarvtu@gmail.com

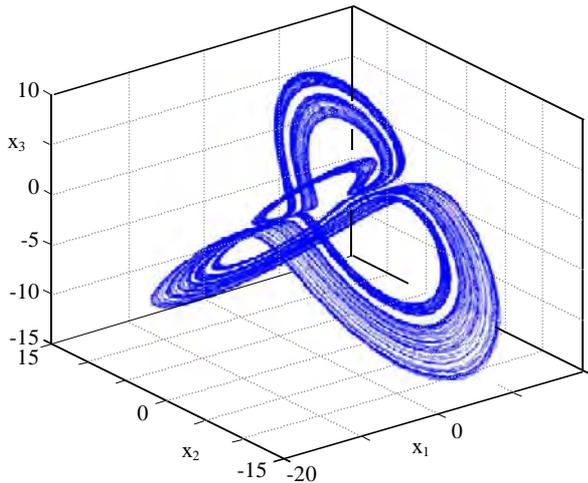


Fig. 1. The four-scroll attractor of the Wang system

scroll chaotic systems are established by the Lyapunov stability theory [44].

This paper has been organized as follows. In Section 2, we give a description of the four-scroll chaotic systems addressed in this paper. In Section 3, we discuss the hybrid synchronization of identical Wang four-scroll systems (2009). In Section 4, we discuss the hybrid synchronization of identical Liu-Chen four-scroll systems (2004). In Section 5, we discuss the hybrid synchronization of non-identical Wang and Liu-Chen four-scroll systems. In Section 6, we summarize the main results obtained in this paper.

2 SYSTEMS DESCRIPTION

The Wang four-scroll system is a new chaotic system derived by L. Wang ([42], 2009).

The Wang system is described by the 3D dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_1 - x_2) - x_2x_3, \\ \dot{x}_2 &= -bx_2 + x_1x_3, \\ \dot{x}_3 &= -cx_3 + dx_1 + x_1x_2\end{aligned}\quad (1)$$

where x_1, x_2, x_3 are the states and a, b, c, d are positive, constant parameters of the system.

The system (1) is chaotic when the parameter values are taken as

$$a = 1, \quad b = 5.7, \quad c = 5 \quad \text{and} \quad d = 0.06.$$

Figure 1 depicts the *four-scroll attractor* of the Wang chaotic system (1).

The Liu-Chen four-scroll system is a new chaotic system derived by W. Liu and G. Chen ([43], 2004). The Liu-Chen system is described by the 3D dynamics

$$\begin{aligned}\dot{x}_1 &= \alpha x_1 - x_2x_3, \\ \dot{x}_2 &= -\beta x_2 + x_1x_3, \\ \dot{x}_3 &= -\gamma x_3 + x_1x_2\end{aligned}\quad (2)$$

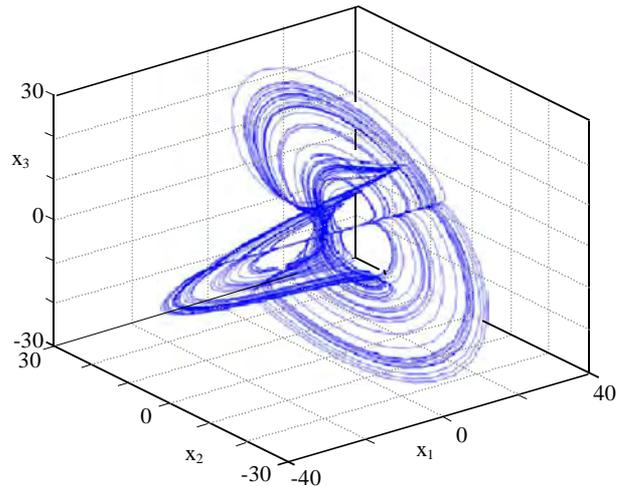


Fig. 2. The four-scroll attractor of the Liu-Chen system

where x_1, x_2, x_3 are the states and α, β, γ are positive, constant parameters of the system. The system (2) is chaotic when the parameter values are taken as

$$\alpha = 0.4, \quad \beta = 12 \quad \text{and} \quad \gamma = 5.$$

Figure 2 depicts the *four-scroll attractor* of the Liu-Chen chaotic system (2).

