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Multi-objective optimal reconfiguration of distribution network

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Configuration of distribution system can be changed manually or automatically, by changing the status of the respective switching elements, with the aim of reducing power losses, increase system reliability, or improving the power quality. When changing the status of switching equipment it is necessary to satisfy the requirement for the radial and connected structure of the distribution network. Using the single criteria optimization it is possible to improve one of the characteristics of the distribution network, on the other hand by using multicriteria optimization it is possible to find a network configuration that enhances multiple distribution system characteristics at the same time. In this paper, a modification of the multi-criteria Gray Wolf optimization algorithm is proposed in order to create an efficient algorithm that can be implemented in the management functions of smart grid concept of modern distribution systems. The proposed reconfiguration algorithm was tested on standard symmetrical IEEE 33 test distribution network.

K e y w o r d s: distribution systems, reconfiguration, load flow, multi-criteria optimization

1 Introduction

The reconfiguration of the distribution network represents the process of changing the topology of individual feeders by opening and closing the tie and sectionalizing switches. The goal of reconfiguration is to improve the individual characteristics of the distribution network. The considered improvements may relate to shorter or longer periods of time. While in the early studies reconfiguration was treated as basic function for planning of distribution networks and connected to the calculation of configuration changes due to seasonal load changes [1, 2], current research is oriented towards finding the solutions for real-time reconfiguration implementations [3,4], especially due to distributed sources and their voltage and power control [5].

Production variability of distributed generators, which are increasingly being used in modern distribution systems, along with daily variations of different load categories, increases daily and even hourly variations of feeder loads in distribution networks. These variations affect not only operational costs in distribution systems, but also the ability to efficiently use all parts of the network and exploit the distributed generation without violating operational constraints. In order to satisfy these constraints, new optimal reconfiguration functions are being developed and implemented in modern smart grid systems. These new functions require fast and reliable algorithms, which need to include as many of distribution network efficiency indicators as possible, as well as all limitations that will ensure system operation within the allowed limits.

The first paper dealing with reconfiguration, in which the potential for exploring this area was noticed, was written by Merlin and Back [6]. Starting from this paper, a new area has been opened in which different authors have tried to create an algorithm for finding the radial structure of a distribution network that will have lower real power losses, but for an acceptable execution time.

Observed from the mathematical point of view, the problem of reconfiguration of the distribution network belongs to the class of complex combinatorial, non-differentiable optimization problems. The requirement for the radial structure of the distribution network further complicates this optimization problem. The iterative structure of the load flow, the thermal constraints of the distribution network components, and the necessity of an exhaustive search of all possible configurations additionally contribute to the complexity of the problem under consideration.

Methods for solving the distribution network reconfiguration problem can be roughly divided into three categories: 1) heuristic, 2) mathematical optimization and 3) meta-heuristic methods. The fourth group could be hybrid combinations of the above methods. The most popular methods are based on heuristic principles, mainly due to the rapid generation of the final solution. Heuristic techniques are in fact optimization processes for finding solutions using system characteristics, which can be defined by appropriate coefficients. These coefficients can be simple (obtained on the basis of author's experience) or, on the other hand, they can be based on solving complex problems by analyzing the sensitivity of a complete or simplified mathematical model [2, 7–10]. However, these methods are highly dependent on the initial configuration of the distribution network, so finding the global optimal solution is not always guaranteed. Mathematical optimization methods, also known as deterministic methods, for solving distribution network reconfiguration

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problem use linear programming methods [11], dynamic programming [12], *etc.*

In the early nineties a considerable number of papers that used meta-heuristic algorithms for solving the distribution network reconfiguration problem appeared [13–19]. Greek prefix "meta" means "higher level". A meta-heuristic approach is a specific search method that starts from an initial point (or from a set of initial points) and which, using certain rules, guide initial point to avoid local optimum. The main difference between various meta-heuristic methods is in the solution search mechanism. These methods can provide an optimal solution regardless of the initial distribution network configuration, but they are time-consuming so they can be used only when it is not necessary to quickly deliver solutions. Different methods were used for distribution network reconfiguration to reduce real power losses and/or to reduce number of switching operations: sequentially encoded genetic algorithm [13], harmony search algorithm [14], modified honey bee mating optimization [15].

In the literature it is possible to find many papers which solve the problem of optimal reconfiguration using the single criteria function. On the other hand, very few papers are dedicated to the simultaneous improvement of several criteria [20, 21].

By applying single criteria optimization, it is possible to improve only certain (considered) characteristics of the distribution network. On the other hand, by applying multi-criteria optimization it is possible to find a solution that will represent a compromise for the improvement of several characteristics. It is justifiable to expect that the compromise solution will be worse than one that was obtained when only single optimal solution is required, on the other hand this is a solution in which all the characteristics are represented, which was not the case for single criteria optimization.

In this paper, the basic Gray Wolf algorithm [22] was used for solving optimal reconfiguration of distribution network. This optimization algorithm has been very little used so far for solving problems in electrical engineering. Also, due to the requirement for the algorithm to make the most comprehensive solution necessary for efficient management in the modern smart grid, a modification of multi-criteria Grey Wolf algorithm [23] has been implemented, enabling the inclusion of several criteria functions. The formed optimization procedure is adapted to the problem of optimal reconfiguration with integer control variables. In this procedure the search is limited only to a set of feasible solutions that do not violate the radial structure and requirement for the connection of all nodes of the distribution network. The proposed multicriteria optimization was used to reduce the real power losses and maximum voltage drop and balance the load across the branches of the observed network. The proposed algorithm, based on the comparison by domination, provides a set of compromise optimal solutions (Pareto front). Using this set of solutions, the decision maker can quickly make a comparison and/or a complete sensitivity analysis and depending on the priority that can be adapted to the management requirements at any time selects an optimal solution.

All calculations were tested on a standard symmetric IEEE 33 test distribution network, while the comparison of the obtained solutions was made by sensitivity analysis using synthetic criteria functions with appropriately chosen parameters.

