A SIMPLE ROBUST PID CONTROLLER DESIGN METHOD BASED ON SINE WAVE IDENTIFICATION OF THE UNCERTAIN PLANT

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The paper deals with the development and application of a new simple empirical approach to the design of robust PID controllers for technological processes in industrial practice. The main advantage of the proposed approach is the possibility to specify the required performance before the design algorithm implementation. Identification of characteristic data of the black-box type plant with varying parameters is carried out using the sine wave excitation signal, thus allowing to design the controller without necessarily knowing the mathematical model of the plant. The proposed approach has been verified on a real-world physical process.

Keywords: empirical methods, characteristic data of the plant, excitation signal, robust stability, robust performance

1 INTRODUCTION

Empirical methods are popular tools to design industrial controllers [9]. Usually, the design proceeds in twosteps: the first step is identification of certain characteristic information about the controlled plant; the second step is the controller design with parameters that directly depend on the identified data. The broad use of these methods is due to their algorithm simplicity and quick controller synthesis as well as to minimum of a priori information about the plant necessary for a successful design [11].

Nevertheless, quite frequently controllers implemented in various industrial technologies are not tuned effectively thus bringing about economic losses. The existing studies have revealed that certain controllers not only do not guarantee the required closed-loop performance but even fail to provide closed-loop stability or are operated just in an open-loop and on achieving setpoint vicinity they are manually switched-off or are further operated by the operation staff [4].

The main advantage of the proposed empirical design method is a quick design of a robust PID controller for specified performance without knowledge of an exact mathematical model of the plant. Characteristic plant data are obtained by performing a small number of experiments on the plant during the design process [1].

2 CONTROL LAW OF THE EMPIRICAL SINE WAVE METHOD

For controller design purpose, stability and performance are usually specified in terms of stability margins [2]. The proposed empirical design method enables the required phase margin ϕ_M to be specified by the control engineer or process technologist. The control law is easy to derive from the closed-loop characteristic equation of the control loop in Fig. 1 when the switch SB is in position "1".

In Fig. 1, G(s) is the plant transfer function with an unknown mathematical model and $G_R(s)$ is the transfer function of the industrial PID controller. From the closedloop transfer function

$$T(j\omega) = \frac{L(j\omega)}{1 + L(j\omega)} = \frac{G(j\omega)G_R(j\omega)}{1 + G(j\omega)G_R(j\omega)}$$
(1)

results the closed-loop characteristic equation

$$1 + L(j\omega_n) = 1 + G(j\omega_n)G_R(j\omega_n) = 0, \qquad (2)$$

which can be easily broken down into the magnitude and phase conditions

$$|G(j\omega_n)||G_R(j\omega_n)| = 1,$$

$$\arg G(\omega_n) + \arg G_R(\omega_n) = -\pi + \phi_M,$$
(3)

where ϕ_M is the required phase margin specified in advance, $L(j\omega)$ is the open-loop transfer function. Introduce the following substitution

$$\varphi = \arg G(\omega_n), \ \Theta = \arg G_R(\omega_n).$$
 (4)

Consider a PID controller

$$G_R(s) = K \left[1 + \frac{1}{sT_i} + sT_d \right],\tag{5}$$

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Fig. 1. Feedback loop for the sine wave experimental method



Fig. 2. Identification procedure of one point of the plant frequency response using the sine wave method



Fig. 3. Mutual position of $G(j\omega_n)$ and $L(j\omega_n)$ and verification of the phase condition (23) in the complex plane

where K is the proportional gain, and T_i and T_d are integral and derivative time constants, respectively. A frequency domain comparison of the right-hand side of (5)

$$G_R(j\omega_n) = K + jK \left[T_d \omega_n - \frac{1}{T_i \omega_n} \right]$$
(6)

with the right-hand side of the PID in polar form

$$G_R(j\omega_n) = |G_R(j\omega_n)|e^{j\Theta} = |G_R(j\omega_n)|\cos\Theta + j|G_R(j\omega_n)|\sin\Theta \quad (7)$$

yields a complex equality

$$K + jK \Big[T_d \omega_n - \frac{1}{T_i \omega_n} \Big] = \frac{\cos \Theta}{|G(j\omega_n)|} + j \frac{\sin \Theta}{|G(j\omega_n)|} \quad (8)$$

from which it is possible to obtain PID controller parameters using the substitution

$$|G_R(j\omega_n)| = \frac{1}{|G(j\omega_n)|} \tag{9}$$

resulting from the magnitude condition (3a).