3 HYBRID SYNCHRONIZATION OF THE IDENTICAL WANG FOUR-SCROLL SYSTEMS

3.1 Theoretical Results

In this section, we deploy the active control method to derive new results for the hybrid synchronization of identical Wang four-scroll systems ([42], 2009).

Thus, the master system is described by the Wang dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_1 - x_2) - x_2x_3, \\ \dot{x}_2 &= -bx_2 + x_1x_3, \\ \dot{x}_3 &= -cx_3 + dx_1 + x_1x_2\end{aligned}\quad (3)$$

where x_1, x_2, x_3 are the states and a, b, c, d are positive, constant parameters of the system.

The slave system is described by the controlled Wang dynamics

$$\begin{aligned}\dot{y}_1 &= a(y_1 - y_2) - y_2y_3 + u_1, \\ \dot{y}_2 &= -by_2 + y_1y_3 + u_2, \\ \dot{y}_3 &= -cy_3 + dy_1 + y_1y_2 + u_3\end{aligned}\quad (4)$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the active controllers to be designed.

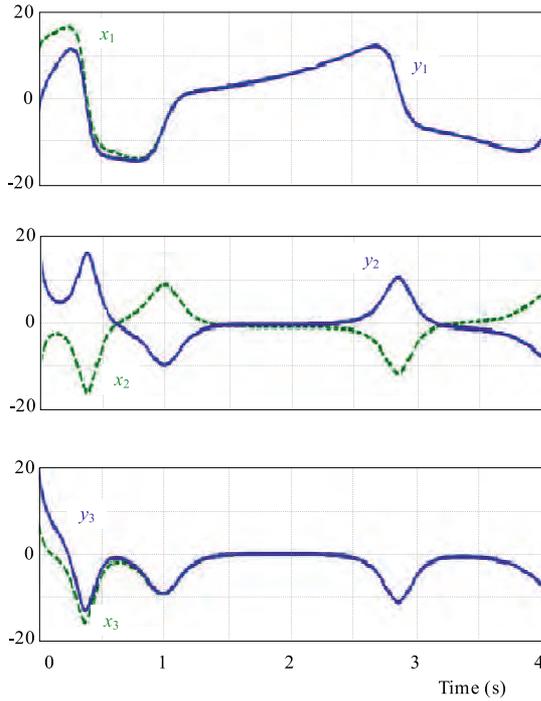


Fig. 3. Hybrid synchronization of the identical Wang four-scroll systems

The hybrid synchronization error is defined as

$$\begin{aligned} e_1 &= y_1 - x_1, \\ e_2 &= y_2 + x_2, \\ e_3 &= y_3 - x_3. \end{aligned} \quad (5)$$

A simple calculation gives the error dynamics as

$$\begin{aligned} \dot{e}_1 &= a(e_1 - e_2 + 2x_2) + u_1, \\ \dot{e}_2 &= -be_2 + y_1y_3 + x_1x_3 + u_2, \\ \dot{e}_3 &= -ce_3 + de_1 + y_1y_2 - x_1x_2 + u_3. \end{aligned} \quad (6)$$

We define the active controls $u_1(t)$, $u_2(t)$ and $u_3(t)$ as

$$\begin{aligned} u_1(t) &= -a(e_1 - e_2 + 2x_2) + y_2y_3 - x_2x_3 - k_1e_1, \\ u_2(t) &= be_2 - y_1y_3 - x_1x_3 - k_2e_2, \\ u_3(t) &= ce_3 - de_1 - y_1y_2 + x_1x_2 - k_3e_3 \end{aligned} \quad (7)$$

where k_1, k_2, k_3 are positive constants.

Substituting (7) into (6), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1e_1, \\ \dot{e}_2 &= -k_2e_2, \\ \dot{e}_3 &= -k_3e_3. \end{aligned} \quad (8)$$

THEOREM 1. *The identical Wang four-scroll systems (3) and (4) with constant, positive parameters are globally and exponentially hybrid-synchronized by the active control law (7), where the gains k_1, k_2, k_3 are positive constants.*

PROOF. This result is a simple consequence of the Lyapunov stability theory.