The paper is organized in the following way: in the second part, the basic concepts of multi-criteria optimization are presented. The basic algorithm of Gray Wolf is presented in the third part, while the fourth part describes the extension of the basic algorithm of Gray Wolf to multi-criteria optimization. The fifth section describes the modifications necessary for the implementation of optimal reconfiguration with description of all algorithm components, while in the sixth section the optimal reconfiguration algorithm of the distribution network is described. The results are shown in the seventh section and finally the conclusion is given in the eighth part.

2 Multi-criteria optimization

Multi-criteria optimization refers to the optimization of a problem that has more than one criteria function. Without loss of generality, this problem can be defined as a minimization problem in the following way

Minimize

$$\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}) \ldots, f_K(\mathbf{x})\}, \tag{1}$$

According to the limitations:

$$g_i(\mathbf{x}) \geq 0\,, \qquad i = 1, \ldots, M\,, \tag{2}$$

$$h_i(\mathbf{x}) = 0\,, \qquad i = 1, \ldots, N\,, \tag{3}$$

$$L_i \leq x_i \leq U_i\,,\ \ i = 1, \ldots, D\,, \tag{4}$$

where D is the number of control variables, K is the number of criteria functions, M is the number of inequality constraints, N is the number of equality constraints, g_i is the i-th inequality constraints, and h_i represents i-th equality constraints. The lower and upper bounds of the i-th control variables are labeled with L_i and U_i.

In single criteria optimization, individual solutions can be easily compared. In the case of minimization problem, the solution $\boldsymbol{x}^*$ is better than the solution $\boldsymbol{x}$ if and only if $f(\mathbf{x}^*) < f(\mathbf{x})$. However, when dealing with multi-criteria problems, it is impossible to compare the solutions with relational operators. Let $\boldsymbol{y} = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_K(\mathbf{x})\}$ and $\boldsymbol{y}^* = \{f_1(\mathbf{x}^*), f_2(\mathbf{x}^*), \ldots, f_K(\mathbf{x}^*)\}$ be given vectors which represent points in the space of the criteria functions $\boldsymbol{y} \in O \subset \Re^K$. Each component of the vector $\boldsymbol{y}$ is the value of an objective function that is calculated based on the assigned point from the search area, $\mathbf{x} \in S \subset \Re^D$. The vector $\boldsymbol{y}^*$ dominates over the vector $\boldsymbol{y}$ $(\boldsymbol{y}^* \preceq \boldsymbol{y})$ if $\boldsymbol{y}^*$ is partially smaller than $\boldsymbol{y}$, *ie*

$$\begin{aligned} &\forall k \in \{1, \ldots, K\} : f_k(\boldsymbol{y}^*) \leq f_k(\boldsymbol{y}) \wedge \\ &\exists k \in \{1, \ldots, K\} : f_k(\boldsymbol{y}^*) < f_k(\boldsymbol{y})\,. \end{aligned} \tag{5}$$

If one of these two conditions is not fulfilled then the solution $\boldsymbol{y}^*$ does not dominate over the solution $\boldsymbol{y}$. In accordance with this definition, three cases are possible:

- the solution $\boldsymbol{y}^*$ dominates the solution $\boldsymbol{y}$,
- the solution $\boldsymbol{y}$ dominates the solution $\boldsymbol{y}^*$,
- the solutions $\boldsymbol{y}^*$ and $\boldsymbol{y}$ are non-dominant one in relation to the other.

A solution that is not dominated by any other feasible solution is called Pareto optimal, or strictly efficient. Pareto dominance refers to the vector in the domain of the criteria functions, O, and not in the domain of search space, S.

The Pareto front is a hyper-surface in the space of the criteria functions, O, made up of a set of strictly efficient solutions. As such, the Pareto front is a set of best compromises that are not dominant in relation to each other.

3 Grey Wolf Optimization

This section will briefly describe a single criteria optimization algorithm based on the behavior of gray wolves. A detailed description of this algorithm can be found in [22]. The social order and the method of hunting of the gray wolf pack were the basic inspiration for this algorithm. To model the hierarchy in the pack, the three best solutions were taken for the (α, β and δ) wolves to make a decision. All other solutions are ω wolves. New locations of ω wolves are determined using the three best wolves in the pack.

A good search mechanism is necessary for the successful operation of optimization algorithms. Roughly speaking, the search is divided into two segments: research and exploitation. In the research process, it is necessary to generate diverse solutions to cover as wide area of search space as possible. On the other hand, in the process of exploitation, it is necessary to search a relatively narrow part around the best solution. The gray wolf algorithm for these purposes uses vectors $\boldsymbol{A}$ and $\boldsymbol{C}$ defined in the following way

$$\begin{aligned} \boldsymbol{A} &= 2\boldsymbol{a} \cdot \boldsymbol{r}_1 - \boldsymbol{a}, \\ \boldsymbol{C} &= 2\boldsymbol{r}_2 \,. \end{aligned} \tag{6}$$

where the members of the vector $\boldsymbol{a}$ linearly decrease from 2 to 0 with the iterations, and $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are random vectors at the interval $[0, 1]$.

Since the position of the prey (optimal solution) is not known in advance, the search for it will be based on the information that the three best wolves possess. All ω wolves will improve their position based on the following expression

$$\boldsymbol{D}_i = |\boldsymbol{C}_i \cdot \boldsymbol{X}_i - \boldsymbol{X}| \,, \quad i = 1, 2, 3 \,, \tag{7}$$

$$\boldsymbol{X}(t+1) = \frac{1}{3} \sum_{i=1}^{3} (\boldsymbol{X}_i - \boldsymbol{A}_i \cdot \boldsymbol{D}_i) \,. \tag{8}$$

The research process is guaranteed when $|\boldsymbol{A}| > 1$, and then wolves move away from prey. Another component that favors research is vector $\boldsymbol{C}$. Since $\boldsymbol{C}$ is an arbitrary vector in the range $[0, 2]$, the wolves will abandon the prey when $\boldsymbol{C} > 1$, and approach the prey when $\boldsymbol{C} < 1$. It is important to note that the vector $\boldsymbol{C}$ can take any value from the specified range at any time of the optimization algorithm. In other words, in the final iterations of the algorithm, it may happen that some wolf goes into research, precisely because of vector $\boldsymbol{C}$, and does not continue exploitation even though the end of the optimization is approaching. This feature helps the optimization algorithm to avoid local optimums.

