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PAPERS

Probabilistic three-phase power flow in a distribution system applying the pseudo-inverse and cumulant method

Jerneja Bogovič[1], Miloš Pantoš[1]

A new, analytical approach using the cumulant method is proposed for the three-phase probabilistic power-flow (PPF) analysis. The approach to forming the sensitivity matrix is based on quantifying the pseudo-inverse instead of the inverse jacobian matrix, since it is commonly singular in a distribution power network. The results are compared with those obtained using the point-estimate method (PEM) and the Monte Carlo (MC) method, which is a commonly used reference method for the PPF analysis in a distribution power network.

K e y w o r d s: cumulants, distribution power network, linearization, probabilistic power flow

1 Introduction

Power-flow (PF) analysis is one of the most important analyses in an electric power system (EPS). As a broad spectrum of operating conditions can be found in an EPS, the PF analysis must consider all variations, such as the production of the power plants, the consumption of the loads as well as the topologies of the power network. This is especially so in recent years, which have witnessed a tremendous growth in renewable energy sources (RESs) that are characterized as extremely intermittent power sources.

Despite this, the deterministic power flow (DPF) is still the most commonly used approach to evaluate the operating and planning of an EPS [1], because it is based on performing multiple PF studies for different scenarios. Knowing all the possible scenarios in the system as well as the value of the power flow and the voltage conditions in the nodes is a key factor in the operating and planning of an EPS. Due to the deep penetration of RESs the uncertainty and the unpredictability of EPSs are increasing [2] and this can lead to an unrealistic assessment of the systems performance obtained by the DPF [3]. To overcome the disadvantages of a DPF involving RESs, different probabilistic approaches have been proposed [1].

The Monte Carlo (MC) method is one of the first probabilistic methods applied in EPS calculations, in which the stochastic nature of the elements of the EPS was incorporated. The method is based on repeated DPF calculations, where the input variables and the parameters of the EPS model for each calculation are determined according to their probability description. In the next step, the results of the repeated DPF calculations are converted to a probabilistic description of the network state. The advantages of the MC method for the probabilistic power flow (PPF) are flexibility, general purpose (appropriate for both, transmission and distribution network), the ability to deal with the large nonlinearity of the EPS and the large variances of the elements parameters, accuracy and robustness, which makes the MC method the most frequently used for PPF calculations [4]. On the other hand, the MC method has some negative characteristics: it is slow, computationally demanding and very time-consuming due the numerous simulations of the DPF calculations subject to the desired accuracy of the results [5]. As reported in [6], the number of simulations estimated as being sufficient for the PPF applying the MC method is around 10,000. To improve the efficiency and to speed up the method, some simplifications and reductions related to sampling the input variables and the parameters have been implemented.

Some approximate methods and faster analytical methods for the PPF calculation have also been developed. The point-estimate method (PEM) is the representative method from a set of approximate methods and is the subject of many research papers [7]. However, the PEM does not give adequate results in all cases due to the fact that the accuracy of the method is decreasing with the increasing order of the estimated moment or cumulant [8].

From analytical methods for the PPF, convolution based methods were initially proposed [4]. However, recent research has tended to focus on the cumulant approach [8]. The cumulant method [9] is based on a determination of the sensitivity matrix, which is most commonly defined by the linearization of the Jacobian matrix for a Newton-Raphson PF calculation at the working point. This approach is characterized by its simplicity and high computational efficiency, and it is appropriate

[1] University of Ljubljana, Faculty of Electrical Engineering, Tržaška cesta 25, Ljubljana, Slovenia, milos.pantos@fe.uni-lj.si, corresponding author: jerneja.bogovic@fe.uni-lj.si

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for a wide range of applications, but in the case of the singularity of the Jacobian matrix, this approach cannot be performed. To visualize the obtained results, several expansions are available, such as CornishFisher, Edgeworth, GaussHermite and Gram–Charlier 10].

Most of the previously mentioned analytical methods for the PPF were first proposed for the transmission system [4] and the DC PF method has been used for their application [11]. The implementation of PPF methods for the transmission system also spread to AC PF problems [1]. However, EPSs with a high R/X ratio, as in the case of distribution power networks, can create ill-conditioned problems for the Newton-Raphson (or Newton-type) PF algorithms. Furthermore, ill-conditioned problems can then seriously affect the PF methods application in the DPF and PPF [12], due to the singularity of the Jacobian matrix, which is most often used to determine the sensitivity matrix for the cumulant method. Instead of Newton-type PF algorithms, the Forward/Backward Sweep (FBS) method with the MC method [13] is used for the PPF in distribution power networks.

In the case of the cumulant method for distribution power networks, several different approaches are proposed to determine the sensitivity matrix, which is frequently singular for distribution power networks, depending on its topology. In [14], the DC PF method is used to consider the correlation in the PPF with the cumulant method. However, using the DC PF method for the PPF only allows a probability estimation of the line power flows. The nodal voltages are predetermined and are not subject to a probabilistic analysis.