Consider the quadratic Lyapunov function V defined by

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2), \quad (9)$$

which is a positive definite function on R^3 .

Differentiating V along the trajectories of (8), we get

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2, \quad (10)$$

Clearly, \dot{V} is a negative definite function on R^3 . Hence, by the Lyapunov stability theory [44], it follows that $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for $i = 1, 2, 3$. This completes the proof.

3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the two systems of differential equations (3) and (4) with the active nonlinear controller (7). We take $k_i = 4$ for $i = 1, 2, 3$.

The parameters of the Wang four-scroll systems are chosen so that the systems (3) and (4) are chaotic, *ie*

$$a = 1, \quad b = 5.7, \quad c = 5, \quad d = 0.06.$$

The initial values of the master system (3) are chosen as

$$x_1(0) = 12, \quad x_2(0) = -9, \quad x_3(0) = 8.$$

The initial values of the slave system (4) are chosen as

$$y_1(0) = -2, \quad y_2(0) = 15, \quad y_3(0) = 20.$$

Figure 3 shows the hybrid synchronization of the Wang four-scroll systems (3) and (4). Figure 4 shows the time-history of the hybrid synchronization errors.

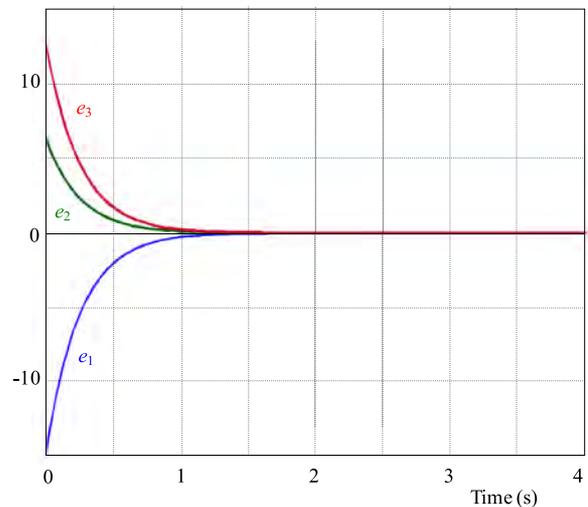


Fig. 4. History of the error states e_1, e_2, e_3

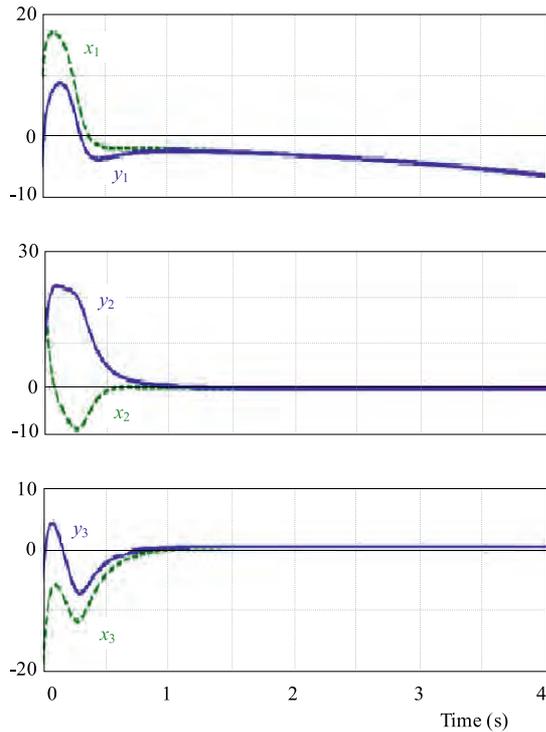


Fig. 5. Hybrid synchronization of the identical Liu-Chen four-scroll systems

4 HYBRID SYNCHRONIZATION OF THE IDENTICAL LIU-CHEN FOUR-SCROLL SYSTEMS

4.1 Theoretical Results

In this section, we deploy the active control method to derive new results for the hybrid synchronization of identical Liu-Chen four-scroll systems ([43], 2004).