The exploitation process is guaranteed when $|\boldsymbol{A}| < 1$. Then the new position of ω wolf will be found between his current position and the position of the prey, which is estimated based on information provided by the leaders of the pack.

Like most other meta-heuristic optimization algorithms, this algorithm begins by creating a randomly selected initial population. At each iteration three best solutions are selected to conduct search in the solution space. The selection of a new location of ω wolves in the search space is done using expressions (6)-(8). With iterations, the vectors $\boldsymbol{a}$ and $\boldsymbol{A}$ decrease, so ω wolves tend to the solution when $|\boldsymbol{A}| < 1$ and they are moving away from the solution when $|\boldsymbol{A}| > 1$. At the end of the optimization process, the last α wolf is the optimal solution.

4 Multi-criteria optimization

In order to implement multi-criteria optimization in the basic algorithm of gray wolves optimizer it is necessary to implement two more components in algorithm. One of these components is the archive in which the best (non-dominant) solutions are located. The second component refers to the method of selecting the leader of a pack (α, β and δ wolf) from the archive.

The archive represents the register in which are stored and deleted, the non-dominant solutions found in the previous search process. One of the important things about the archive is the control of the entry of new vectors into the archive. It should also know the archive has a predefined number of places, so it is not possible to enter more vectors than the predefined maximum allowed number of archive.

In order to reduce the number of comparisons, the concept of fronts from the NSGA-II algorithm was introduced [24]. These solutions, which are not dominated by any member in the population, are called non-dominant solutions of the first level. Figure 1 shows one minimization problem where the solutions are divided into fronts. The first front in this case is composed of a set of vectors $\{1, 5, 8, 9, 10\}$. If the solutions from the first dominant front are eliminated for a moment, and the principle of sorting according to domination is applied to the remaining solutions, the next dominant front is obtained

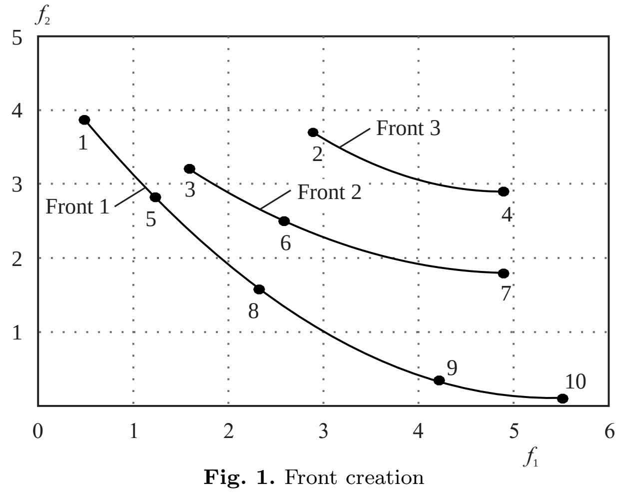

Fig. 1. Front creation

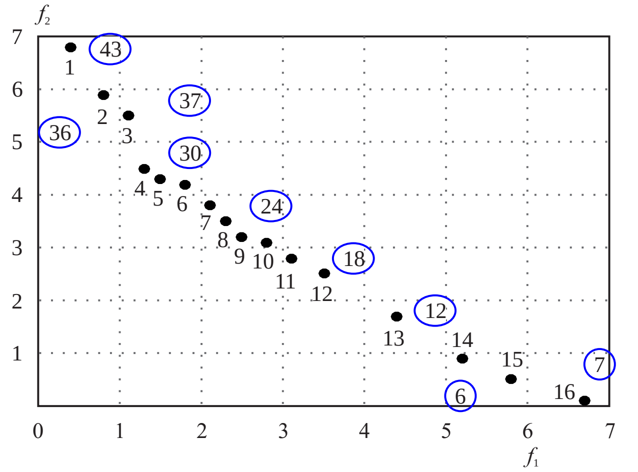

Fig. 2. Division of the criteria functions space in the hyper region

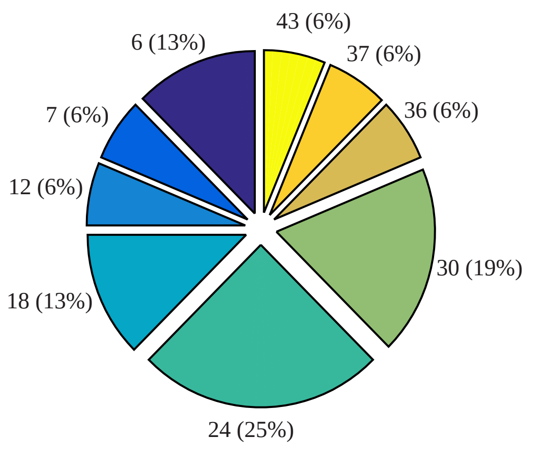

Fig. 3. Roulette wheel

$\{3, 6, 7\}$, Fig. 1. The process continues until all the solutions in the population are processed.

At the very beginning of the optimization process, after creating the initial population, it is necessary to place the first front of the initial population in the archive. If the first front has less than three members, the corresponding number of vectors must be randomly selected from the second front. It is necessary that the archives have at least three members. This is conditioned by the basic algorithm of gray wolves since the search process is navigated with the three best solutions.

As already noted, in order to avoid unnecessary comparison of worse solutions with vectors located in the archive, only vectors from the first front of the current population will be compared. Comparison is carried out by combining a complete archive with the first front of the current population, and then the comparison is repeated according to the principle of domination. The new archive consists of the first front of this formed set. If the maximum number of vectors that can be stored in the archive is exceeded, it is necessary to reject the corresponding number of vectors. The decision which member of the archive will be rejected is based on the density of the number of solutions in certain parts of the space of the criteria functions, O.

Criteria function space can be uniformly divided as shown in Fig. 2. After that, it is necessary to find occupied areas (numbers within the circle in Fig. 2) as well as the number of vectors in these areas. Segment selection from which the vector will be discarded is done using the roulette wheel. First, it is necessary to separate the areas on the roulette wheel. The surface of the roulette slice for each hyper surface is determined using the following expression

$$p_i = \frac{n_i^b}{\sum_{j=1}^{m} n_j^b} \tag{9}$$

where p_i is the probability of selecting the i-th segment, b is the gain coefficient, n_i is the number of vectors in the i-th hyper surface, while m is the total number of hyper surface in which there is at least one vector. Figure 3 shows segments division of the roulette wheel for the example given in Fig. 2. Due to the visibility of the diagram itself, a unit value is assigned for the gain coefficient.