The controller gain K can be expressed directly from the complex equation (8)

$$K = \frac{\cos\Theta}{|G(j\omega_n)|} \tag{10}$$

and the derivative time constant T_d can be specified from the quadratic equation with respect to $T_d \omega_n$

$$K\left[T_d\omega_n - \frac{1}{\beta T_d\omega_n}\right] = \frac{\sin\Theta}{|G(j\omega_n)|}.$$
 (11)

The ratio of integral and derivative time constants is set by an appropriate choice of coefficient β (according to the most frequently used empirical methods $\beta = 4$ [9])

$$\beta = \frac{T_i}{T_d} \implies T_i = \beta T_d \,. \tag{12}$$

Substituting (10) into (11) yields a quadratic equation with respect to $T_d \omega_n$

$$\left[T_d\omega_n - \frac{1}{\beta T_d\omega_n}\right] = \operatorname{tg}\Theta\tag{13}$$

that can simply be modified as follows

$$T_d^2 \omega_n^2 - T_d \omega_n \operatorname{tg} \Theta - \frac{1}{\beta} = 0.$$
 (14)

From the solution of (14) directly results an expression for calculation of the derivative time constant T_d

$$T_d = \frac{\operatorname{tg}\Theta}{2\omega_n} \pm \frac{1}{\omega_n} \sqrt{\frac{\operatorname{tg}^2\Theta}{4} + \frac{1}{\beta}}.$$
 (15)

Hence, PID controller parameters are calculated according to the following expressions

$$K = \frac{\cos\Theta}{|G(j\omega_n)|}, \ T_i = \beta T_d,$$

$$T_d = \frac{\operatorname{tg}\Theta}{2\omega_n} \pm \frac{1}{\omega_n} \sqrt{\frac{\operatorname{tg}^2\Theta}{4} + \frac{1}{\beta}},$$
 (16)

where angle Θ is obtained from the phase condition (3b)

1

$$\Theta = -\pi + \Phi_M - \arg G(\omega_n) \,. \tag{17}$$



Fig. 4. Dispersion circles M_G , M_L and the prohibited area delimited by the circle M_s



Fig. 5. Control loop of the DC motor with uncertainties

3 IDENTIFICATION OF THE SINGLE POINT OF THE PLANT FREQUENCY RESPONSE USING THE SINE WAVE METHOD

The proposed PID controller synthesis is based on the knowledge of a single point of the frequency response $G \equiv G(j\omega_n)$ of the unknown plant

$$G(j\omega_n) = |G(j\omega_n)|e^{j\varphi(\omega_n)}.$$
(18)

If the reference variable in the control loop in Fig. 1 equals zero, *ie* w(t) = 0, and the switch SB is in position "2, the control variable injected into the unknown plant G(s) is generated by a sine wave signal

$$u(t) = U_0 \sin(\omega_n t), \qquad (19)$$

where U_0 is the amplitude of the excitation signal u(t)and ω_n is the excitation frequency of the sine wave generator. The output signal y(t) is also of sine wave type with the same frequency and amplitude Y_0

$$y(t) = Y_0 \sin(\omega_n t + \varphi) \tag{20}$$

 φ is the phase shift with respect to the excitation signal u(t). The corresponding time response is depicted in Fig. 2b. After reading off the values Y_0 and φ from the recorded output, the particular point of the plant frequency characteristics corresponding to the excitation frequency ω_n

$$G(j\omega_n) = \frac{Y_0}{U_0} e^{j\varphi(\omega_n)} \tag{21}$$

is plotted in the complex plane as depicted in Fig. 2c.

The advantage of the sine wave identification is that the sine signal amplitude Y_0 of the output y(t) can be affected by the amplitude U_0 of the excitation sine signal u(t) generated by the sine wave generator.

