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CALCULATION OF INDUCTANCES OF INDUCTION MACHINES UNDER AXIAL NON–UNIFORMITY CONDITIONS

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A modified model for calculating the inductances of induction machines, considering axial non-uniformity is proposed. By means of the proposed model, inductances of a three-phase squirrel-cage induction machine under eccentricity and misalignment conditions are calculated. The effects of stator and rotor slots, rotor skewing and several rotor asymmetries on inductances in these conditions are investigated.

K e y w o r d s: induction motors, inductances, axial non-uniformity, eccentricity

1 INTRODUCTION

Transient analysis of induction machines by means of the conventional d-q model is based on the well-known assumptions. One of them is that the stator windings are sinusoidally distributed. This implies that all harmonics of the stator windings distribution, except for the fundamental one, are neglected. A model based on the basic geometry and winding layout of n stator windings and m rotor loops machine was proposed by Tolyiat and Lipo and has been proved suitable for time domain simulation of ac motors [1]. An essential part of this theory is the calculation of motor inductances. These inductances are evaluated using winding function and other equations within the theory. By this approach all space harmonics are taken into account without any restriction concerning symmetry of stators or rotors windings. Hence this model has found application in the analysis of asymmetrical and fault conditions in machines, Such as broken rotor bars and cracked end rings [2,3] and fault condition in stator windings [4, 5]. The modified winding function approach (MWFA) for non-symmetrical air-gap in a salient pole synchronous machine has been proposed in [6]. This method has been applied to analyze static, dynamic and mixed eccentricity in induction and synchronous machines [7–10]. In [11] effects of skew on inductances are analyzed. In [12], different geometrical models for calculation of inductances are evaluated and the best approximation is recommended. A new comprehensive method for the calculation of inductances of induction machine under healthy and fault conditions, based on combined winding function approach (WFA) and magnetic equivalent circuit (MEC) is presented in [13].

In all of the mentioned works, axial air gap non-uniformity is not considered in the analysis of the machines. Therefore these models do not allow the analysis of effects produced by axial air-gap non-uniformity. In [14] an extension of the MWFA for the inductance calculation considering axial non-uniformity is proposed. This method was used for calculation of mutual inductance of a three phase induction machine under static eccentricity condition. In this extension, mean radius of the machine is considered constant. Also for calculation of inductances, length of the air-gap of the machine is approximated and therefore the geometrical model of the machine is simplified.

All available techniques based on MWFA approximate the inverse length and mean radius of the air-gap and simplify the geometrical model of the machine. These approximations are applied in geometrical evaluation of these functions and determining the inverse of the air-gap length. This paper proposes model of the squirrel cage induction machine under axial non-uniformity conditions. In the proposed model, precise functions of mean radius and air-gap length, considering slots effects and other asymmetries, are included. Proposed model is more precise than the available models without increasing the computation time. By means of modified model, inductances of three-phase squirrel cage induction machine are calculated. Rotor skewing, several rotor asymmetries and stator and rotor slots effects on calculated inductances in these conditions are shown.

2 INDUCTANCES OF INDUCTION MACHINE UNDER NON–UNIFORMITY CONDITIONS

In [14] an extension of the MWFA for the inductance calculation considering axial non-uniformity was proposed. This method "2-D Modified Winding Function" (2-DMWF), has been applied to calculate the mutual inductance (L_{sr}) of an induction machine under static eccentricity condition.

In the induction machine, the 2-DMWF, $N(\phi, z, \theta_r)$, can be defined for each stator winding and each rotor loop composed by two bars.

$$N(\phi,z,\theta_r) = n(\phi,z,\theta_r) - \frac{1}{2\pi L\big(g^{-1}(\phi,z,\theta_r)\big)} \times \int_{0_1}^{2\pi}\int_0^L n(\phi,z,\theta_r) g^{-1}(\phi,z,\theta_r)\,\mathrm{d}z\,\mathrm{d}\phi \quad (1)$$

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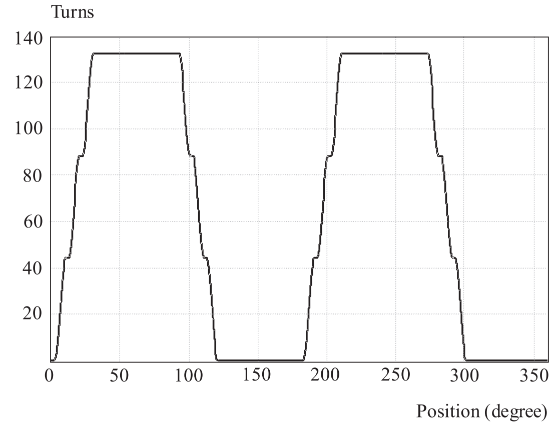

Fig. 1. Turn function of the stator phase A

where g, r and n are the air gap length, mean radius and turn functions respectively; L, θ_r, ϕ and z are rotor length, rotor angle, arbitrary angle in stator reference frame and position of axial length respectively.