The area with the highest number of vectors, segment number 24, has a 25 % chance of being selected. This percentage would be higher if a higher value for b is adopted. For example, for $b = 2$, this segment would occupy 42 % of the surface of the roulette wheel, while for $b = 4$ would be 68 %. After the division, a random number from the range $[0, 1]$ is generated. Based on this number, the segment from which arbitrarily selected vector will be ejected is chosen. If the number of archive members is still greater than allowed, it is necessary to repeat the described procedure until the size of the archive becomes acceptable.

The second additional component is the mechanism for selecting the leader of the pack. With single criteria optimization, the three best solutions were chosen to conduct a search with a objective to find a solution that is close to the global optimum. However, in multi-criteria optimization it is not possible to determine which solutions are the best. Therefore, it was necessary to select a mechanism that would enable the selection of the leaders of the pack. Roulette wheel rule is also applied for this purpose. In this case it is necessary to emphasize

Algorithm 1

Formation of the initial population $\boldsymbol{X}_i, (i = 1, \ldots, N_p)$
Calculating the value of the criteria functions of each vectors of the initial population
Test dominance and form fronts
Fill the archive with the first front and with the required number of vectors from the higher fronts.
while ($t <$maximum number of iterations)
 Calculate the vectors $\boldsymbol{a}$, $\boldsymbol{A}$ and $\boldsymbol{C}$
 for each vector from the current population
  Select the α leader and temporarily remove it from the archive
  Select the β leader and temporarily remove it from the archive
  Select the δ leader and temporarily remove it from the archive
  Return α, β and δ vectors to the archive
  Improve the position of the current vector by expression (6)–(8)
 end for
 Calculate the values of the criteria functions of each newly formed vector
 Test dominance and form first front
 Combine the archive and the first front of the current population into an ancillary registry
 Test dominance of the auxiliary register and form first front
 Place the first front in the archive
 if archive size greater than allowed
  Determine the number of vectors, k, which need to be removed from the archive
  for $i = 1$ to k
   Form the network and determine the number of vectors in hyper areas
   Determine from which hyper area the vector will be discarded
   From a selected hyper area, randomly select a single vector which will be discarded
  end for
 end if
 $t = t + 1$
end while

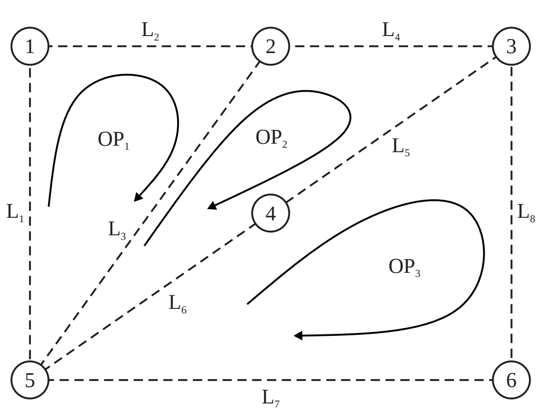

Fig. 4. An example of meshed grid

the exploitation process to cover the whole Pareto front evenly. For this purpose, the expression (9) can be used, with the difference that a negative value for the gain coefficient b is assigned. One vector is arbitrarily selected from a selected segment using a roulette wheel rule and temporarily removed from the archive. The same procedure is repeated for the other two leading members of the pack. When all three wolves (α, β and δ) that navigate search process are selected, it is necessary to return the deleted vectors to the archive. The further process of improving all ω wolves is performed using the expressions (6)–(8). Since the best solutions cannot be determined when solving the problem of multi-criteria optimization, the new leaders of the pack are selected for each member of the population. Now it is possible to summarize the multi-criteria algorithm of gray wolves (Algorithm 1).

5 Components of algorithm

This chapter will describe the basic components used in the process of optimal reconfiguration. Load flow is one of the basic tools in solving the problem of reconfiguration. For the purposes of this paper, it is sufficient to use the most basic variant of power flow calculation [25], which does not respect modifications due to the presence of loops and/or distributed generators.

5.1 Fundamental loops

The feasible configuration of the distribution network is formed using fundamental loops [26]. When the network is meshed, the number of fundamental loops, FL, can be calculated using the expression

$$FL = N_{\text{br}} - N_{\text{nod}} + 1, \tag{10}$$

Table 1. Branch determination of the first fundamental loop

B		Node					
		1	2	3	4	5	6
Branch	1	-1	0	0	0	1	0
	3	0	-1	0	0	1	0
	6	0	0	0	-1	1	0
	5	**0**	**0**	**-1**	**1**	**0**	**0**
	7	**0**	**0**	**0**	**0**	**1**	**-1**
	2	1	-1	0	0	0	0
Step 1	$\lvert\Sigma\rvert$	2	2	1	2	4	1
Step 2	$\lvert\Sigma\rvert$	2	2	0	1	3	0
Step 3	$\lvert\Sigma\rvert$	2	2	0	0	2	0

where $N_{\rm br}$ is the number of branches in the observed network, and $N_{\rm nod}$ is the number of nodes of the observed network. The number of fundamental loops also gives information on how many branches should be opened to have a radial configuration of the network. Elements of one fundamental loop constitute of a set of all graph branches that form the observed loop. Branches of the fundamental loops for the network shown in Fig. 4 are: $FL_1 = [L_1, L_2, L_3]$, $FL_2 = [L_3, L_4, L_5, L_6]$, $FL_3 = [L_5, L_6, L_7, L_8]$. In order to create radial topology, it is necessary to select exactly one branch from each fundamental loop. However, in this approach, it should be careful not to select the same branches for different fundamental loops. As shown in Fig. 4 branch L_3 belongs to the fundamental loops 1 and 2, while branches L_5 and L_6 belong to the fundamental loops 2 and 3. This means that if the branch L_3 is selected in the first fundamental loop, for the second control variable branch 2, 3 and 4 (L_4, L_5 and L_6) can be selected.