Substituting coordinates of the identified point of the frequency response of the unknown plant G (21) into (16a) yields final expressions for PID controller parameters

$$K = \frac{U_0}{Y_0} \cos \Theta , \ T_i = \beta T_d ,$$

$$T_d = \frac{\operatorname{tg} \Theta}{2\omega_n} \pm \frac{1}{\omega_n} \sqrt{\frac{\operatorname{tg}^2 \Theta}{4} + \frac{1}{\beta}} ,$$
 (22)

where for the particular excitation frequency ω_n the following phase condition holds

$$\Theta = -\pi + \Phi_M - \varphi \,. \tag{23}$$

The identified point G of the plant frequency response $G(j\omega)$

$$G = G(j\omega_n) = [|G(j\omega_n)|, \arg G(j\omega_n)] = [|G(j\omega_n)|, \varphi]$$
(24)

determines the amplitude crossover L of the open-loop $L(j\omega)$

$$L = L(j\omega_n) = [|L(j\omega_n)|, \arg L(\omega_n)] = [|L(j\omega_n)|, \Phi_M]$$
(25)

in which the designed PID controller guarantees the required phase margin Φ_M . Therefore under the excitation frequency ω_n it holds $|L(j\omega_n)| = 1$. Mutual position of the points $G(j\omega_n)$ and $L(j\omega_n)$ is depicted in Fig. 3.

The empirical sine wave method evolves from the fulfilment of the phase condition (23) for the angular frequency ω_n of the identified point G and simultaneously of the amplitude crossover of $L(j\omega)$. Similarly as a PID controller designed using Ziegler-Nichols tuning rules [1] compensating the critical point of the plant $G(j\omega c)$ by shifting it into the open-loop point $C_{PID} =$ [-0.6, -j0.28], the PID controller tuned according to the empirical sine wave method moves the identified point $G(j\omega_n)$ of the plant frequency response into the openloop amplitude crossover $L(j\omega_n)$ lying on the unit circle M_1 .

Note that in general a critical point of any plant is a point of its frequency characteristics with a phase shift $\varphi = -180^{\circ}$, where ω_c is the critical frequency of the plant.

It is advantageous to derive the frequency of the sine wave generator from the plant critical frequency ω_c that can be determined by the relay experiment according Rotač [8]. The experiment is carried out by switching the switch in the block diagram in Fig. 1 to the position "3". The sine wave generator frequency is chosen from the interval

$$\omega_n \in \langle 0.3\omega_c, \omega_c \rangle \,. \tag{26}$$

As the choice of the frequency ω_n influences the closed-loop dynamics, the interval (26) enables to modify the closed-loop dynamics [10].



Fig. 6. Identification of DC motor in three working points using the sine wave method (for Z_{\min}, Z_{\max})

4 IDENTIFICATION OF THE UNCERTAIN PLANT AND DESIGN OF ROBUST PID CONTROLLER

If the plant model is uncertain (due to variations in humidity, ambient temperature, linearization *etc*), it cannot be represented by a single frequency characteristics but by a set of frequency characteristics. The basic idea of identification of such an uncertain system consists in repeating the sine wave identification at the excitation signal frequency ω_n for individual uncertainty changes yielding a set of identified points G_i , i = 1, 2, ..., N of the uncertain plant frequency responses

$$G_i(j\omega_n) = |G_i(j\omega_n)| e^{j \arg G_i(\omega_n)}.$$
(27)

Plant parameter changes are reflected in magnitude and phase changes $|G_i(j\omega_n)|$ and $\arg G_i(\omega_n)$, respectively, of the identified points $G_i(j\omega_n)$ in the complex plane.

If the multiple identification of individual points G_i of the uncertain plant frequency characteristics is performed using sine wave excitation signals with the same frequency ω_n in each identification experiment, then each identified point G_i in the complex plane corresponds to a different frequency characteristics from the set of plant models and simultaneously, each identified point G_i in the complex plane corresponds to the same angular frequency ω_n for $i = 1, 2, \ldots, N$; where N = p + 1 is the number of identification experiments and p is the number of varying technological quantities of the plant [6].

