MODEL REFERENCE ADAPTIVE CONTROL OF PERMANENT MAGNET SYNCHRONOUS MOTOR

Marián Tárník — Ján Murgaš *

In this paper the classical theory of the direct Model Reference Adaptive Control is used to develop a control algorithm for Permanent Magnet Synchronous Motor (PMSM). A PMSM model widely used in electric drives community is considered as base for control system development. Conventionally used controllers are replaced by adaptive ones. The resulting control system adapts to changes in any of PMSM parameters.

K e y w o r d s: model reference adaptive control, permanent magnet synchronous motor

1 INTRODUCTION

The most common control scheme for Permanent Magnet Synchronous Motor (PMSM) is rotor oriented Vector Control with a voltage source inverter [1, 2]. The commanded voltages are modulated using a pulse-width modulation (PWM). In order to get variables in the rotor dq reference frame the rotor position is required. The rotor position can be obtained with an encoder or, in the case of the so called sensorless control, by estimation of the rotor position. The rotor speed is then derived from the rotor position measurement.

The speed regulator gives the commanded torque, this is considered to be proportional to q-axis current component reference value. When the PMSM with the surface mounted magnets is considered, the d-axis current component does not contribute to the torque production and its reference value is set to zero. In order to get the two current closed loops decoupled a decoupling block is used. After adding the decoupling terms the commanded voltage is transformed from the dq reference frame to the polar values in stationary reference frame. Commonly the PI controller is used in the current control loops and also in the speed control loop.

The current controller design is usually based on the PMSM model, which is developed by using the physical laws. Commonly used model describes the fundamental dynamics of the PMSM [3]. For the purposes of the high-precision drive controller design the PMSM with non-sinusoidal flux density distribution is considered [4, 5]. The parameter values of the PMSM model can be derived from the motor datasheet or obtained using the appropriate identification method.

The current closed loop dynamics is required to be fast. Then the torque response of the drive is fast. In high-performance drives the torque rise time is around 1 ms. In such a case the exact values of motor model parameters has to be known. Otherwise the designed current controller may not achieve the desired performance. Knowing the exact values is even more important in the design of decoupling block. Inaccurate decoupling causes significant deterioration in the performance of current control system [6]. However, an exact identification of the electromagnetical subsystem parameters is challenging, especially when more comprehensive PMSM model with higher number of parameters is considered. Moreover, the electromagnetical subsystem parameters are sensitive to changes in the working conditions, particularly to changes in the temperature.

The disturbances caused by the electromagnetical parameters variation can be estimated on-line using a Model Reference Adaptive System (MRAS) technique and compensated by a feedforward manner [6]. This makes the decoupling more accurate, so the dynamic performance of current controller designed for some nominal model remains almost unchanged.

The estimation of disturbances caused by the variation of electromagnetical parameters can be also used for torque ripple minimization [7]. The undesired torque ripple is mainly due to the non-sinusoidal flux density distribution of PMSM. This PMSM property can be modeled using the appropriate model [4, 5]. The primary purpose of disturbance estimation is to ensure the accuracy of decoupling but this disturbance signal also contains information that can be used for the current reference signal shaping. The tracking of the resulting current reference waveform ensures minimization of the torque ripple. This principle is used also in [8]. The MRAS is used for the on-line estimation of flux linkage disturbance. The estimated disturbance is then processed to obtain certain PMSM model parameter values. These values are then used as parameters of block which is designed for the current reference signal shaping. However, this approach does not ensure the accuracy of decoupling. Both, the decoupling

* Institute of Control and Industrial Informatics, Slovak University of Technology in Bratislava, Faculty of Electrical Engineering and Information Technology, Ilkovicova 3, 812 19 Bratislava, Slovakia, marian.tarnik@stuba.sk, jan.murgas@stuba.sk

DOI: 10.2478/v10187-011-0020-4, ISSN 1335-3632

accuracy and the current reference signal shaping are addressed in [9]. But not all of the current control law parameters are directly adapted, some of them are considered to be known.

As mentioned above the speed regulator gives the commanded torque. In many cases this commanded torque is converted to the current reference using the so called motor torque constant. Together with considering of the standard time separation principle, when current dynamics is considered to be infinitely fast, the controlled system reduces to motor mechanical subsystem. In this case the main unknown parameters are moment of inertia and viscous friction coefficient. The load torque acts as control input disturbance. Simple example of MRAC in such a case can be found in [10]. In many cases the MRAS based estimators of motor mechanical parameters are used, mainly the load torque estimator see [11, 12, 13].