Furthermore, in [12] a modified system-admittance matrix is applied in the PPF with the cumulant method as a sensitivity matrix. This approach makes it possible to observe line power flows and the nodal voltage through the injection current, although the final PPF is obtained by integration of the DPF solution and the stochastic variations of the nodal voltages. Additionally, in [8] the authors proposed a determination of the sensitivity matrix using the multiple integral method that is only suitable for a restricted number of random variables and parameters.

To observe the probabilistic nature of line power flows and nodal voltages in three-phase system and to overcome the long and complicated procedure for their determination, which is also independent of the number of random variables and parameters, an upgraded cumulant method is proposed in this paper. The novelty of this upgraded method is that the pseudo-inverse of the Jacobian matrix for the three-phase AC PF method is used as a sensitivity matrix, instead of the inverse of the Jacobian matrix for the AC PF method to spread the application of the cumulant method on balanced and unbalanced systems in radial and weakly meshed three-phase networks. This new, upgraded approach enables rapid and straightforward probabilistic analyses without compromising the solutions accuracy. Furthermore, it makes it possible to observe active and reactive line power flows as well as the nodal voltages. Not only the magnitude, but also the phase angle in both balanced and unbalanced systems in radial, weakly meshed or mashed power three-phase networks.

2 Upgraded cumulant method for PPF calculation

The proposed method for an analytical calculation of the PPF applies a cumulant method that consists of three main segments, as presented in Fig. 1.

The first section is data preparation and consists of two main steps, which are network modelling that includes network topology and parameters, and converting the probability density functions (PDF) of the random variables and parameters into cumulants, Section 2.1. The PPF formulation is the second section and is composed of three main steps. The first step is linearization of the PF equations using a Taylor-series-expansion linearization technique. The second step is the determination of the sensitivity matrix, for which a new analytical approach is proposed. The last step is the determination of the cumulant of the nodal voltages and the line PF variables.

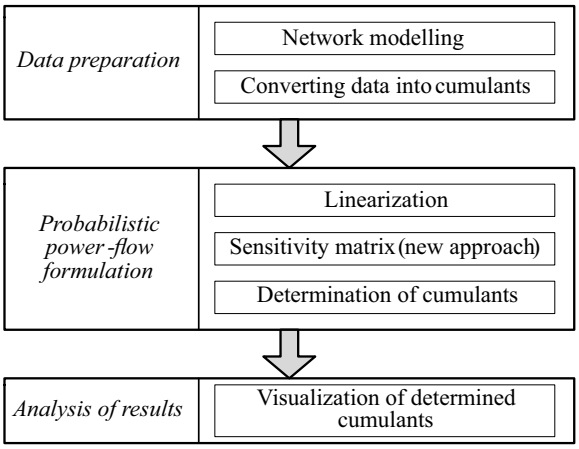

Fig. 1. Main segments of the proposed cumulant method

2.1 Data preparation

Data preparation is the first (and in some aspects the most important) segment in the whole procedure. Errors caused during the data preparation reflect in the errors in the results. For this reason, network models should represent the physical power system accurately in terms of the network topology as well as the parameters and variables of all the components in the EPS.

A common model of an EPS consists of some major components: nodal productions, consumptions and linkages, *ie*, a network required for transmission and distribution. Here, the models and a standard PF formulation with nonlinear equations are applied in order to obtain a

solution. The equivalent probabilistic models of all the elements are also needed for the PPF analysis, such as the network configuration, line parameters, loads consumption and generation injections, where the stochastic nature of the RESs is the main reason for applying the PPF.

For the PPF analysis the network configuration is assumed to be a discrete, stochastic variable with network uncertainties modelled to account for the availability of each network component. The line parameters for the PPF analysis are in most cases defined with fixed parameters. In some cases, related to the variation of the temperature, the elements properties, such as the impedance of the series line, are assumed with uniform distributions. The distribution of the susceptance of a line is assumed to be a uniform distribution. However, the impact of the line parameters for the PPF is almost negligible [1].

The consumption or load patterns have great impact on the PPF calculations and the probability models are obtained from a statistical analysis of historical data, which allows the construction of a probability model of the load [13]. If historical data are not available, a Gaussian distribution with 5—10 % of the variability is mostly used to represent the PDF of the loads and conventional power generation [1].

As already mentioned, the main reason for the development of different PPF methods is the stochastic nature of RESs. The probability model of the generation injection can be obtained from historical data. If there is no data available, the different distributions are used to approximate the model. For modelling wind-power production a Gaussian distributed is assumed [15]. Otherwise, modelling of the wind-power generation based on the uncertainty of the wind speed and the Weibull distribution is used [1]. However, recently the Gaussian mixture model and also beta and gamma distributions were used to approximate the PDF of wind-power generation [16]. For photovoltaic power generation the beta distribution was used [17].

