

# Quaternion-valued single-phase model for three-phase power system

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In this work, a quaternion-valued model is proposed in lieu of the Clarke's  $\alpha, \beta$  transformation to convert three-phase quantities to a hypercomplex single-phase signal. The concatenated signal can be used for harmonic distortion detection in three-phase power systems. In particular, the proposed model maps all the harmonic frequencies into frequencies in the quaternion domain, while the Clarke's transformation-based methods will fail to detect the zero sequence voltages. Based on the quaternion-valued model, the Fourier transform, the minimum variance distortionless response (MVDR) algorithm and the multiple signal classification (MUSIC) algorithm are presented as examples to detect harmonic distortion. Simulations are provided to demonstrate the potentials of this new modeling method.

**Key words:** harmonics detection, Fourier transform, minimum variance distortionless response, multiple signal classification, quaternion, three-phase power system

## 1 Introduction

Power quality control is one of the major concerns for power delivery systems to function reliably, and it requires measurements of voltage characteristics, among which the frequency measurement is a non-trivial task due to the presence of voltage sags and voltage harmonics mostly caused by nonlinear loads [1]. In the particular case of three-phase power systems, the Clarke's  $\alpha, \beta$  transformation is widely used as the preprocessing method to create a complex-valued single-phase signal from the real-valued three-phase signals [2], so that traditional complex-valued spectrum estimation methods can be applied, such as the MVDR method or the recently proposed Iterative MVDR (I-MVDR) method [3, 4]. To improve the resolution, we can further apply the subspace methods and one representative example is the MUSIC method [5].

However, all the zero sequence voltages will be cancelled out in the complex-valued signal and hence can not be detected. Although these harmonic voltages would simply be blocked by a delta transformer, they will add up in the neutral, leading to overheating in the transformer and potential fire hazards [6]. To detect these harmonics, as well as harmonics of other orders, we propose a quaternion-valued model and all the traditional spectrum estimation methods can be extended to this domain, such as MVDR and MUSIC. We will show that harmonics of all orders will be reserved in the resulting quaternion-valued signal and will be detected by relevant estimation methods. This quaternion-valued model has been previously presented for the tracking of the fundamental frequency in the presence of voltage sags using the Kalman filter

algorithm [7]. In this paper, we demonstrate a complementary merit of this model when dealing with harmonic voltages.

## 2 Complex-valued frequency estimation for three-phase power systems

### 2.1 A brief review

We consider the following discrete-time balanced three-phase power system in the presence of  $H - 1$  harmonic distortions

$$\begin{aligned} v_a(n) &= \sum_{h=1}^H V_h \cos(h(\Omega n T_s + \phi)) + \varepsilon_a(n), \\ v_b(n) &= \sum_{h=1}^H V_h \cos\left(h\left(\Omega n T_s + \phi - \frac{2\pi}{3}\right)\right) + \varepsilon_b(n), \\ v_c(n) &= \sum_{h=1}^H V_h \cos\left(h\left(\Omega n T_s + \phi + \frac{2\pi}{3}\right)\right) + \varepsilon_c(n) \end{aligned} \quad (1)$$

where  $\{V_h\}_{h=1}^H$  are the amplitudes of the harmonic signals,  $\Omega$  is the fundamental (angular) frequency,  $T_s$  is the sampling interval,  $\phi$  is the signal phase, and  $\varepsilon_a(n), \varepsilon_b(n), \varepsilon_c(n)$  are the measurement noise.

Traditionally, the three-phase signals will be converted to a complex-valued single-phase signal via the Clarke's  $\alpha, \beta$  transformation. Firstly, the three-phase signals are mixed into two parts, namely  $v_\alpha(n)$  and  $v_\beta(n)$ , where

$$\begin{bmatrix} v_\alpha(n) \\ v_\beta(n) \end{bmatrix} = \mathbf{T} \begin{bmatrix} v_a(n) \\ v_b(n) \\ v_c(n) \end{bmatrix}, \quad (2)$$

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$$\mathbf{T} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}. \quad (3)$$

Then these two parts will be merged as a complex-valued signal  $v_{cv}(n) = v_{\alpha}(n) + iv_{\beta}(n)$ .

With this complex-valued signal, we can exploit the MVDR spectrum to locate the frequencies, and it is given by

$$S_{\text{MVDR}}(\Omega) = \frac{1}{\mathbf{s}^H(\Omega)\mathbf{R}^{-1}\mathbf{s}(\Omega)} \quad (4)$$

where  $(\cdot)^H$  is the Hermitian-transpose operation,  $\mathbf{R}$  is the covariance matrix of dimension  $M \times M$ , and

$$\mathbf{s}(\Omega) = \left[ 1, e^{i\Omega T_s}, \dots, e^{-i\Omega T_a(M-1)} \right]^T \quad (5)$$

is the frequency sweeping vector.