The Speed and current control loops are designed simultaneously using the MRAC technique in [14].

Generally speaking, the PMSM consist of the two subsystems. The electromagnetical subsystem and the mechanical subsystem. The control algorithms for each of subsystems can be designed separately. In electromagnetical subsystem the following properties are important. The current dynamics has to be relatively fast. The time constant around 1 ms is usually considered. The decoupling of the two current control loops has to be accurate, otherwise the performance of current controllers is degraded. When the torque ripple minimization is required the additional claims to the decoupling block and to the conversion of torque reference to current reference has to be satisfied. Exact values of electromagnetical subsystem parameters are unknown, hard to identify and they can vary with working conditions. Parameters of mechanical subsystem can vary in large range as load changes, mainly the moment of inertia and the load torque. When the speed closed loop dynamics is given for example by the reference model or by the pole placement the standard PI controller may not be able to cope with such a disturbances. Although, some steady state requirements can be still satisfied. But, in high-performance applications also the speed and position dynamics is important.

In this paper the classical theory of the direct Adaptive Control discussed for example in textbooks [15, 16, 17] is used to develop a control algorithm for each PMSM subsystem. Purpose is to examine the resulting closed loop systems properties by means of simulations. A PMSM model widely used in electric drives community is considered as a base for control laws development.

2 MODEL OF PMSM IN dq REFERENCE FRAME

The Permanent Magnet Synchronous Motor with surface mounted magnets and with an isotropic rotor, i.e., its self and mutual inductances of stator windings do not depend on rotor position, has in dq reference frame the form

$$u_d = Ri_d + \frac{\mathrm{d}\psi_d}{\mathrm{d}t} - \omega_e\psi_q\,, \tag{1}$$

$$u_q = Ri_q + \frac{\mathrm{d}\psi_q}{\mathrm{d}t} + \omega_e\psi_d\,, \tag{2}$$

$$\psi_d = L_d i_d + \psi_{d_{rot}}\,, \tag{3}$$

$$\psi_q = L_q i_q + \psi_{q_{rot}}\,, \tag{4}$$

$$M_m = \frac{3}{2}p\frac{P_{mech}}{\omega_e}\,, \tag{5}$$

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = M_m - M_z - B_f\omega\,, \tag{6}$$

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}t} = \omega \tag{7}$$

where ω is the rotational speed of the rotor in rad/s, ϑ is the rotor position measured in radians, p is number of pole-pairs, and $\omega_e = p\omega$, $\vartheta_e = p\vartheta$ are electrical angular speed and position. M_m is the torque produced by the motor, P_{mech} is part of electrical power which is converted to mechanical power, M_z is the load torque, J is the moment of inertia and B_f is the viscous friction coefficient. u_d and u_q are the d-axis and q-axis stator voltages, i_d and i_q are stator currents, R is the stator winding resistance, L_d and L_q are the stator inductances in dq reference frame, and $L_d = L_q$, ψ_d and ψ_q are the total flux linkages and $\psi_{d_{rot}}$ and $\psi_{q_{rot}}$ are the flux linkages established by the permanent magnets.

When the rotor and stator magnetic field distributions are not sinusoidal, the permanent magnet flux linkages can be viewed as the sum of a fundamental component and the series of higher harmonics, see [4, 5]

$$\psi_{d_{rot}} = \psi_{d0} + \psi_{d6}\cos(6\vartheta_e) + \psi_{d12}\cos(12\vartheta_e) + \dots \tag{8}$$

$$\psi_{q_{rot}} = \psi_{q6}\sin(6\vartheta_e) + \psi_{q12}\sin(12\vartheta_e) + \dots \tag{9}$$

where ψ_{d0}, ψ_{d6}, ψ_{d12}, ψ_{q6} and ψ_{q12} are amplitudes of corresponding higher harmonics. For good approximation of the permanent magnet flux linkages it is sufficient to keep only series members up to 12th harmonics.