5.2 Radial structure and connectivity constraints

With a random selection of open branches in the observed distribution network, many generated combinations will be unfeasible. To prevent to work with unacceptable distribution network configurations, it is necessary to check connectivity and radial structure constraints. The process of checking the radial structure of distribution network can also be used before the start of the optimization process to determine the branches of each individual fundamental loop. The algorithm for checking the radial structure is proposed in [27] and will be briefly explained here. For the observed network, it is necessary to establish a connection matrix $\mathbf{B}$. The dimensions of this matrix are $N_{\rm br} \times N_{\rm nod}$. Non-zero elements of the matrix $\mathbf{B}$ are 1 and -1. When the branch i is oriented from the node j then the value 1 is entered at position $B(i,j)$, otherwise -1 is entered. When determining the branches of the fundamental loops, the branches of tree are entered first, and in the end only one branch which forms the observed loop is added. After the formation of the connectivity matrix it is necessary to sum absolute values per columns of this matrix. If the sum of the absolute values of some column is equal to 1, it is necessary to find this row and delete it from the matrix $\mathbf{B}$. After that, the described procedure is repeated until the moment when there is no longer a column in which the sum of the absolute values of the elements is equal to 1. The remaining branches are the branches of the loop. In the case of testing the radial structure of the network at the end of the described procedure matrix $\mathbf{B}$ should remain empty. The described process can be understood in the case of determining the branch of the first fundamental loop, Fig. 4, which is shown in Tab. 1. Since in the first step there are two columns in which sum of absolute values is equal to 1, this means that the two branches must be ejected from the matrix $\mathbf{B}$. The ejected branches are in bold and underlined. After this change and after summing the absolute value by columns, step 2, there is only one column that has one element. Branch number 6, marked with rectangles, will be ejected from the matrix $\mathbf{B}$. After a new summation of absolute values per columns, step 3, there will no longer be a column that has only one element. All branches that are left in the matrix $\mathbf{B}$ form the branches of the first fundamental loop. This only confirms the results that can be seen from Fig. 4, which are described in the previous text.

The connection of the observed network is checked using the value of determinant of the matrix $\mathbf{B}$. Since matrix $\mathbf{B}$ is not square it is necessary to remove the column corresponding to the reference node and then calculate its determinant. If the value of the determinant of the so formed matrix B is equal to 1 or -1 then the network is radial and connected otherwise, this condition is not satisfied, and such a solution needs to be rejected.

6 Algorithm

The optimal reconfiguration algorithm in many ways coincides with the described multi-criteria optimization algorithm, as can be seen in Fig. 5. The calculations required for the reconfiguration of the distribution network are performed as part of the steps in which the values of the criteria functions are calculated.

First, it is necessary to round the control variables to the nearest integer. After that, it is necessary to decode the control variables. For example, if the following vector of control variables 1, 3, 4 is created for the network shown in Fig. 4, it means that it is necessary to open the following branches: $FL_1(1) = L_1$, $FL_2(3) = L_5$ and $FL_3(4) = L_8$. In the next step, the condition of the radial structure and network connection is checked. If this condition is not satisfied, the solution is immediately rejected and a new one is generated. After this, it is necessary to properly orient the branches of the distribution network and to carry out the load flow. With the results of this calculation, the values of the considered criteria functions can be calculated. In this paper three criteria are considered: real power losses, maximal voltage drop

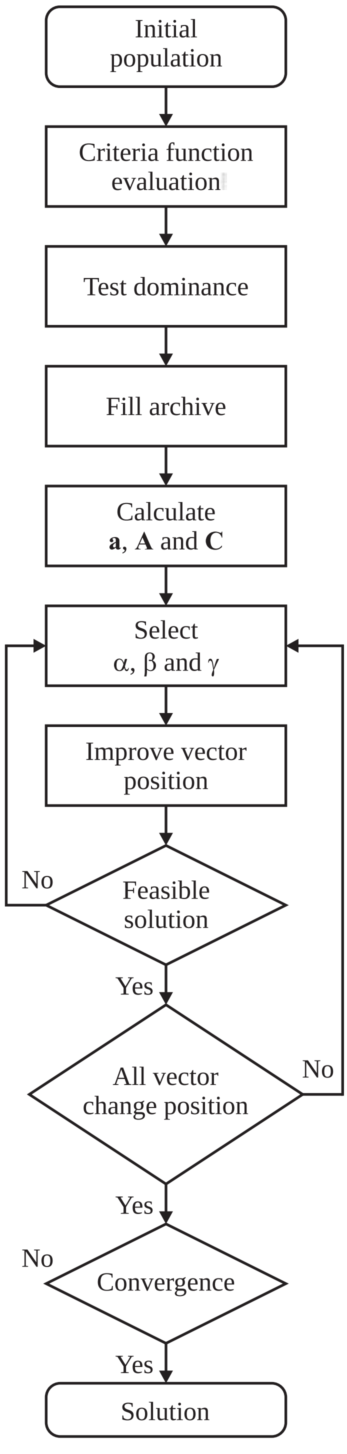

Fig. 5. Algorithm flow chart

and minimization of load balancing index. These criteria functions can be represented in mathematical form

$$f_1 = \min \sum_{i=1}^{N_{\rm br}} R_i \frac{P_{r,i}^2 + Q_{r,i}^2}{|U_{r,i}|^2}\,,$$
$$f_2 = \min\Big(\max_{i\in[1,N_{\rm nod}]} \big(|U_n| - |U_i|\big)\Big), \qquad (11)$$
$$f_3 = \min \frac{|I_i|}{I_{\max}}\,, \quad i = 1,\ldots,N_{\rm br}$$

where R_i is the resistance of the i-th branch, $P_{r,i}$ and $Q_{r,i}$ are the real and reactive power of the receiving end of the i-th branch, respectively, U_r is voltage of the receiving end of the i-th branch, U_n is the nominal value of the voltage, U_i is the voltage of the i-th node, I_i is the current of the i-th branch, and I_{max} represents the highest value of the current flowing through a segment of considered distribution network.