We can also use the MUSIC spectrum which is expressed as

$$S_{\text{MUSIC}}(\Omega) = \frac{1}{\|\mathbf{s}^H(\Omega)\mathbf{U}_N\|^2} \quad (6)$$

where  $\|\cdot\|$  denotes the Euclidean norm,  $\mathbf{U}_N$  represents the noise subspace and comprises the eigenvectors of the covariance matrix  $\mathbf{R}$  which are corresponding to the  $M_0$  smallest eigenvalues, where  $M_0$  is assumed to be known or can be estimated using the information theory methods [9].

In practice, the covariance  $\mathbf{R}$  needs to be updated and estimated from the average of samples

$$\hat{\mathbf{R}}(n) = \frac{1}{K} \mathbf{V}_{cv}(n) \mathbf{V}_{cv}^H(n) \quad (7)$$

where

$$\mathbf{V}_{cv}(n) = \begin{bmatrix} v_{cv}(n) & \dots & v_{cv}(n-K+1) \\ \vdots & \ddots & \vdots \\ v_{cv}(n-M+1) & \dots & v_{cv}(n-K-M+2) \end{bmatrix} \quad (8)$$

and  $K$  is the number of observations.

## 2.2 Missing harmonic signals in the complex-valued signal

In detail,  $v_{cv}(n)$  is composed of complex-domain harmonic signals that can be divided into two categories plus noise,

$$v_{cv}(n) = v_{cv1}(n) + v_{cv2}(n) + \varepsilon_{cv}(n) \quad (9)$$

where  $v_{cv1}(n)$  is the summation of all positive sequence voltages,

$$v_{cv1}(n) = \sum_{p=1}^{\lfloor \frac{H+2}{3} \rfloor} V_{3p-2} e^{i(3p-2)(\Omega n T_s + \phi)} \quad (10)$$

and  $v_{cv2}(n)$  is the summation of all negative sequence voltages,

$$v_{cv2}(n) = \sum_{p=1}^{\lfloor \frac{H+1}{3} \rfloor} V_{3p-1} e^{-i(3p-1)(\Omega n T_s + \phi)} \quad (11)$$

and  $\lfloor x \rfloor$  denotes the largest integer not greater than  $x$ .

All the zero sequence voltages have been cancelled out. Zero sequence voltages of the same order are cophasal in the three voltage channels and will be eliminated since both rows of the transformation matrix  $\mathbf{T}$  are zero-mean vectors.

To solve this problem, we propose our quaternion-valued approach in the next section.

## 3 Quaternion-valued frequency estimation for three-phase power systems

We construct a quaternion-valued signal from the three-phase signals as [7]

$$v_{qv}(n) = iv_a(n) + jv_b(n) + kv_c(n) \quad (12)$$

where  $i, j, k$  are the three imaginary units of the quaternion algebra which are constrained by [8]

$$\begin{aligned} i^2 &= j^2 = k^2 = ijk = -1 \\ ij &= -ji = k \\ jk &= -kj = i \\ ki &= -ik = j \end{aligned} \quad (13)$$

This quaternion-valued signal contains quaternion-domain harmonic signals that belong to three categories  $v_{qv1}(n)$ ,  $v_{qv2}(n)$ ,  $v_{qv3}(n)$  plus noise,

$$v_{qv}(n) = iv_a(n) + jv_b(n) + kv_c(n) = v_{qv1}(n) + v_{qv2}(n) + v_{qv3}(n) + \varepsilon_{qv}(n) \quad (14)$$

where  $v_{qv1}(n)$  is the summation of all the positive sequence voltages

$$\begin{aligned} v_{qv1}(n) &= \sum_{p=1}^{\lfloor \frac{H+2}{3} \rfloor} \frac{2i-j-k}{2} V_{3p-2} \left\{ \cos[(3p-2)(\Omega n T_s + \phi)] \right. \\ &\quad \left. - \frac{i+j+k}{\sqrt{3}} \sin[(3p-2)(\Omega n T_s + \phi)] \right\} \\ &= \sum_{p=1}^{\lfloor \frac{H+2}{3} \rfloor} \frac{2i-j-k}{2} V_{3p-2} e^{-\frac{i+j+k}{\sqrt{3}}(3p-2)(\Omega n T_s + \phi)}, \end{aligned} \quad (15)$$