3 MRAC OF THE CURRENT SUBSYSTEMS

The design of the current controller is based on the electromagnetical subsystem model which is described by equations 1–5. Equations governing the i_d current and i_q current dynamics can be written in the convenient form

$$\frac{\mathrm{d}i_d}{\mathrm{d}t} = -\frac{R}{L_d}i_d + \frac{1}{L_d}u_d + \frac{1}{L_d}\boldsymbol{\Omega}_d^\top\varphi_d\,, \tag{10}$$

$$\frac{\mathrm{d}i_q}{\mathrm{d}t} = -\frac{R}{L_q}i_q + \frac{1}{L_q}u_q - \frac{1}{L_q}\boldsymbol{\Omega}_q^\top\varphi_q \tag{11}$$

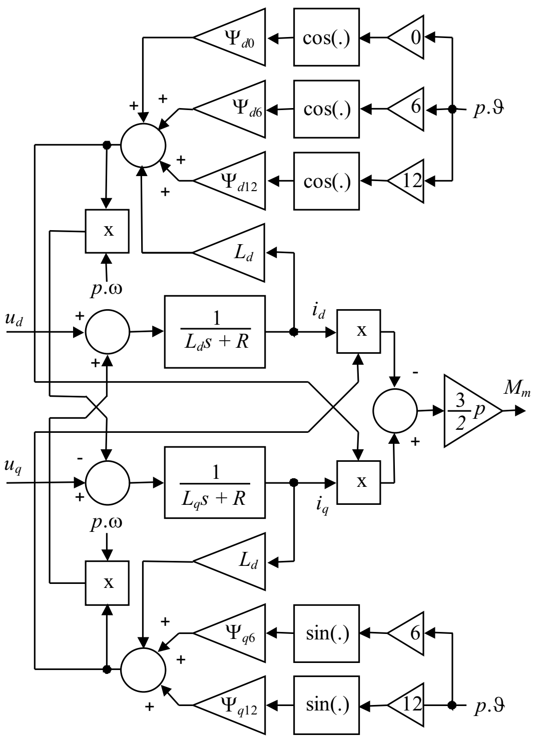

Fig. 1. Block scheme of the PMSM model

where the terms 8 and 9 are used and

$$
\begin{aligned}
\mathbf{\Omega}_d &= [\, L_q \quad \Psi_{q6} \quad \Psi_{q12} \,]^\top , \\
\varphi_d &= [\, \omega_e i_q \quad \omega_e \sin(6\vartheta_e) \quad \omega_e \sin(12\vartheta_e) \,]^\top , \\
\mathbf{\Omega}_q &= [\, L_d \quad \Psi_{d0} \quad \Psi_{d6} \quad \Psi_{d12} \,]^\top , \\
\varphi_q &= [\, \omega_e i_d \quad \omega_e \quad \omega_e \cos(6\vartheta_e) \quad \omega_e \cos(12\vartheta_e) \,]^\top
\end{aligned}
$$

where

$$
\begin{aligned}
\Psi_{d0} &= \psi_{d0} \,, & \Psi_{q6} &= 6\psi_{d6} + \psi_{q6} \,, \\
\Psi_{d6} &= 6\psi_{q6} + \psi_{d6} \,, & \Psi_{q12} &= 12\psi_{d12} + \psi_{q12} \,, \\
\Psi_{d12} &= 12\psi_{q12} + \psi_{d12} \,.
\end{aligned}
$$

The block scheme of the electromagnetical subsystem model is in Fig. 1.

Consider the reference models for each of currents in the form

$$\dot{i}_{dm} = -a_{dm} i_{dm} + b_{dm} r_d \tag{12}$$

$$\dot{i}_{qm} = -a_{qm} i_{qm} + b_{qm} r_q \tag{13}$$

where a_{dm}, b_{dm}, a_{qm} and b_{qm} are constants given by the refence models designer, r_d and r_q are the current reference signals and i_{dm}, i_{qm} are the reference model outputs to be tracked by the actual currents. The control law substitution to equations (10) and (11) has to lead to the closed loop equations of same form as the form of corresponding reference models. Equations (10) and (11) are coupled by last terms. Therefore, the decoupling terms has to be added too.