Thus, the master system is described by the Liu-Chen dynamics

$$\begin{aligned}\dot{x}_1 &= \alpha x_1 - x_2 x_3, \\ \dot{x}_2 &= -\beta x_2 + x_1 x_3, \\ \dot{x}_3 &= -\gamma x_3 + x_1 x_2\end{aligned}\quad (11)$$

where x_1, x_2, x_3 are the states and α, β, γ are positive, constant parameters of the system.

The slave system is described by the controlled Wang dynamics

$$\begin{aligned}\dot{y}_1 &= \alpha y_1 - y_2 y_3 + u_1, \\ \dot{y}_2 &= -\beta y_2 + y_1 y_3 + u_2, \\ \dot{y}_3 &= -\gamma y_3 + y_1 y_2 + u_3\end{aligned}\quad (12)$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the active controllers to be designed.

The hybrid synchronization error is defined as

$$\begin{aligned}e_1 &= y_1 - x_1, \\ e_2 &= y_2 + x_2, \\ e_3 &= y_3 - x_3.\end{aligned}\quad (13)$$

A simple calculation gives the error dynamics as

$$\begin{aligned}\dot{e}_1 &= \alpha e_1 - y_2 y_3 + x_2 x_3 + u_1, \\ \dot{e}_2 &= -\beta e_2 + y_1 y_3 + x_1 x_3 + u_2, \\ \dot{e}_3 &= -\gamma e_3 + y_1 y_2 - x_1 x_2 + u_3.\end{aligned}\quad (14)$$

We define the active controls and as

$$\begin{aligned}u_1(t) &= -\alpha e_1 + y_2 y_3 - x_2 x_3 - k_1 e_1, \\ u_2(t) &= \beta e_2 - y_1 y_3 - x_1 x_3 - k_2 e_2, \\ u_3(t) &= \gamma e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3\end{aligned}\quad (15)$$

where k_1, k_2, k_3 are positive constants.

Substituting (15) into (14), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1, \\ \dot{e}_2 &= -k_2 e_2, \\ \dot{e}_3 &= -k_3 e_3.\end{aligned}\quad (16)$$

THEOREM 2. *The identical Liu-Chen four-scroll systems (11) and (12) with constant, positive parameters are globally and exponentially hybrid-synchronized by the active control law (15), where the gains k_1, k_2, k_3 are positive constants.*

PROOF. This result is a simple consequence of the Lyapunov stability theory.

Consider the quadratic Lyapunov function

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2),\quad (17)$$

which is a positive definite function on R^3 .

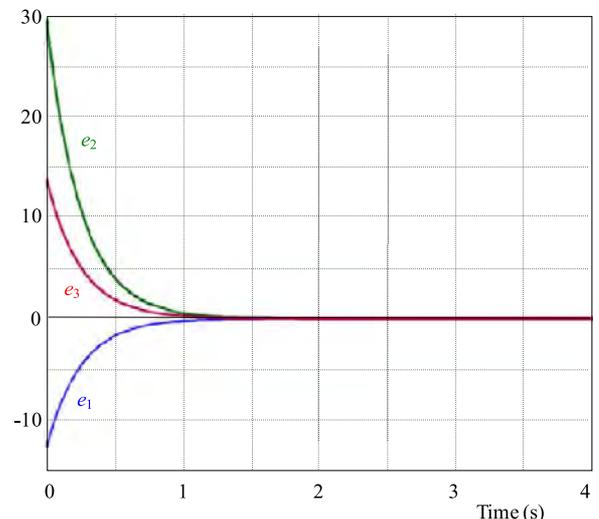


Fig. 6. History of the error states e_1, e_2, e_3

Differentiating V along the trajectories of (16), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \quad (18)$$

which is a negative definite function on R^3 . Hence, by the Lyapunov stability theory [44], it follows that $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for $i = 1, 2, 3$. This completes the proof.

4.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the two systems of differential equations (11) and (12) with the active nonlinear controller (15). We take $k_i = 4$ for $i = 1, 2, 3$. The parameters of the Liu-Chen four-scroll systems are chosen so that the systems (11) and (12) are chaotic, *ie*

$$\alpha = 0.4, \quad \beta = 12, \quad \gamma = 5.$$

The initial values of the master system (11) are chosen as

$$x_1(0) = 7, \quad x_2(0) = 20, \quad x_3(0) = -19.$$

The initial values of the slave system (12) are chosen as

$$y_1(0) = -6, \quad y_2(0) = 10, \quad y_3(0) = -5.$$

Figure 5 shows the hybrid synchronization of the Liu-Chen four-scroll systems (11) and (12). Figure 6 shows the time-history of the hybrid synchronization errors.