Table 2. Possible values of control variables

No	FL_1	FL_2	FL_3	FL_4	FL_5
1	3-4	2-3	9-10	2-3	6-7
2	4-5	3-4	10-11	3-4	7-8
3	5-6	4-5	11-12	4-5	8-9
4	3-23	5-6	12-13	5-6	9-10
5	23-24	6-7	13-14	6-7	10-11
6	24-25	7-8	14-15	7-8	11-12
7	6-26	2-19	9-15	8-9	12-13
8	26-27	19-20		9-10	13-14
9	27-28	20-21		10-11	14-15
10	28-29	8-21		11-12	15-16
11	25-29			2-19	16-17
12	19-20			19-20	17-18
13	20-21			20-21	6-26
14	21-22			21-22	26-27
15	12-22			12-22	27-28
16					28-29
17					29-30
18					30-31
19					31-32
20					32-33
21					18-33

7 Results

The described algorithm was tested on a standard symmetric IEEE 33 test distribution network [28]. In order to make the sensitivity analysis, the results were analyzed for 7 cases with different combination of criteria functions:

1) real power losses,
2) voltage drop,
3) load balancing index,
4) real power losses and voltage drop,
5) real power losses and load balancing index,
6) voltage drop and load balancing index,
7) all proposed criteria.

The number of control variables of the optimization problem is equal to the number of loops in the observed network. The possible values of the individual control variables are given in Tab. 2. The first column of Tab. 2 presents the ordinal number or branch code in the respective loop. This approach makes it easy to apply the optimization algorithm. The upper limits of the control variables are defined by the total number of branches in the observed fundamental loop, respectively $\{11, 10, 7, 15, 21\}$. The maximum number of iterations was set to 1000. The number of population members was amounted to 50, while the coefficient of reinforcement, b, was assigned with a value of 4. Table 3 shows the solutions for all cases using proposed multi criteria optimization algorithm.

Because of different range of changes of distribution network characteristics, it is necessary to normalize them in order to enable a quality comparison of the solution from the Pareto front. Normalization can be done using the following expression

$$f_{iT,j} = \frac{f_{i,j} - f_{i\min}}{f_{i\max} - f_{i\min}} \tag{12}$$

where $f_{i,j}$ and $f_{iT,}$ are value and transformed value of the i-th criteria function $(i = 1, 2, 3)$ of the j-th solution from the Pareto front, $f_{i\min}$ and $f_{i\max}$ are the minimum and maximum values of the i-th criteria function of all the solutions found on the Pareto front, respectively. Using the appropriate weight factors it is possible to extract only those solutions from hyper surfaces, Fig. 6, which are in a certain direction. The weight factors can be considered using the following expression

$$F = \overline{\omega}_1 f_1 + \overline{\omega}_2 f_2 + \overline{\omega}_3 f_3 \tag{13}$$

where ω_i, $(i = 1, 2, 3)$ represents the weight factors related to the real power losses, voltage drop and load balancing index, respectively, and serve to determine only one solution that is optimal according to this function.

Using different combinations of weighting factors it is possible to choose some solution from the Pareto front. The basic economic indicator of distribution networks is usually related to the costs associated with power losses. In order to emphasize these criteria the weight factor ω_1 should take higher values than other weight factors. This might be the main criteria when the network components are not overloaded. However, during peak load times, voltage drops can become critical from the point of providing high quality power supply, especially to critical consumers. In this case, a greater value can be given to the coefficient ω_2. On the other hand, in case if some segments of the distribution network are overloaded, it is necessary to give priority to the third criteria function to provide that all the elements of the network are loaded more uniformly. The operator can decide, using collected measurements from advanced meters in the smart grid, and according to the determined level of load of the whole network or individual parts, which weighting factors to choose. Since multi-criteria optimization allows faster search in different directions, the operator can, based on the available results of the multi-criteria optimization, perform the sensitivity analysis, like it is done in the following text.

Single criteria optimization is just a special case of multi-criteria optimization. In other words, if the optimization of only one distribution network characteristic is considered, in expression (13) only one of the functions should be retained, like it was done in the first three cases in Tab. 3. These points can also be seen in Fig. 6 on the appropriate axes because they represent the best solutions to the considered criteria function when all others are neglected. Cases 4, 5 and 6 correspond to the two criteria optimizations. Due to the correlation of the first and second criteria functions, case 4 ($\omega_3 = 0$), only a small number of close solutions dominate over everything else and make Pareto front. While in the case of opposing criteria, as in case 5 and case 6, all compromise solutions are included in the Pareto front. For the case 5, Pareto front is shown in Fig. 6(b), and for case 6 in Fig. 6(c).

Knowing all optimal solutions, decision maker can make an assessment which finds the best compromise

Table 3. Results

Case	Weighting factors			Branch code	Criteria functions		
	ω_1	ω_2	ω_3		f_1	f_2	f_3
1	1	0	0	11 6 7 10 20	136.76	0.0621	0.4246
2	0	1	0	10 6 7 10 20	137.19	0.0587	0.5434
3	0	0	1	10 4 1 14 18	243.20	0.1342	0.2714
4	0.7	0.3	0	11 6 7 10 20	136.76	0.0621	0.4246
4	0.3	0.7	0	10 6 7 10 20	137.19	0.0587	0.5434
5	0.8	0	0.2	11 5 7 10 20	137.87	0.0587	0.3719
5	0.5	0	0.4	10 4 7 7 21	157.61	0.0607	0.2929
6	0	0.7	0.3	10 6 7 8 20	138.84	0.0587	0.5431
6	0	0.3	0.7	10 4 6 14 20	182.13	0.0798	0.2890
7	0.33	0.33	0.33	11 5 3 7 20	144.29	0.0675	0.3712
7	0.3	0.4	0.3	10 4 1 7 21	158.61	0.0621	0.2928
7	0.5	0.2	0.3	11 5 7 9 20	138.10	0.0602	0.3719