Magneto-motive force (MMF) distribution in the air gap, produced by a current i_A flowing in any coil A (stator windings or rotor loops) is as follows

$$F_A(\phi, z, \theta_r) = N_A(\phi, z, \theta_r) i_A \,, \tag{2}$$

and so, a differential flux through a differential area in the air gap, $r(\phi, z, \theta_r)\mathrm{d}z\mathrm{d}\phi$, can be written as follows

$$\mathrm{d}\phi = \mu_0 F_A(\phi, z, \theta_r) g^{-1}(\phi, z, \theta_r) r(\phi, z, \theta_r)\mathrm{d}z\mathrm{d}\phi \,. \tag{3}$$

Integrating the differential flux in the region covered by either a stator coil or rotor loop, yields

$$\phi_B = \mu_0 \int_{\phi_1}^{\phi_2} \int_{z_1(\phi)}^{z_2(\phi)} r(\phi, z, \theta_r) N_A(\phi, z, \theta_r) i_A(\phi, z, \theta_r)\mathrm{d}z\mathrm{d}\phi \,. \tag{4}$$

$n_B(\phi, z, \theta_r)$ is equal to the coil turns in the region ($\phi_1 < \phi < \phi_2$, $z_1(\phi) < z < z_2(\phi)$) and zero otherwise. Therefore the total flux linking coil B, λ_B is obtained from multiplying (4) by $n_B(\phi, z, \theta_r)$ and integrating it over the whole surface

$$\lambda_B = \mu_0 \int_{0_1}^{2\pi} \int_0^L r(\phi, z, \theta_r) n_B(\phi, z, \theta_r) N_A(\phi, z, \theta_r) \times i_A g^{-1}(\phi, z, \theta_r)\mathrm{d}z\mathrm{d}\phi \,. \tag{5}$$

The mutual inductance of windings A and B, due to current i_A in the coil A ($L_{BA}(\theta_r)$), is

$$L_{BA}(\theta_r) = \frac{\lambda_B}{i_A} \,, \tag{6}$$

thus,

$$L_{BA}(\theta_r) = \mu_0 \int_{0_1}^{2\pi} \int_0^L \frac{r(\phi, z, \theta_r) n_B(\phi, z, \theta_r) N_A(\phi, z, \theta_r)}{g(\phi, z, \theta_r)}\mathrm{d}z\mathrm{d}\phi \,. \tag{7}$$

Mutual inductance between windings B and A, due to current i_B in coil B ($L_{AB}(\theta_r)$), is as follows

$$L_{AB} = \mu_0 \int_{0_1}^{2\pi} \int_0^L \frac{r(\phi, z, \theta_r) n_B(\phi, z, \theta_r) N_A(\phi, z, \theta_r)}{g(\phi, z, \theta_r)}\mathrm{d}z\mathrm{d}\phi \,. \tag{8}$$

Equations (7) and (8) have cumulative property. Thus the condition of $L_{AB} = L_{BA}$ is always satisfied. Inductances of induction machine can be calculated from (8) using geometrical characteristic of the machine.

It is possible to model the rotor and stator slots effects by variation of reluctance on the slots [15]. In this method the variation of reluctance on the slots can be easily modelled by means of the air gap, $g(\phi, z, \theta_r)$, and mean radius, $r(\phi, z, \theta_r)$, functions.

A salient feature of this technique is its capability to simulate the mechanical asymmetry and fault of stator and rotor which has no restrictions about axial and radial non-uniformity. Considering the Eq. (8); mean radius, air gap, winding and turn functions are the function of ϕ, z and θ_r. Therefore non-uniformity in all directions can be modelled by these functions. Also space harmonics of the windings MMF and slots harmonics are taken into account in the model. The winding function used in this paper takes into account the sinusoidal variation of MMF on the slots. Fig. 1 shows the turn function of the stator phase A.