For the PPF calculation we propose using the cumulant method for all the input variables and parameters of the EPS model, determined according to their probability description, which can be described or approximated with cumulants. Cumulants, also called semi-invariants of the distribution, were introduced and studied by the scientist T. N. Thiele, who showed that many properties of random variables or parameters can be represented by cumulants. They are used to describe or approximate the PDF and the distribution function and can also be approximated from the elements in the list, not only symbolic distributions. General probabilistic descriptions of the random variables or parameters can be adopted in the procedure and no limitation on the normal distribution is enforced. Cumulants of the random variable or parameter are defined as derivatives of the logarithm of a moment-generating function or as a logarithm of the Fourier transformation of a random variable PDF expanded in a Maclaurin series [18] and are

$$\kappa_n = \left.\frac{\mathrm{d}^n}{\mathrm{d}t^n}\right|_{t=0} \ln\left(E\left(e^{t\chi}\right)\right) = K^{(n)}(0)\,, \tag{1}$$

$$K(t) = \ln\left(E\left(e^{t\chi}\right)\right) = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}\,, \tag{2}$$

where κ_n is a cumulant of the n^{th} order, E is the expectation of the random variable $e^{t\chi}$, here t is a real value and $K^{(n)}(0)$ is the n-times-differentiated, moment-generating function $K(t)$ of the random variable χ at the real value $t = 0$.

The first cumulant is the mean value of the variable or parameter, the second cumulant is defined as the variance, the third cumulant is related to the skewness, the fourth cumulant is related to the kurtosis, and cumulants of higher orders have no mathematical significance [19]. In the case of a fixed value, the value is represented only by the first cumulant, with the mean value and all the higher cumulants being zero.

The properties of the cumulants that simplify and make the application efficient are the translation invariance

$$\begin{aligned} &\kappa_1(\chi + c) = \kappa_1(\chi) + c\,, \\ &\kappa_n(\chi + c) = \kappa_n(\chi)\,; n \geq 2\,, \end{aligned} \tag{3}$$

the homogeneity

$$\kappa_n(c\chi) = c^n \kappa_n(\chi)\ , \tag{4}$$

and the additivity

$$\kappa_n(\chi + \xi) = \kappa_n(\chi) + \kappa_n(\xi)\,, \tag{5}$$

where κ_n is a cumulant of the n^{th} order, c is a constant, χ and ξ are random variables.

The cumulants change in a very simple way when the random variable is subject to an affine transformation and the sums of the independent random variables have a very simple relationship to the cumulants of the addends [10].

2.2 PPF formulation

After the data preparation, the next step in the cumulant method is the PPF formulation, which consists of the linearization of the PF equations, the determination of the sensitivity matrix and the determination of the cumulants that follow the written order in the PPF calculations. Due to the fact that the cumulant method is based on the linearized PF equations and the determination of the sensitivity matrix, this section is very significant in the PPF calculations.

An EPS is a non-linear system and has non-linear AC PF equations to describe it. To linearize the non-linear AC equations for the PF and PPF formulation, a Taylor-series-expansion linearization technique is used to find the linear approximation to a function at a given point. A linearization of the AC equations for the Newton-Raphson method

$$\underline{S}_i^p = \underline{U}_i^p \sum_{j=1}^{N} \sum_{q=a,b,c} (\underline{U}_i^q)^* (\underline{Y}_{ij}^{pq})^*\,, \tag{6}$$

where $\underline{S}_i^p$ is the nodal complex power at bus i for a given phase p, N is the number of buses, $\underline{U}_i^p$ and $\underline{U}_j^q$ are the complex voltages at buses i and j for a given phase p and q and $\underline{Y}_{ij}^{pq}$ is the element of the admittance matrix.

To obtain a sensitivity matrix between the injected power and the nodal voltage (an inverse Jacobian matrix) the equations for the Newton-Raphson method (6) are developed in a Taylor series expansion

$$\boldsymbol{C} + \Delta\boldsymbol{C} = f(\boldsymbol{U} + \Delta\boldsymbol{U}) = f(\boldsymbol{U}) + \boldsymbol{J}\Delta\boldsymbol{U} + \cdots , \quad (7)$$

in which only the first derivatives

$$\boldsymbol{J}^{-1} = \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{C}}, \quad (8)$$

are taken into consideration, [9] and the high-order terms are very small and are in most cases ignored.

$$\boldsymbol{T} = \boldsymbol{G} \cdot \boldsymbol{J}^{-1} = \frac{\partial \boldsymbol{S}}{\partial \boldsymbol{U}} \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{C}} . \quad (9)$$

This procedure makes it possible to obtain a linear relationship or sensitivity matrix $\boldsymbol{J}^{-1}$ between the nodal voltage $\boldsymbol{U}$ and the nodal active and reactive powers $\boldsymbol{C}$. Following this procedure we also obtain the linear relationship or sensitivity matrix $\boldsymbol{T}$ (9), only in this case between the line power flows S from (10) and the nodal active and reactive powers $\boldsymbol{C}$, which also consist of the inverse Jacobian matrix $\boldsymbol{J}^{-1}$ and the sensitivity matrix $\boldsymbol{G}$, which presents the relationship between the nodal voltage $\boldsymbol{U}$ and the line power flows $\boldsymbol{S}$

$$\underline{S}_{ij}^p = \underline{U}_i^p \left(\underline{I}_{ij}^p\right)^* , \quad (10)$$

where $\underline{S}_{ij}^p$ is the complex power flow in line ij for a given phase, $\underline{U}_i^p$ is the complex voltages at bus i for a given phase p and $\underline{I}_{ij}^p$ is the complex current at line ij.