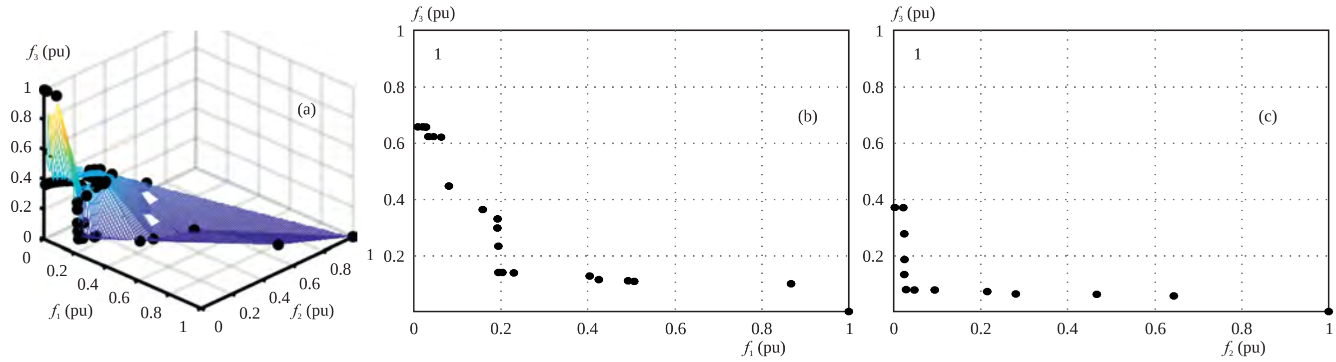

Fig. 6. Pareto front: (a) – case 7, (b) – case 5, (c) – case 6

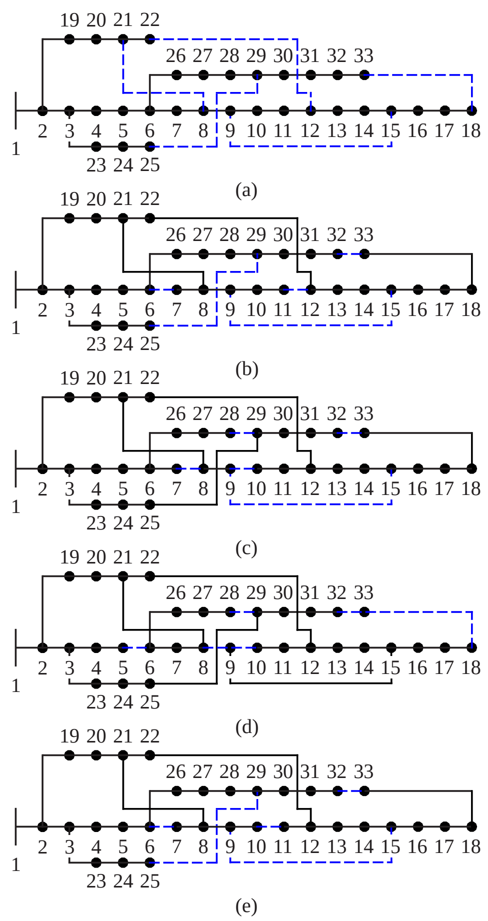

Fig. 7. Configuration of distribution network: (a) – base case, (b) – case 5; $\omega_1 = 0.8$, $\omega_2 = 0$, $\omega_3 = 0.2$, (c) – case 6; $\omega_1 = 0$, $\omega_2 = 0.7$, $\omega_3 = 0.3$, (d) – case 7; $\omega_1 = 0.3$, $\omega_2 = 0.4$, $\omega_3 = 0.3$, (e) – case 7; $\omega_1 = 0.5$, $\omega_2 = 0.2$, $\omega_3 = 0.3$

solution, or can choose a solution using weight factors. In the case of multi-criteria optimizations (with two and three criteria), only the results for some directions are given in Tab. 3. Strictly speaking, all the solutions presented here represent only a small number of special cases, but sufficient to compare the gains that correspond to different configurations.

In addition to considering these criteria, this analysis allows the operator to determine the final solution based on the number of control activities for each specific optimal solution in relation to the previous operational condition. In Fig. 7, besides the base configuration, in which only the tie lines are open, the configurations of the four solutions from Tab. 3 are shown. In addition, the insight into the proposed topology and the knowledge of the characteristics of the available equipment (possession of communication devices, possibility of remote control, cumulative number of performed manipulations, *etc*) can additionally influence the choice of the network configuration that will be selected. These characteristics and criteria can also be implemented in the proposed algorithm by expanding the number of criteria, but due to the 3D capabilities of the graphic representation of the Pareto front, the authors have selected the three criteria mentioned earlier.

8 Conclusion

The reconfiguration of the distribution network can contribute to significant savings through reduction of power losses. This paper presents a meta-heuristic method for solving problem of distribution network reconfiguration. Three criteria functions are considered: real power losses, maximal voltage drop and minimization of load balancing index. In addition to a single criteria optimization that can only improve one distribution network characteristic, and others to exacerbate, in this paper the multi-criteria optimization was proposed. For this purpose, a modified method of gray wolves was used. By appropriate projections of multi-criteria hyper surfaces to the coordinate axis or plane, it is possible to select the best solution for single criteria or two criteria optimization. The efficiency of the optimization process depends on the size of the observed network and on the number of the possible contours. The proposed algorithm provides a useful tool for an operator that can determine the control actions based on the current information of network and the optimization results. Normally, according to the predefined rules, choice of weight factors can be automated and used in the smart grid concept. The proposed algorithm was tested on a standard symmetric IEEE 33 test network.

References

[1] S. Civanlar, J. Grainger, H. Yin and S. Lee, "Distribution feeder reconfiguration for loss reduction", *IEEE Transactions on Power Delivery*, vol. 3, no. 3, pp. 1217–1223, 1988.

[2] D. Shirmohammadi and H. Hong, "Reconfiguration of Electric Distribution Networks for Resistive Line Losses Reduction", *IEEE Transactions on Power Delivery*, vol. 4, no. 2, pp. 1492–1498, 1989.

[3] M. Andervazh, M. Haghifam and J. Olamaei, "Adaptive Multi-Objective Distribution Network Reconfiguration using Multi-Objective Discrete Particles Swarm Optimisation Algorithm and Graph Theory", *IET Generation*, Transmission & Distribution, vol. 7, no. 12, pp. 1367–1382, 2013.

[4] F. Ding and K. Loparo, "Hierarchical Decentralized Network Reconfiguration for Smart Distribution Systems-Part I: Problem Formulation and Algorithm Development", *IEEE Transactions on Power Systems*, vol. 30, no. 2, pp. 734–743, 2015.