3 CALCULATION OF INDUCTANCES

In this section, inductances of a 3 hp, 220 V three phase induction machine are calculated under different non-uniformity conditions. Obviously, the winding functions of the stator winding and rotor loops of the machine don't change in non-uniformity conditions compared to the symmetrical conditions. However, the functions of the air-gap length and the mean radius will change with respect to the symmetrical case. It is intended to calculate these functions which express the geometrical model of the machine.

Inductances are calculated using the (8) and geometrical models. Also effects of stator and rotor slots and several rotor asymmetries on these inductances are shown. It should be noted that saturation and leakage flux have not been considered.

3.1 Skew

To model the rotor skewing, the rotor is divided to the cross sections axially. Figure 2 shows the turn function of the rotor loop1, considering rotor skewing. The length of the air-gap and the mean radius functions are as follows

$$g_s(\phi, z, \theta_r) = g\big(\phi, \theta_r - \frac{\lambda}{r} z\big) \,, \tag{9}$$

$$r_{av}(\phi, z, \theta_r) = r_{sta} - \frac{g_s(\phi, z, \theta_r)}{2} \,. \tag{10}$$

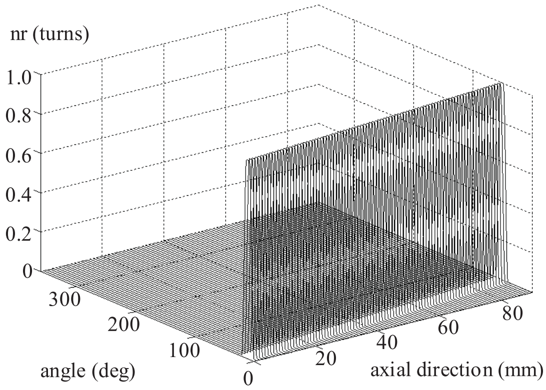

Fig. 2. Turn function of the rotor loop1, considering rotor skewing

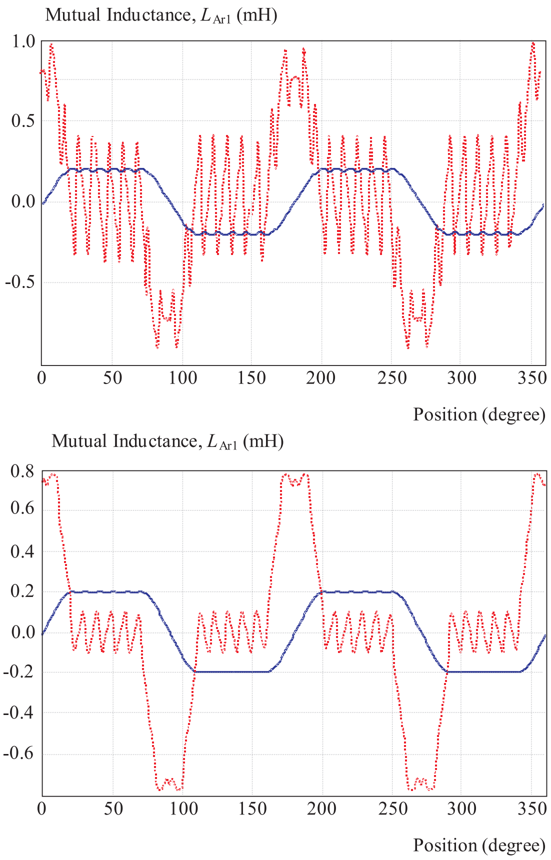

Fig. 3. Mutual inductance between stator phase A and rotor loop1 (solid line) and its derivative (dotted line): without skew (top), and with skew (bottom)

Where λ is the mechanical angle of skewing, $g(\phi, z, \theta_r)$ is the air-gap function in the cross section in front of the rotor and $r_{sta}(\phi)$ is the inner stator radius.

Figure 3 shows the calculated mutual inductance in thick line and its derivative in thin line. Figure 3 top shows the inductance when no skew is present whereas Fig. 3 bottom shows the corresponding inductance when rotor skewing is considered. As shown in Fig. 3 bottom, due to skew effect, the derivative of mutual inductance between stator phases and rotor loops has lower variations. On the other hand, shape of inductance gets smooth in presence of skew.