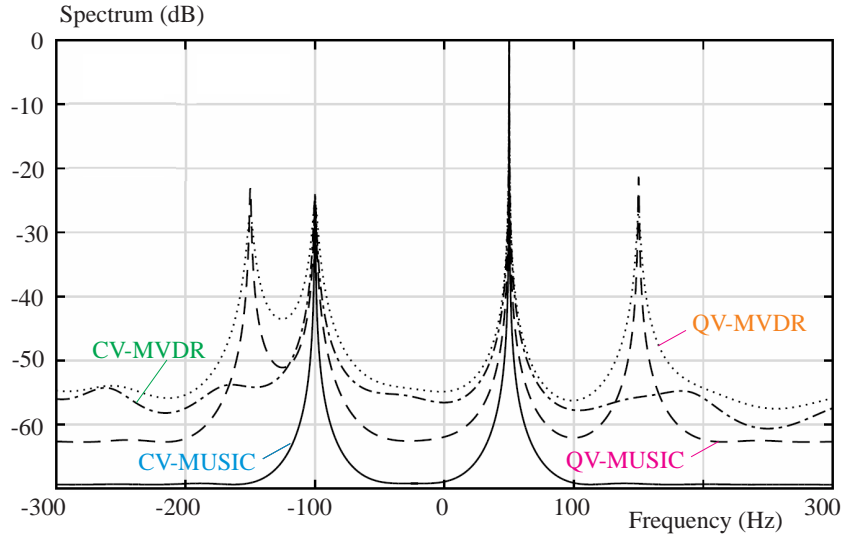


Fig. 1. MVDR and MUSIC spectra of quaternion-and complex-valued models

$v_{qv2}(n)$  is the summation of all the negative sequence voltages

$$v_{qv2}(n) = \sum_{p=1}^{\lfloor \frac{H+1}{3} \rfloor} \frac{2i - j - k}{2} V_{3p-1} \left\{ \cos[(3p-1)(\Omega n T_s + \phi)] + \frac{i + j + k}{\sqrt{3}} \sin[(3p-1)(\Omega n T_s + \phi)] \right\}$$

$$= \sum_{p=1}^{\lfloor \frac{H+1}{3} \rfloor} \frac{2i - j - k}{2} V_{3p-1} e^{\frac{i+j+k}{\sqrt{3}}(3p-1)(\Omega n T_s + \phi)} \quad (16)$$

and  $v_{qv3}(n)$  is the summation of all the zero sequence voltages

$$v_{qv3}(n) = \sum_{p=1}^{\lfloor \frac{H}{3} \rfloor} V_{3p} (i + j + k) \cos[3p(\Omega n T_s + \phi)]$$

$$= \sum_{p=1}^{\lfloor \frac{H}{3} \rfloor} V_{3p} \frac{i + j + k}{2} \left[ e^{\frac{i+j+k}{\sqrt{3}}3p(\Omega n T_s + \phi)} + e^{-\frac{i+j+k}{\sqrt{3}}3p(\Omega n T_s + \phi)} \right]. \quad (17)$$

Hence all the harmonic signals will be reserved in the quaternion-valued signal. We may observe from (15)–(17) that the frequencies of the harmonic signals have been mapped into the frequencies of the quaternion-valued signal associated with the  $\frac{i+j+k}{\sqrt{3}}$  axis.

The frequency estimation problem from a sum of quaternion-valued exponentials is similar to the problem in the complex domain. Then we can adopt the MVDR spectrum in (4) and the MUSIC spectrum in (6) by substituting the frequency sweeping vector as

$$\mathbf{s}(\Omega) = \left[ 1, e^{-\frac{1}{\sqrt{3}}(i+j+k)\Omega T_s}, \dots, e^{-\frac{M-1}{\sqrt{3}}(i+j+k)\Omega T_s} \right]^T. \quad (18)$$

NOTE-1: Not to be confused with the Quaternion-MVDR (Q-MVDR) algorithm [10] for the adaptive beamforming with vector-sensor array beamforming or the Quaternion-MUSIC (Q-MUSIC) algorithm [11] for the direction-of-arrival estimation with vector-sensor arrays. Their steering vectors are complex-valued vectors multiplied by quaternion-valued scalars, which are conceptionally different from the quaternion-valued frequency sweeping vector defined in this paper. We marked our algorithms by QV-MVDR and QV-MUSIC for clarification.

The frequencies detected in the spectrum are either the original real-domain angular frequencies or their additive inverses, namely

- (1) If a peak is detected in the spectrum in the absence of its additive inverse, it corresponds to a positive or negative sequence voltage signal and this spectrum peak indicates its angular frequency or its additive inverse.
- (2) If two "mirrored" peaks are detected in the spectrum, they correspond to a zero sequence voltage signal and they indicate the signal's angular frequency and its additive inverse, respectively.

## 4 Simulations

In this section, we provide a numerical example to illustrate the performance of the proposed quaternion model. The fundamental frequency is 50 Hz, the sampling frequency is  $f_s = 1/T_s = 20$  kHz, the initial phase is  $\phi = \pi/7$ , and  $K = 80$ ,  $M = 32$ . There exist a second-order and a third-order harmonic signals, both set to be 6% in amplitude.

We test the capability of the two modelings. The MVDR and MUSIC spectra of the quaternion- and complex-valued models are plotted in Fig. 1, where SNR = 40 dB. It can be observed that the proposed model is able to detect all the harmonic signals, namely

50 Hz (the fundamental frequency),  $-100$  Hz (the second-order harmonic), and  $\pm 150$  Hz (the third-order harmonic), while the complex-valued model fails at the third-order harmonic frequency.

## 5 Conclusion

We have presented a quaternion-valued model as an alternative preprocessing approach to convert the three-phase signals into a single-phase system. Compared with the Clarke's transformation, the proposed model can additionally detect the zero sequence voltages. Simulated results show that the proposed model can detect all-order voltage harmonics effectively.

### Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant Nos. 61331019 and 61490691).

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Received 19 October 2017

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