[5] L. Xu, R. Cheng, Z. He, J. Xiao and H. Luo, "Dynamic Reconfiguration of Distribution Network Containing Distributed Generation", *presented at the 2016 9th International Symposium on Computational Intelligence and Design (ISCID 2016)*, Hangzhou, Zhejiang, China, 10-11 December 2016.

[6] A. Merlin and H. Back, "Search for a Minimal Loss Operating Spanning Tree Configuration an Urban Power Distribution System", *presented at the 5th Power System Computing Conference*, Cambridge, UK, 1975.

[7] S. Goswami and S. Basu, "A New Algorithm for the Reconfiguration of Distribution Feeders for Loss Minimization", *IEEE Transactions on Power Delivery*, vol. 7, no. 3, pp. 1484–1491, 1992.

[8] Whei Min, Lin and Hong Chan Chin, "A New Approach for Distribution Feeder Reconfiguration for Loss Reduction and Service Restoration", *IEEE Transactions on Power Delivery*, vol. 13, no. 3, pp. 870–875, 1998.

[9] F. Hosseinzadeh, B. Alinejad and K. Pakfar, "A New Technique Distribution Network Reconfiguration for Loss Reduction and Optimum Operation", *presented at the CIRED 2009. 20th International Conference and Exhibition on Electricity Distribution*, Prague, Czech Republic, 8-11 June 2009.

[10] A. Bayat, "Uniform Voltage Distribution Based Constructive Algorithm for Optimal Reconfiguration of Electric Distribution Networks", *Electric Power Systems Research*, vol. 104, pp. 146–155, 2013.

[11] F. Llorens-Iborra, J. Riquelme-Santos and E. Romero-Ramos, "Mixed-Integer Linear Programming Model for Solving Reconfiguration Problems Large-Scale Distribution Systems", *Electric Power Systems Research*, vol. 88, pp. 137–145, 2012.

[12] M. Shariatkhah, M. Haghifam, J. Salehi and A. Moser, "Duration Based Reconfiguration of Electric Distribution Networks using Dynamic Programming and Harmony Search Algorithm", *International Journal of Electrical Power & Energy Systems*, vol. 41, no. 1, pp. 1–10, 2012.

[13] H. de Macedo-Braz and B. de Souza, "Distribution Network Reconfiguration Using Genetic Algorithms with Sequential Encoding: Subtractive and Additive Approaches", *IEEE Transactions on Power Systems*, vol. 26, no. 2, pp. 582–593, 2011.

[14] R. Rao, K. Ravindra, K. Satish and S. Narasimham, "Power Loss Minimization Distribution System Using Network Reconfiguration the Presence of Distributed Generation", *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 317–325, 2013.

[15] J. Olamaei, T. Niknam and S. Badali Arefi, "Distribution Feeder Reconfiguration for Loss Minimization Based on Modified Honey Bee Mating Optimization Algorithm", *Energy Procedia*, vol. 14, pp. 304–311, 2012.

[16] E. Carreno, R. Romero, and A. Padilha-Feltrin, "An Efficient Codification to Solve Distribution Network Reconfiguration for Loss Reduction Problem", *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1542–1551, 2008.

[17] Q. Zhou, D. Shirmohammadi and W. Liu, "Distribution Feeder Reconfiguration for Operation Cost Reduction", *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 730–735, 1997.

[18] S. Bahadoorsingh, J. Milanović, Y. Zhang, C. Gupta and J. Dragović, "Minimization of Voltage Sag Costs by Optimal Reconfiguration of Distribution Network Using Genetic Algorithms", *IEEE Transactions on Power Delivery*, vol. 22, no. 4, pp. 2271–2278, 2007.

[19] D. Bernardon, V. Garcia, A. Ferreira and L. Canha, "Multicriteria Distribution Network Reconfiguration Considering Subtransmission Analysis", *IEEE Transactions on Power Delivery*, vol. 25, no. 4, pp. 2684–2691, 2010.

[20] A. Eldurssi and R. O'Connell, "A Fast Nondominated Sorting Guided Genetic Algorithm for Multi-Objective Power Distribution System Reconfiguration Problem", *IEEE Transactions on Power Systems*, vol. 30, no. 2, pp. 593–601, 2015.

[21] A. Asrari, S. Lotfifard and M. Payam, "Pareto Dominance-Based Multiobjective Optimization Method for Distribution Network Reconfiguration", *IEEE Transactions on Smart Grid*, vol. 7, no. 3, pp. 1401–1410, 2016.

[22] S. Mirjalili, S. Mirjalili and A. Lewis, "Grey Wolf Optimizer", *Advances Engineering Software*, vol. 69, pp. 46–61, 2014.

[23] S. Mirjalili, S. Saremi, S. Mirjalili and L. Coelho, "Multi-Objective Grey Wolf Optimizer: A Novel Algorithm for Multi-Criterion Optimization", *Expert Systems With Applications*, vol. 47, pp. 106–119, 2016.

[24] K. Deb,, *Multi-Objective Optimization using Evolutionary Algorithms*, 1st ed.: John Wiley & Sons, 2001.

[25] D. Rajicic, R. Ackovski and R. Taleski, "Voltage Correction Power Flow", *IEEE Transactions on Power Delivery*, vol. 9, no. 2, pp. 1056–1062, 1994.

[26] J. Mendoza, R. López, D. Morales, E. López, P. Dessante and, R. Moraga, "Minimal Loss Reconfiguration using Genetic Algorithms with Restricted Population and Addressed Operators: Real Application", *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 948–954, 2006.

[27] T. Nguyen, A. Truong and T. Phung, "A Novel Method based on Adaptive Cuckoo Search for Optimal Network Reconfiguration and Distributed Generation Allocation Distribution Network", *Electrical Power and Energy Systems*, vol. 78, pp. 801–815, 2016.

[28] J. Mendoza, M. Lopez, C. Coello and E. Coello Lopez, "Microgenetic multiobjective reconfiguration algorithm considering power losses and reliability indices for medium voltage distribution network", *IET Generation*, Transmission & Distribution, vol. 3, no. 9, pp. 825–840, 2009.

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