It is noted that the skewing of the rotor bars has no influence on the self and inter-phase mutual inductances on the stator side or rotor side. The skewing only affects the mutual inductance between stator and rotor.

3.2 Eccentricity

There are three types of air-gap eccentricity; Static, dynamic and mixed eccentricity [12]. Static eccentricity (SE) occurs when the rotor rotates about its own centreline, but this centreline does not coincide with that of the stator bore. Dynamic eccentricity (DE) occurs when the rotor geometric centre is not at the centre of rotation, producing consequently an air-gap periodic variation as a rotor position function.

In mixed eccentricity (ME) both rotor symmetrical and mechanical rotation centrelines are displaced individually in respect to the stator symmetrical centreline.

In the case of eccentricity without skew and with uniform air-gap along the rotor, we can use the MWFA used in [12]. However for the calculation of machine inductances under eccentricity conditions, considering rotor skewing and slot effects, Eq. (8) should be used. In ME condition, air-gap and mean radius functions can be represented by

$$g_e(\phi, z, \theta_r) = g_h(\phi, z, \theta_r)\big(1 - \delta_s \cos(\phi - \phi_0) - \delta_d \cos(\phi - \phi_0 - \theta_r)\big)\,, \qquad (11)$$

$$r_{av}(\phi, z, \theta_r) = r_{sta}(\phi) - \frac{g_e(\phi, z, \theta_r)}{2}\,. \qquad (12)$$

In the case of SE, $\delta_d = 0$ and in DE, $\delta_s = 0$.

Figure 4 shows that static eccentricity causes an asymmetrical mutual inductance between stator phases and rotor loops and dynamic eccentricity causes a symmetrical mutual inductance with a larger magnitude compared to the non-eccentric condition.

In the case of static eccentricity, self and mutual inductances of the stator windings are constant whereas those of the rotor loops change with rotor position. For dynamic eccentricity, the stator self and mutual inductances are rotor position functions. Also self and mutual inductances of the rotor loops are constant. For example, self inductance of phase A, under different healthy, static and dynamic eccentricity conditions is shown in Fig. 5.

The mutual inductance of stator phase A, and rotor loop1, with a 20 % DE and 50 % SE, is shown in Fig. 6, without skew in Fig. 6 top and with skew in Fig. 6 bottom. It is obvious that skew smooth the inductance variation due to mixed eccentricity. This is evident in the inductance derivative.

3.3 Misalignment

Relatively small amount of misalignment can have a significant impact on the operational life of bearings. As shown in Fig. 7, misalignment is caused by improper alignment of right and left bearing centres so that centrelines of rotor shaft and stator bore are not parallel. In the case of static misalignment rotor rotates about its

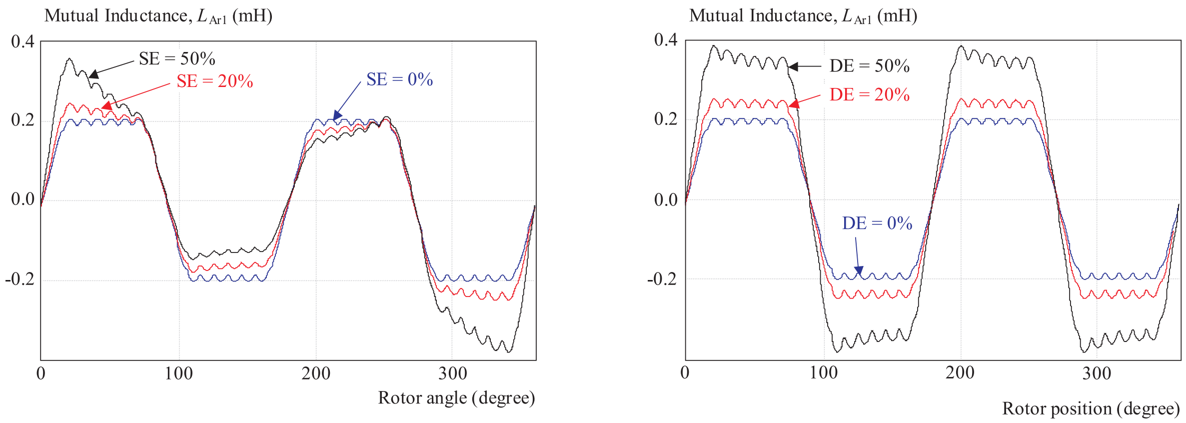

Fig. 4. Mutual inductance between stator phase A and rotor loop 1 as a function of rotor position and static eccentricity (left) and dynamic eccentricity (right)