For the line power flows it was proposed to solve the PPF, as was the case in some others [20].

The sensitivity matrix for the PPF is based on the Jacobian matrix. Because the Jacobian matrix is often singular for a radial system, a new approach is presented to determine the sensitivity matrix. This new approach proposes a Moore–Penrose pseudo-inverse to analytically define the sensitivity matrix for a radial system. The Moore–Penrose pseudo-inverse was introduced by Moore in 1920 and rediscovered by Penrose in 1955 and is a generalization of the inverse matrix and brings conceptual clarity to the study of solutions to arbitrary systems of linear equations and linear least-squares problems. However, the Moore–Penrose pseudo-inverse always exists and is unique. For matrix $\boldsymbol{A}$ the Moore–Penrose pseudo-inverse vector is generally defined as

$$\boldsymbol{A}^+ = \left(\boldsymbol{A}^\top \boldsymbol{A}\right)^{-1} \boldsymbol{A}^\top , \quad (11)$$

when $\boldsymbol{A}$ has linearly independent columns and by

$$\boldsymbol{A}^+ = \boldsymbol{A}^\top \left(\boldsymbol{A}\boldsymbol{A}^\top\right)^{-1} , \quad (12)$$

when $\boldsymbol{A}$ has linearly independent rows.

The Moore–Penrose pseudo-inverse has some properties of the inverse matrix, but not necessarily all of them. In the case of a non-singular matrix the Moore-Penrose pseudo-inverse is the same as the ordinary inverse of the matrix. Also in the case of weak systems, the use of the pseudo-inverse is justified, since it does not change the sensitivity matrix. But in the case of a singular matrix the Moore-Penrose pseudo-inverse is uniquely determined by the minimum Euclidean norm and is the best approximation of the inverse matrix. The influence of small singular values of the matrix is neglected and replaced by zero to prevent a strong response to a small change. For the singular matrix and the non-square matrix the Moore–Penrose pseudo-inverse is so calculated from the singular-value decomposition of the matrix, [21]. The pseudo-inverse of the matrix is defined as

$$\boldsymbol{A} = \boldsymbol{V}_1 \boldsymbol{D} \boldsymbol{V}_2^\top , \quad (13)$$

with singular-value decomposition

$$\boldsymbol{A}^+ = \boldsymbol{V}_2 \boldsymbol{D}^+ \boldsymbol{V}_1^\top , \quad (14)$$

where $\boldsymbol{V}_1$ is a unitary matrix where the columns are called the left-singular vectors, $\boldsymbol{V}_2$ is a unitary matrix where the columns are called the right singular vectors of matrix $\boldsymbol{A}$ and $\boldsymbol{D}$ is a diagonal matrix of $\boldsymbol{A}$.

The Moore–Penrose pseudo-inverse has already been applied in many areas such as mathematics, physics and engineering [22], where the Moore–Penrose pseudo-inverse is also employed in data analysis. After defining the sensitivity matrix, the next step in the PPF formulation is a determination of the cumulants of unknown variables from the random variables or the parameters and sensitivity matrix.

The output variable in the PPF calculation is considered as a function of the input variables or parameters. Invariance, equivariance, homogeneity and additivity enables mapping the vector of cumulants (second and higher orders) of random variables or parameters $\boldsymbol{\kappa}_{n,k}$ through the sensitivity matrix $\boldsymbol{X}$ (in the case of PPF $\boldsymbol{J}^+$ and $\boldsymbol{T}$) at the point of the mean value of the random variable and parameters into the vector of cumulants of unknown variables $\boldsymbol{\kappa}_{n,u}$ for each n^{th} order of cumulant, is

$$\boldsymbol{\kappa}_{n,u} = (\boldsymbol{X})^n \boldsymbol{\kappa}_{n,k} . \quad (15)$$

However, it is more appropriate to use one of the PF methods (Newton's method, Newton-Raphson method, FBS) to calculate the first cumulant.

In the case of $\boldsymbol{J}^+$ as a sensitivity matrix is a transform

$$\boldsymbol{\kappa}_n, \boldsymbol{U} = (\boldsymbol{J}^+)^n \boldsymbol{\kappa}_n, \boldsymbol{C} , \quad (16)$$

where the unknown variables are nodal voltages $\boldsymbol{U}$ that are dependent on the nodal power $\boldsymbol{C}$.