Location of identified points $G_i(j\omega_n)$ of the unknown uncertain plant can be expressed in the complex number standard form

$$G_i(j\omega_n) = a_i + jb_i, i = 1, 2, \dots, N.$$
 (28)

The real and imaginary parts of the nominal plant model $G_0(j\omega_n)$

$$G_0(j\omega_n) = a_0 + jb_0 \tag{29}$$

can be obtained as mean values of real and imaginary parts of identified points according to

$$G_0(j\omega_n) = a_0 + jb_0 = \frac{1}{N}\sum_{i=1}^N a_i + j\frac{1}{N}\sum_{i=1}^N b_i.$$
 (30)

The nominal point $G_0(j\omega_n)$ expressed in polar form is

$$G_0(j\omega_n) = |G_0(j\omega_n)|e^{j\varphi_0(\omega_n)},\tag{31}$$

where the magnitude $|G_0(j\omega_n)|$ and the phase $\varphi_0(\omega_n) = \arg G_0(j\omega_n)$ are calculated as follows

$$|G_0(j\omega_n)| = \sqrt{a_0^2 + b_0^2}, \ \varphi_0(\omega_n) = \operatorname{arctg} \frac{b_0}{a_0}.$$
 (32)

The points G_i representing plant uncertainties can be enclosed in the circle M_G centred in $G_0(j\omega_n)$ with the radius $R_G \equiv R_G(\omega_n)$ that can be obtained as a maximum distance between the *i*-th identified point $G_i(j\omega_n)$ and the nominal point $G_0(j\omega_n)$ according to

$$R_G = \max\{\sqrt{(a_i - a_0)^2 + (b_i - b_0)^2}\},\$$

$$i = 1, 2, \dots, N.$$
 (33)

The dispersion circle M_G centred in the nominal point G_0 with the radius R_G encircles all identified points G_i of the uncertain plant. Figure 4 illustrates the situation for N = 3 identifications.

The proposed control law generated by the robust controller $G_{\text{Rrob}}(s)$ designed for the nominal point $G_0(j\omega_n)$ actually carries out the transformation

$$\Re: \left\{ R_G \to R_L \colon R_L = |G_{\text{Rrob}}| R_G \right\}$$
(34)

of the set of identified points $G_i(j\omega_n)$ encircled by M_G with the radius R_G into the set of points $L_i(j\omega_n)$ delimited by M_L and also calculates the radius $R_L \equiv R_L(\omega_n)$ of the dispersion circle M_L corresponding to the points $L_i(j\omega_n)$ of the Nyquist plot so as to guarantee fulfilment of the robust stability condition.



Fig. 7. DC motor closed-loop time responses for different ϕ_M

Table 1. Robust PID controller coefficients for various ϕ_M

type	ϕ_M	K	T_i	T_d	δ_0
PID PID	50° 70°	$1.1883 \\ 0.8236$	$\begin{array}{c} 0.3448 \\ 0.4754 \end{array}$	$1.3793 \\ 1.9018$	50° 70°

The robust PID controller is designed using the empirical sine wave method; the input data for the nominal model $G_0(j\omega_n)$ are its following coordinates

$$A_0 = |G_0(j\omega_n)|, \ \varphi_0 = \arg G_0(\omega_n). \tag{35}$$

Substituting coordinates of the nominal model $G_0(j\omega_n)$ into (22) following expressions for calculating robust PID controller parameters are obtained

$$K_{\rm rob} = \frac{\cos \Theta_0}{A_0}, \ T_{i_{\rm rob}} = \beta T_{d_{\rm rob}},$$

$$T_{d_{\rm rob}} = \frac{\operatorname{tg} \Theta_0}{2\omega_n} \pm \frac{1}{\omega_n} \sqrt{\frac{\operatorname{tg}^2 \Theta_0}{4} + \frac{1}{\beta}}.$$
 (36)

For the parameter Θ_0 and excitation frequency ω_n the modified phase condition holds

$$\Theta_0 = -\pi + \delta_0 - \varphi_0 \,. \tag{37}$$

Thus, δ_0 is a modified phase margin and at the same time a robust PID controller tuning parameter appearing in (36) and (37) for calculation of its parameters that guarantee the necessary phase margin required for robust stability; δ_0 does not influence the radius of the dispersion circle M_L , just the distance between L_0 and the critical point (-1, 0). This enables to draw the circle M_L apart from the critical point (-1, 0) thus improving robust closed-loop performance.