5 HYBRID SYNCHRONIZATION OF THE NON-IDENTICAL WANG AND LIU-CHEN FOUR-SCROLL SYSTEMS

5.1 Theoretical Results

In this section, we deploy the active control method to derive new results for the hybrid synchronization of non-identical four-scroll systems, *viz.* Wang system ([42], 2009) and Liu-Chen system ([43], 2004).

Thus, the master system is described by the Wang dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_1 - x_2) - x_2 x_3, \\ \dot{x}_2 &= -bx_2 + x_1 x_3, \\ \dot{x}_3 &= -cx_3 + dx_1 + x_1 x_2 \end{aligned} \quad (19)$$

where x_1, x_2, x_3 are the states and a, b, c are positive, constant parameters of the system.

The slave system is described by the controlled Liu-Chen dynamics

$$\begin{aligned} \dot{y}_1 &= \alpha y_1 - y_2 y_3 + u_1, \\ \dot{y}_2 &= -\beta y_2 + y_1 y_3 + u_2, \\ \dot{y}_3 &= -\gamma y_3 + y_1 y_2 + u_3 \end{aligned} \quad (20)$$

where y_1, y_2, y_3 are the states, α, β, γ are positive, constant parameters of the system and u_1, u_2, u_3 are the active controllers to be designed.

The hybrid synchronization error is defined as

$$\begin{aligned} e_1 &= y_1 - x_1, \\ e_2 &= y_2 - x_2, \\ e_3 &= y_3 - x_3. \end{aligned} \quad (21)$$

A simple calculation gives the error dynamics as

$$\begin{aligned} \dot{e}_1 &= \alpha y_1 - a(x_1 - x_2) - y_2 y_3 + x_2 x_3 + u_1, \\ \dot{e}_2 &= -\beta y_2 - bx_2 + y_1 y_3 + x_1 x_3 + u_2, \\ \dot{e}_3 &= -\gamma y_3 + cx_3 - dx_1 + y_1 y_2 - x_1 x_2 + u_3. \end{aligned} \quad (22)$$

We define the active controls $u_1(t), u_2(t)$ and $u_3(t)$ as

$$\begin{aligned} u_1(t) &= -\alpha y_1 + a(x_1 - x_2) + y_2 y_3 - x_2 x_3 - k_1 e_1, \\ u_2(t) &= \beta y_2 + bx_2 - y_1 y_3 - x_1 x_3 - k_2 e_2, \\ u_3(t) &= \gamma y_3 - cx_3 + dx_1 - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{aligned} \quad (23)$$

where k_1, k_2, k_3 are positive constants.

Substituting (23) into (22), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1, \\ \dot{e}_2 &= -k_2 e_2, \\ \dot{e}_3 &= -k_3 e_3. \end{aligned} \quad (24)$$

THEOREM 3. *The non-identical Wang four-scroll system (19) and Liu-Chen four-scroll system (20) with constant, positive parameters are globally and exponentially hybrid-synchronized by the active control law (23), where the gains k_1, k_2, k_3 are positive constants.*

PROOF. This result is a simple consequence of the Lyapunov stability theory. Consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2), \quad (25)$$

which is a positive definite function on R^3 .

Differentiating V along the trajectories of (24), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \quad (26)$$

which is a negative definite function on R^3 . Hence, by the Lyapunov stability theory [44], it follows that $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for $i = 1, 2, 3$. This completes the proof.