The same transformation from (15) is made in the case of $\boldsymbol{T}$ as a sensitivity matrix, where the unknown variables

are the line power flows $\boldsymbol{S}$ and are also dependent on the nodal power $\boldsymbol{C}$

$$\boldsymbol{\kappa}_{n,\boldsymbol{S}} = (\boldsymbol{T})^n \boldsymbol{\kappa}_{n,\boldsymbol{C}} = \left(\boldsymbol{G} \cdot \boldsymbol{J}^{+}\right)^n \boldsymbol{\kappa}_{n,\boldsymbol{C}}. \tag{17}$$

However, the determined cumulants of the unknown variables do not give recognition about the probability distribution of a specified variable. For this reason, an analysis of result is carried out.

2.3 Analysis of results

The main task in the analysis of the result to characterize the determined cumulants, due to the fact that the third- and higher order of the cumulants effect the shape of the PDF and the cumulative distribution function (CDF) and cannot be as easily visualized as the first two cumulants.

There are several types of expansion, such as Cornish–Fisher, Edgeworth, Gauss–Hermite and Gram–Charlier that can be used to visualize the results or to approximate a probability distribution in terms of its cumulants. In this paper the Gram–Charlier series is chosen to perform the PDF of the nodal voltages and the line power flows, because the Cornish–Fisher, Edgeworth and Gauss–Hermite expansions have convergence problems and might have an error in the tail regions. On the other hand, the first few terms of the Gram–Charlier expansion give a good fit, although that is based on a normal distribution [10]. The Gram–Charlier expansion is determined for the PDF by

$$f(\chi) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \sum_{m=0}^{\infty} a_m \cdot H_m(\chi)\,, \tag{18}$$

based on a standard zero mean and unit variance normal density and Hermite polynomials, as in [23]. Here a_m are predefined m^{th}-order coefficients of the Gram–Charlier expansion and H_m are the Hermite polynomials of m^{th} order of random variable χ.

3 Case study

The new, proposed method for the PPF in the distribution networks was tested on the three-phase IEEE 37-node radial distribution test system, Fig. 2. This system contains three-phase lines and a mixture of single- and three-phase loads, making it suitable for evaluating the proposed method. For analyses purposes, some modifications are made to the test system. The voltage regulator included in line segment 799-701 is out of operation and modelled as a transformer to simplify the systems modelling. As described in [24], the voltage regulator additionally extends the simulation time for the MC method.

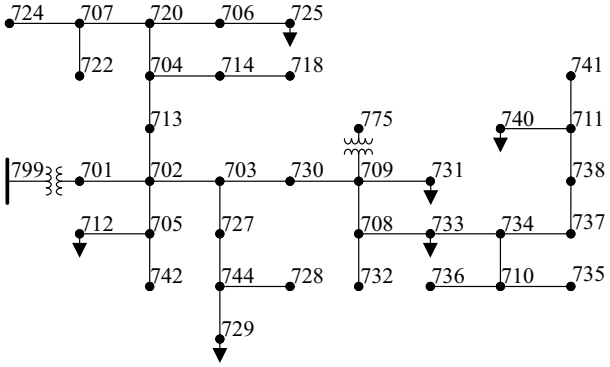

Fig. 2. IEEE 37-node test system

All the loads are single-phase spot loads and are, for analysis proposes, modelled as Y-PQ (Y connection, constant power). The loads, presented with a probability model, are located at the nodes 712, 725, 729, 731, 733 and 740 at different phases (a, b and c) are listed in Tab. 1.

Table 1. 1 Loads data with beta distributions parameters

Node/phase	Active power (kW)	Reactive power (kVAr)
712/c	65 + 40 B(2, 2)	35 + 10 B(2, 2)
725/b	32 + 20 B(2, 2)	16 + 10 B(2, 2)
729/a	32 + 20 B(4, 4)	16 + 10 B(4, 4)
731/b	65 + 40 B(4, 4)	35 + 10 B(4, 4)
733/a	65 + 40 B(6, 6)	35 + 10 B(6, 6)
740/c	65 + 40 B(6, 6)	35 + 10 B(6, 6)

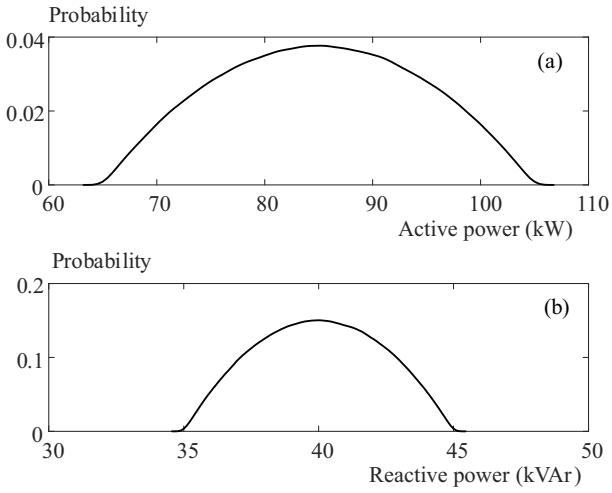

Fig. 3. PDF of load power in node 712/c: (a) – active power, and (b) – reactive power

The active and reactive powers of the loads consist of a fixed value and a multiplied beta distribution ($B(\alpha, \beta)$) with different parameters α and β, *ie*, for the load active power in node 712/c 65 kW + 40 kWB(2,2) and the reactive power in node 712/c 35 kVAr + 10 kVArB(2,2).