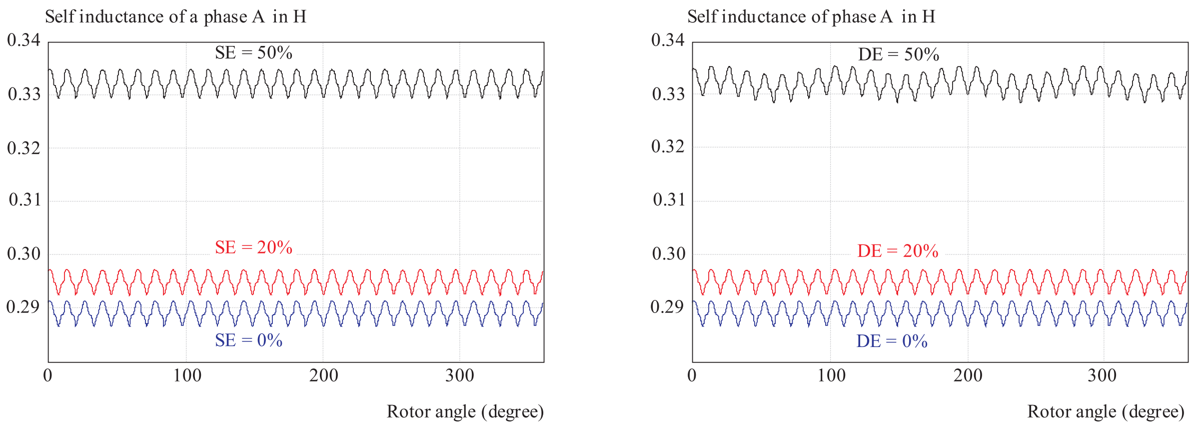

Fig. 5. Self inductance of phase A, under different healthy, static (left) and dynamic eccentricity (right)

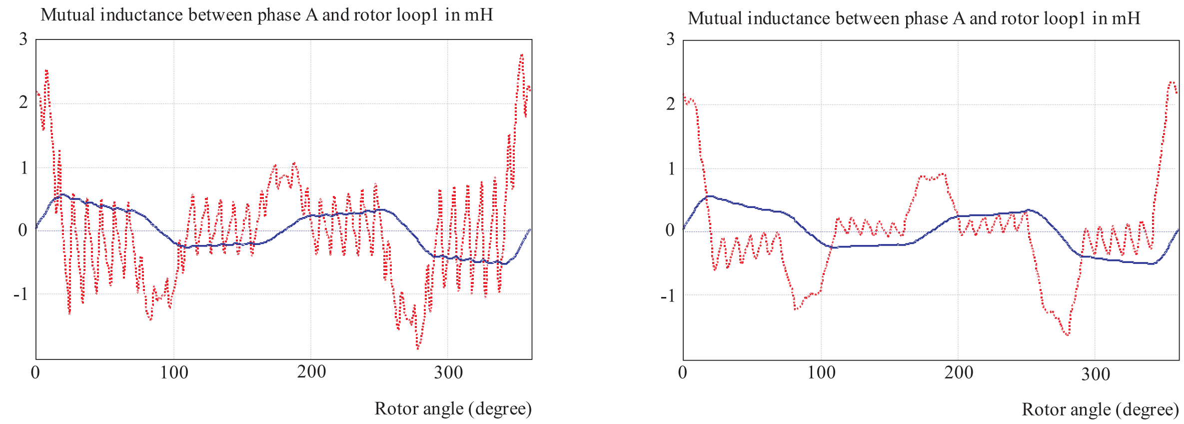

Fig. 6. Mutual inductance of stator phase A, and rotor loop 1 (solid line) and its derivative (dotted line), with a 20 % DE and 50 % SE: without skew (left), and with skew (right)

own centreline but in dynamic misalignment rotor geometric centre is not at the centre of rotation and air-gap varies periodically as a rotor position function.