5 ROBUST STABILITY CONDITION

If the nominal open-loop

$$L_0(s) = G_R(s)G_0(s), (38)$$

is stable, then according to the Nyquist stability criterion [7] the closed-loop with uncertain system will be stable if the distance between L_0 and the point $(-1, j_0)$, ie $|1 + L_0(j\omega_n)|$, will be greater than the radius $R_L(\omega_n)$ of the dispersion circle M_L centred in L_0 [3], [6]

$$R_L(\omega_n) < |1 + L_0(j\omega_n)|, \qquad (39)$$

where ω_n is the frequency of the sine wave generator. The distance between the point (-1,0) and the openloop Nyquist plot with the nominal model L_0 can be calculated according to Fig. 3 by applying the cosine rule to the triangle $(-1,0,L_0)$

$$|1 + L_0|^2 = |1|^2 + |L_0|^2 - 2 \cdot 1 \cdot |L_0| \cos \delta_0, \qquad (40)$$

where δ_0 is the modified phase margin.

According to the robust stability condition the distance

$$|1 + L_0| = \sqrt{1 + |L_0|^2 - 2|L_0|\cos\delta_0}, \qquad (41)$$

has to be greater than the radius R_L of the dispersion circle centred in L_0 , *ie* the following inequality has to be satisfied

$$R_L \stackrel{!}{<} |1 + L_0|.$$
 (42)

Substituting the distance (41) into (42) yields the robust stability condition in the form

$$R_L \stackrel{!}{<} \sqrt{1 + |L_0|^2 - 2|L_0|\cos\delta_0} \,. \tag{43}$$

From the concept of the proposed empirical sine wave PID controller tuning method results, that the robust controller shifts the nominal point of the plant frequency response G_0 to the point L_0 of the unit circle at frequency ω_n . Thus ω_n becomes amplitude crossover frequency for the open loop. As the point L_0 is lying on the circle M_1 , the magnitude $|L_0(j\omega_n)|$ equals one

$$|L_0| = |G_0||G_R| = 1, (44)$$



Fig. 8. Robust stability for different ϕ_M

yielding the transformation ratio $|G_R| = |G_0|^{-1}$ between the radii R_G and R_L of the circles M_G and M_L , respectively. The radius R_L of the dispersion circle M_L can be expressed as

$$R_L = R_G |G_R| = R_G |G_0|^{-1}.$$
 (45)

Substituting (45) and (44) in (41) yields the robust stability condition in the following form

$$R_G^2 |G_R^2|^{-1} \stackrel{!}{<} 2 - 2\cos\delta_0.$$
(46)

After minor manipulations, condition for calculating the angle δ_0 is obtained

$$\cos \delta_0 < \left(1 - \frac{R_G^2}{2|G_0^2|}\right).$$
 (47)

According to the robust stability condition the chosen value δ_0 is subsequently substituted into the phase condition (37) and subsequently parameters of the robust PID controller are calculated from (36) for $\delta_0 \ge \phi_M$.

6 VERIFICATION OF THE EMPIRICAL SINE WAVE METHOD ON A REAL PLANT

The empirical sine wave method was applied for robust control of a physical model — the DC motor. In case of the DC motor the controlled output y(t) is the speed, the input variable is the armature voltage generated by the control system implemented in Matlab-Realtime Workshop. To sense the output y(t) a tachogenerator (TG) is used. The disturbance affecting the motor operation is the load torque z(t), as depicted in the feedback control loop in Fig. 5.

Three identification experiments were carried out on the DC motor, for the minimum, medium and the maximum loads. Corresponding time responses are in Fig. 6. Parameters of the designed robust PID controllers for two phase margin values $\phi_M = 50^\circ$ and 70° are in Table 1.

Robust properties for both cases were verified by applying DC motor reference speed step change and after the transient response died out the load step change was applied. Corresponding time responses are in Fig. 7.

7 CONCLUSION

Stability and performance of designed robust control loops [5] have been proved by the time responses in Fig. 7 and by the position of the dispersion circle M_L in Fig. 8. Moreover, Figure 8 shows that the DC motor control loop designed for the phase margin $\phi_M = 70^\circ$ determines a larger prohibited area in the complex plain delineated by the circle M_s than the control-loop designed for a phase margin $\phi_M = 50^\circ$. Both control loops have successfully coped with the step change of the motors load torque z(t) occurred at 65 s with keeping a 100% steady-state accuracy.

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