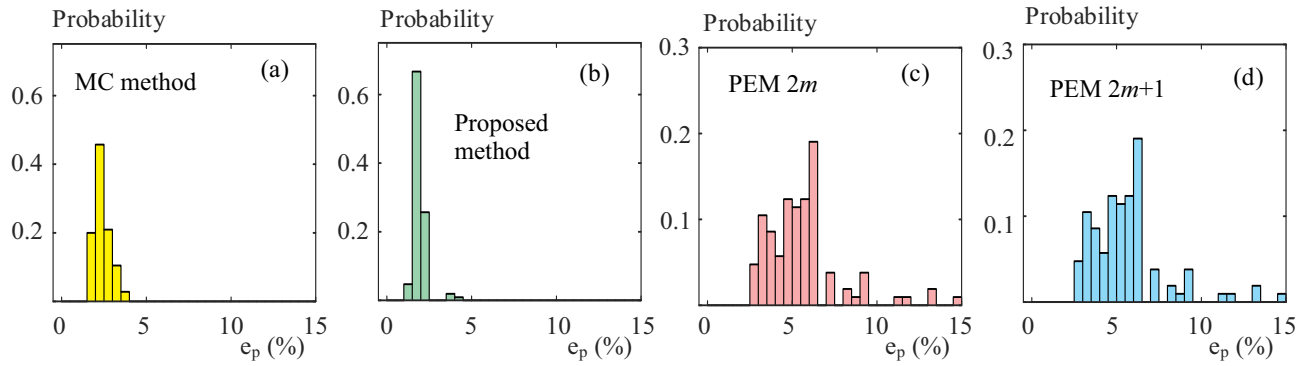

Fig. 4. Absolute percentage error of voltage magnitudes for: (a) – MC method, (b) – proposed method, (c) – PEM $2m$, and (d) – PEM $2m+1$

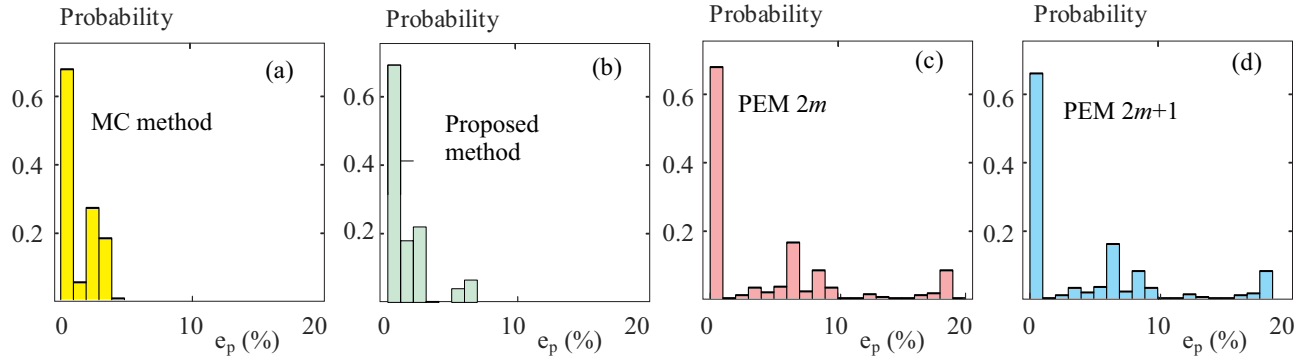

Fig. 5. Absolute percentage error of line power-flows for: (a) – MC method, (b) – proposed method, (c) – PEM $2m$, and (d) – PEM $2m+1$

To visualize, Fig. 3(a) presents the PDF of the load active power and Fig. 3(b) the load reactive power in node 712/c.

To represent the accuracy of the proposed method, a comparison with the MC method is used. The MC method with 25 000 simulations (MC-25000) is proposed as a reference. The MC method with 10,000 simulations (MC-10000) is also used to compare the accuracy, because MC-10000 guarantees good accuracy of the results [9]. Also, the PEM with two points (PEM $2m$) and the PEM with two points plus one (PEM $2m+1$) are used to compare the accuracy.

Voltage magnitudes and line power flows were the topic of a comparison to verify the accuracy of the proposed method. The PDF of the voltage magnitudes and the line power flows at each phase for the all methods were drawn and the absolute percentage error

$$e_p = \left| \frac{f_{MC_{25000}}(\chi) - f(\chi)}{f_{MC_{25000}}(\chi)} \right| \cdot 100\%, \qquad (19)$$

were observed MC-25000 is proposed as a reference method.