To model the rotor misalignment, rotor is divided axially in to cross sections. In each cross section, coefficient of eccentricity is the function of cross section position. Variation of coefficients along the rotor axis is linear respect to the axial length. Fig. 8 shows the variation of air gap eccentricity coefficients in different positions. δ_{mis} is the static eccentricity coefficient in the back side or front side of the rotor. The air gap function is as follows

if $(0 \le z \le l/2)$

$$g_{mis}(\phi, z, \theta_r) = g_h(\phi, z, \theta_r)\big(1 - \delta_s(z)\cos(\phi - \phi_0) - \delta_d \cos(\phi - \theta_r - \phi_0)\big) \quad (13)$$

where

$$\delta_s(z) = \frac{-2\delta_{mis}}{l} z + \delta_{mis} \,. \quad (14)$$

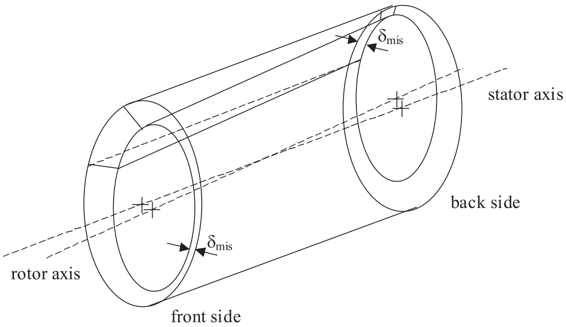

Fig. 7. Misalignment in the induction machine

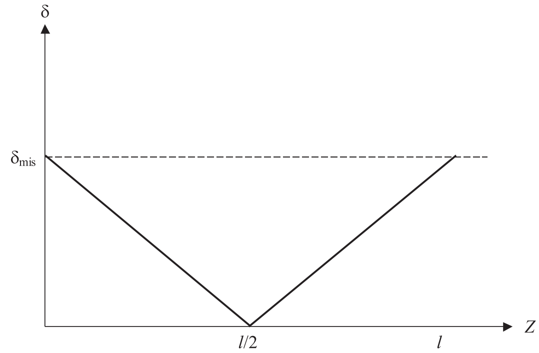

Fig. 8. Variation of eccentricity coefficients in different positions

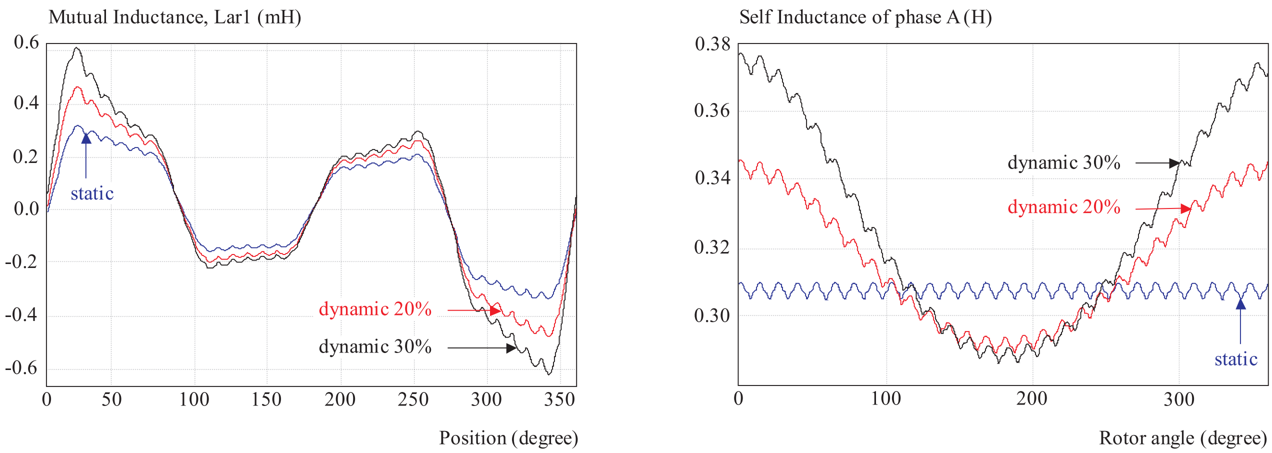

Fig. 9. Mutual inductance of stator phase A and rotor loop1, L_{ar1}, (left) and self inductance of phase A, L_{ma}, (right) in misalignment condition