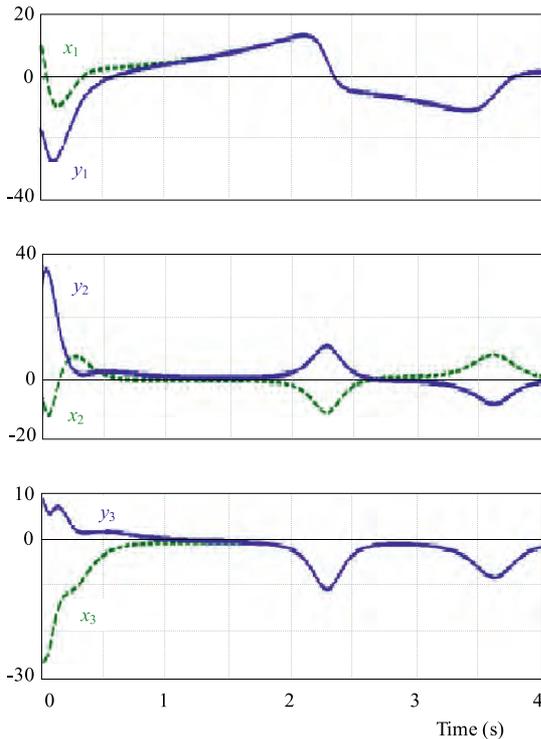


Fig. 7. Hybrid synchronization of the non-identical Wang and Liu-Chen four-scroll systems

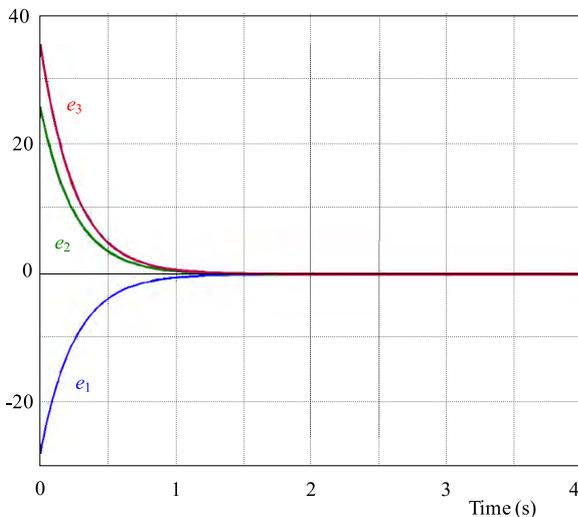


Fig. 8. History of the error states e_1 , e_2 , e_3

5.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the two systems of differential equations (19) and (20) with the active nonlinear controller (23).

We take $k_i = 4$ for $i = 1, 2, 3$.

The parameters of the Wang and Liu-Chen four-scroll systems are chosen as in the chaotic case, *ie*

$$a = 1, \quad b = 5.7, \quad c = 5, \quad d = 0.06, \\ \alpha = 0.4, \quad \beta = 12, \quad \gamma = 5.$$

The initial values of the master system (19) are chosen as

$$x_1(0) = 12, \quad x_2(0) = -4, \quad x_3(0) = -27.$$

The initial values of the slave system (20) are chosen as

$$y_1(0) = -16, \quad y_2(0) = 30, \quad y_3(0) = 9.$$

Figure 7 shows the hybrid synchronization of the Wang system (19) and the Liu-Chen system (20). Figure 8 shows the time-history of the hybrid synchronization errors.

6 CONCLUSIONS

In this study, we have applied active control method for the hybrid chaos synchronization of the identical Wang four-scroll systems (2009), the identical Liu-Chen four-scroll systems (2004) and non-identical Wang and Liu-Chen four-scroll systems. The hybrid synchronization results derived in this paper have been proved using the Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient for achieving hybrid synchronization of the four-scroll systems addressed in this paper. Numerical simulations are shown to validate and illustrate the hybrid synchronization results derived in this paper for the four-scroll chaotic systems.

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Karthikeyan Rajagopal is a Professor and the Head of the Department of Electronics and Communication Engineering at Velammal Institute of Technology, Chennai, India since 2012. He has 14 publications in Scopus-indexed journals and conferences. His areas of interest include Wireless Networks, Security Algorithms, Embedded Systems and Control Systems.

Sundarapandian Vaidyanathan is a Professor and the Dean at the Research and Development Centre, Vel Tech Dr RR & Dr SR Technical University, Chennai, India. His current research focuses on Control Systems, Chaos Theory and Computational Science. He has published over 100 Scopus-indexed research publications. He has delivered many plenary talks on Control Engineering and Chaos in international conferences.