Above, $f_{\mathrm{MC25000}}(\chi))$ represents the PDF of MC-25000 and $f(\xi)$ represents the PDF of MC-10000.

As is clear from Fig 4(a), the e_p for the nodal voltage magnitude is in range 1—4 % for MC-10000. Similarly, the e_p for the nodal-voltage magnitude is obtained for the proposed method, where the e_p is 1—5 % (see Fig. 4(b)). Despite this, the range of e_p for the proposed method is larger than in the case of MC-10000, the majority of e_p for the proposed method is less than 2 %, with the mean of e_p being 1.9 %. For the MC-10000 method, the majority of e_p is 2—3 % and the mean of e_p is 2.4 %.

On the other hand, the PEM $2m$ (see Fig. 4(c)) and PEM $2m+1$ (see Fig. 4(d)) have an e_p for the nodal-voltage magnitude in the range 3—15 % with a mean value of 5.6 % for both methods. As is also clear from Fig. 4, there is no major difference between the results of PEM $2m$ and PEM $2m+1$.

In the case of the e_p the line power flows for MC-10000 is 0—4 %, see Fig. 5(a). The e_p for line power flows for the proposed method, Fig. 5(b), is in the range 0—7 %. Then again, the majority of e_p for the proposed method is in range 0—3 % and the mean of e_p for the MC method and the proposed method is 1.2 %. However, the e_p for line power flows for PEM $2m$, Fig. 5(c), and PEM $2m+1$, Fig. 5(d), is 0—20 %, with a mean value of 4.0 % for both methods.

Besides the fact that the proposed method has a similar accuracy to MC-10000, it is also less time consuming. The simulation time for the proposed method is around 2.5 s and the simulation time for MC-10000 is 4330 s, which means that the proposed method is 1732 times faster than MC-10000.

On the other hand, the simulation times for PEM $2m$ and PEM $2m+1$ are 26.4 and 27.9 s, respectively. The simulation times are around ten times higher than the

simulation time for the proposed method and the e_p for the voltage magnitude and the line power flows are also higher for PEM $2m$ and PEM $2m+1$ in comparison with the proposed method.

4 Conclusion

This paper proposes a new, analytical approach using the cumulant method for the thre-phase PPF analysis, where the cumulant method is upgraded so that the pseudo-inverse of the Jacobian matrix for the AC PF method is used as a sensitivity matrix. This enables the cumulant method to be used with both balanced and unbalanced systems in radial, weakly meshed or mashed power networks, considering the probability of power consumption and production.

The proposed approach is verified using the MC method where 25,000 simulations were carried out on the IEEE 37-node test system and in most cases more accurate results are obtained for the nodal voltages and for the line power flows with the proposed method than with the MC method, where 10,000 simulations were carried out. Furthermore, the time and space consumption is incomparably larger in the case of the MC method.

To compare the proposed method with the other analytical method, a comparison with the PEM $2m$ and PEM $2m+1$ was performed. The proposed method is more accurate and less time consuming than both PEM $2m$ and PEM $2m+1$.

In conclusion, accuracy and simulation time allow system operators to evaluate the possible power-flow and voltage levels based on their probability of occurrence quickly enough in the case of sudden changes to the system by using the new, proposed approach.

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References

[1] B. R. Prusty, and D. Jena, "A critical review on probabilistic load flow studies in uncertainty constrained power systems with photovoltaic generation and a new approach", *Renewable and Sustainable Energy Reviews*, vol. 69, pp. 1286–1302 2017.

[2] W. Su, J. Wang, and J. Roh, "Stochastic Energy Scheduling in Microgrids with Intermittent Renewable Energy Resources", *IEEE Transactions on Smart Grid*, vol. 5, no. 4, pp. 1876–1883, 2014.

[3] B. Brinkmann, and M. Negnevitsky, "A Probabilistic Approach to Observability of Distribution Networks", *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1169–1178, 2017.

[4] Y. Wang, N. Zhang, Q. Chen, J. Yang, C. Kang, and J. Huang, "Dependent Discrete Convolution Based Probabilistic Load Flow for the Active Distribution System", *IEEE Transactions on Sustainable Energy*, vol. 8, no. 3, pp. 1000–1009, 2017.

[5] J. Bogovic, U. Rudez, and R. Mihalic, "Probability-based approach for parametrisation of traditional underfrequency load-shedding schemes", *IET Generation, Transmission & Distribution*, vol. 9, no.16, pp. 2625–2632, 2015.

[6] P. Caramia, G. Carpinelli, and P. Varilone, "Point estimate schemes for probabilistic three-phase load flow", *Electric Power Systems Research*, vol. 80, no. 2, pp. 168–175, 2010.

[7] M. Zare, T. Niknam, R. Azizipanah-Abarghooee, and B. Amiri, "Multi-objective probabilistic reactive power and voltage control with wind site correlations", *Energy*, vol. 66, pp. 810–822, 2014.