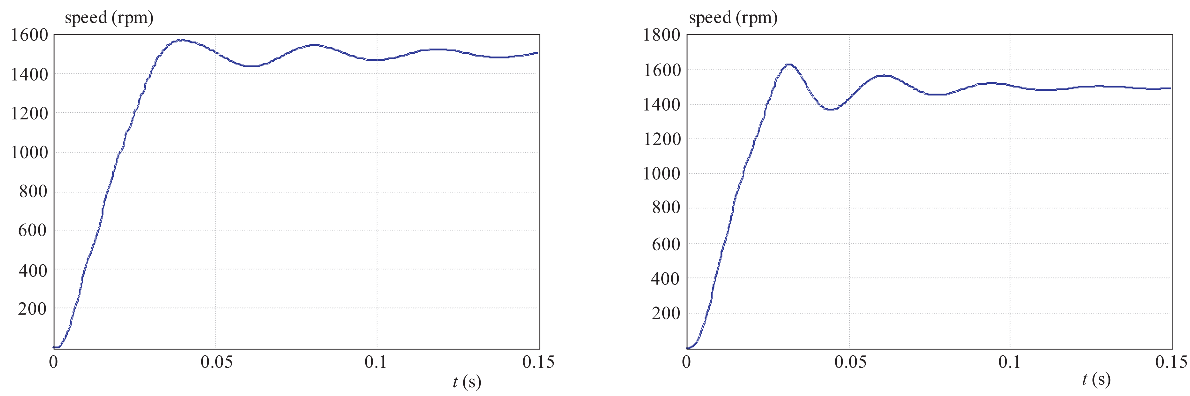

Fig. 10. Motor speed from simulation under healthy (left) and misalignment (right) conditions

If $(l/2 \le z \le l)$

$$g_{mis}(\phi, z, \theta_r) = g_h(\phi, z, \theta_r)\big(1 - \delta_s(z)\cos(\phi - \phi_0 + \pi) - \delta_d \cos(\phi - \theta_r - \phi_0)\big) \quad (15)$$

where

$$\delta_s(z) = \frac{\delta_{mis}(2z - l)}{l} z\,. \quad (16)$$

$g_h(\phi, z, \theta_r)$ is the air gap function in healthy machine. It is clear that in the case of static misalignment, δ_d is equal to zero. Mean radius is as follows

$$R_{av}(\phi, z, \theta_r) = r_{sta}(\phi) - \frac{g_{mis}(\phi, z, \theta_r)}{2}\,. \quad (17)$$

To calculate the machine inductances under misalignment condition, these equations should be used in (8).

In the case of static misalignment, self and mutual inductances of the stator windings are constant whereas those of the rotor loops change with rotor position. For dynamic misalignment, all of the inductances are the function of θ. As an example, mutual inductance of stator phase A and rotor loop1 (L_{ar1}) and self inductance of phase A (L_{ma}) are shown in Fig. 9.

4 SIMULATION RESULTS

In order to validate the modified method and proposed models, a three phase squirrel cage induction machine is simulated. Induction machine is simulated by the coupled electromagnetic model of the machine [1]. The electromagnetic coupling model of the machine circuits is solved using a 4^{th}, 5^{th} order Runge-Kutta method. The simulation study was perform on a 3 phase, 3 hp induction machine.

Figure 10 shows the motor speed from simulation under healthy and misalignment conditions. As shown, in the case of misalignment condition, motor speed has more overshoot than in the healthy condition. Electromagnetic motor torque depends on derivative of mutual inductances between stator phases and rotor loops. In misalignment condition, the derivative of mutual inductance has lower magnitude respect to the healthy condition. This in turn leads to increased torque magnitude in misalignment condition. For the simulation of induction machine:

1) Modified method for calculation of inductances was used. In this method, the turn, winding, mean radius and air-gap functions are functions of ϕ, z and θ_r. Therefore non-uniformity in all directions can be modelled precisely.
2) A more precise geometrical model of machine was included in all conditions.
3) Stator and rotor slots effects were considered in the model.
4) A more precise distribution of the turn function of windings and sinusoidal rise of MMF across the slots was considered.

5 CONCLUSIONS

In this paper, a modified model has been presented to calculate the inductances of induction machines, considering radial and axial non-uniformity. Then more precise geometrical models of three phase squirrel cage induction machine under healthy, eccentricity and misalignment conditions, considering rotor skewing and Stator and rotor slots effects are determined. In the modified method, turn, winding, air-gap and mean radius, are function of ϕ, z and θ_r. Therefore it has no restriction about non-uniformity in all directions. By means of the modified method and proposed geometrical models, inductances of a three-phase squirrel-cage induction machine are calculated in these conditions.

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