[8] W. Wu, K. Wang, G. Li, X. Jiang, and Z. Wang, "Probabilistic load flow calculation using cumulants and multiple integrals", *IET Generation, Transmission & Distribution*, vol. 10, no. 7, pp. 1703–1709, 2016.

[9] X. Ran, and S. Miao, "Three-phase probabilistic load flow for power system with correlated wind, photovoltaic and load", *IET Generation, Transmission & Distribution*, vol. 10, no. 12, pp. 3093–3101, 2016.

[10] M. Fan, V. Vittal, G. T. Heydt, and R. Ayyanar, "Probabilistic Power Flow Studies for Transmission Systems with Photovoltaic Generation Using Cumulants", *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2251–2261, 2012.

[11] N. Gupta, and N. Daratha, "Probabilistic three-phase load flow for unbalanced electrical systems with wind farms", *International Journal of Electrical Power & Energy Systems*, vol. 87, pp. 154–165, 2017.

[12] Y.-Y. Hong, F.-J. Lin, Y.-C. Lin, and F.-Y. Hsu, "Chaotic PSO-Based VAR Control Considering Renewables Using Fast Probabilistic Power Flow", *IEEE Transactions on Power Delivery*, vol. 29, no. 4, pp. 1666–1674, 2014.

[13] F. J. Ruiz-Rodriguez, J. C. Hernandez, and F. Jurado, "Probabilistic load flow for radial distribution networks with photovoltaic generators", *IET Renewable Power Generation*, vol. 6, no. 2, pp. 110–121, 2012.

[14] D. Villanueva, A. E. Feijóo, and J. L. Pazos, "An analytical method to solve the probabilistic load flow considering load demand correlation using the DC load flow", *Electric Power Systems Research*, vol. 110, pp. 1–8, 2014.

[15] W. C. B. Vicente, R. Caire, and N. Hadjsaid, "Probabilistic load flow for voltage assessment in radial systems with wind power", *International Journal of Electrical Power & Energy Systems*, vol. 41, no. 1, pp. 27–33, 2012.

[16] Z. O. Olaofe, and K. A. Folly, "Statistical Analysis of Wind Resources at Darling for Energy Production", *International Journal of Renewable Energy Research (IJRER*,), vol. 2, no. 2, pp. 250–261, 2012.

[17] F. Diop, and M. Hennebel, "Probabilistic load flow methods to estimate impacts of distributed generators on a LV unbalanced distribution grid", *2017 IEEE Manchester PowerTech*, pp. 1–6, 2017.

[18] P. Amid, and C. Crawford, "Cumulant-based probabilistic load flow analysis of wind power and electric vehicles", *2016 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS*,), pp. 1–6, 2016.

[19] T.-H. Kim, and H. White, "On more robust estimation of skewness and kurtosis", *Finance Research Letters*, vol. 1, no. 1, pp. 56–73, 2004.

[20] A. C. Melhorn, and A. Dimitrovski, "A method for modeling voltage regulators in probabilistic load flow for radial systems", *2014 International Conference on Probabilistic Methods Applied to Power Systems* (PMAPS), pp. 1–6, 2014.

[21] J. C. A. Barata, and M. S. Hussein, "The Moore–Penrose Pseudoinverse. A Tutorial Review of the Theory", *Brazilian Journal of Physics*, vol. 42, no. 1–2, pp. 146–165, 2012.

[22] P. S. Stanimirović, V. N. Katsikis, and I. Stojanović, "Computing the Pseudoinverse of Specific Toeplitz Matrices Using Rank-One Updates", *Mathematical problems in engineering*, 2016.

[23] N. Mori, and T. Yasuda, "Weakly non-gaussian model of wave height distribution for nonlinear random waves", *Coastal Engineering Proceedings*, vol. 1, no. 25, 1996.

[24] B. Cetindag, I. Kocar, A. Gueye, and U. Karaagac, "Modeling of Step Voltage Regulators in Multiphase Load Flow Solution of Distribution Systems Using Newton's Method and Augmented Nodal Analysis", *Electric Power Components and Systems*, vol. 45, no. 15, pp. 1667–1677, 2017.

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Jerneja Bogovič received BSc and PhD degrees in electrical engineering from the Faculty of Electrical Engineering, University of Ljubljana, Ljubljana, Slovenia, in 2011 and 2017, respectively. She is currently an assistant with the Faculty of Electrical Engineering, University of Ljubljana. Her areas of interest include power-system modeling and analysis and FACTS devices.

Miloš Pantoš received BSc and PhD degrees in electrical engineering from the Faculty of Electrical Engineering, University of Ljubljana, Ljubljana, Slovenia, in 2001 and 2005, respectively. He is currently a professor with the Faculty of Electrical Engineering, University of Ljubljana. His current research interests include integration of renewable energy sources into power systems, energy markets and ancillary services, demand-side management, and optimization and control of distributed energy sources. From 2010 to 2016 he was an Associate Editor for IEEE Transactions on Smart Grid.