Consider the control laws in the vector form

$$u_d = \mathbf{\Theta}_d^\top \chi_d \,, \tag{14}$$

$$u_q = \mathbf{\Theta}_q^\top \chi_q \tag{15}$$

where

$$
\begin{aligned}
\mathbf{\Theta}_d &= [\, \Theta_{d1} \quad \Theta_{d2} \quad \Theta_{d3} \quad \Theta_{d4} \quad \Theta_{d5} \,]^\top , \\
\chi_d &= [\, i_d \quad r_d \quad \varphi_d \,]^\top , \\
\Theta_q &= [\, \Theta_{q1} \quad \Theta_{q2} \quad \Theta_{q3} \quad \Theta_{q4} \quad \Theta_{q5} \quad \Theta_{q6} \,]^\top , \\
\chi_q &= [\, i_q \quad r_q \quad \varphi_q \,]^\top .
\end{aligned}
$$

The closed loop system is formed by substituting (14) and (15) to (10) and (11) respectively. Equality of the closed loop systems and reference models (12) and (13) is obtained using the ideal values of control law parameters. The ideal values of these parameters for d-axis current closed loop are $\Theta_{d1}^\star = -a_{dm} L_d + R$, $\Theta_{d2}^\star = L_d b_{dm}$ and $[\, \Theta_{d3}^\star \quad \Theta_{d4}^\star \quad \Theta_{d5}^\star \,]^\top = -\mathbf{\Omega}_d$. Further, for q-axis current closed loop: $\Theta_{q1}^\star = -a_{qm} L_q + R$, $\Theta_{q2}^\star = L_q b_{qm}$ and $\left[\, \Theta_{q3}^\star \quad \Theta_{q4}^\star \quad \Theta_{q5}^\star \quad \Theta_{q6}^\star \,\right]^\top = \mathbf{\Omega}_q$. Since the PMSM parameters are unknown the ideal control law parameters can not be calculated and are replaced by the estimates. An adaptive law to generate the control law parameter estimates on-line has to be determined. In following the adaptive law for d-axis current control loop is derived. The adaptive law for the q-axis current control loop can be derived analogically.

A current tracking error is defined as $e_d = i_d - i_{dm}$. Parametrization of the controlled plant equation (10) in terms of the ideal control law parameters can be done by adding and subtracting the ideal control law term which is in the form $\frac{1}{L_d} \mathbf{\Theta}_d^{\star\top} \chi_d$ where superscripts $\star$ denotes that the vector contains the ideal parameters. Then the plant equation is in the form

$$\dot{i}_d = -a_{dm} i_d + b_{dm} r_d + \frac{1}{L_d} \Big(u_d - \mathbf{\Theta}_d^{\star\top} \chi_d \Big) \,. \tag{16}$$

The tracking error dynamics equation is obtained by subtracting (12) from (16), then

$$\dot{e}_d = -a_{dm} e_d + \frac{1}{L_d} \Big(u_d - \mathbf{\Theta}_d^{\star\top} \chi_d \Big) \,. \tag{17}$$

The use of control law (14) is considered, then (17) can be written in the form

$$\dot{e}_d = -a_{dm} e_d + \frac{1}{L_d} \big(\boldsymbol{\theta}_d^\top \chi_d \big) \tag{18}$$

where the control law parameters estimation error is introduced

$$\boldsymbol{\theta}_d = \mathbf{\Theta}_d - \mathbf{\Theta}_d^\star \,. \tag{19}$$

Notice that the equation (18) can be expressed in the form when the parameters estimation error is related to the current tracking error trough the Strictly Positive Real (SPR) transfer function. This motivates the use of SPR Lyapunov design approach. Consider the candidate for Lyapunov-like function in the form

$$V_d = \frac{e_d^2}{2} + \frac{1}{2L_d}\boldsymbol{\theta}_d^\top \boldsymbol{\Gamma}_d^{-1}\boldsymbol{\theta}_d \tag{20}$$

where $L_d > 0$ because negative inductance is meaningless and $\boldsymbol{\Gamma}_d = \boldsymbol{\Gamma}_d^\top > 0$ is the so called adaptation gain matrix. The time derivative $\dot{V}_d$ along the trajectory of (18) is given by

$$\dot{V}_d = -a_{dm}e_d^2 + e_d\frac{1}{L_d}\boldsymbol{\theta}_d^\top \chi_d + \frac{1}{L_d}\boldsymbol{\theta}_d^\top \boldsymbol{\Gamma}_d^{-1}\dot{\boldsymbol{\theta}}_d\,. \tag{21}$$

Choosing $\dot{\boldsymbol{\theta}}_d = -e_d\boldsymbol{\Gamma}_d\chi_d$ leads to $\dot{V}_d \le 0$. Therefore, the adaptive law which generates the control law parameter estimates on-line is given by

$$\dot{\boldsymbol{\Theta}}_d = -e_d\boldsymbol{\Gamma}_d\chi_d \tag{22}$$

where ideal parameters are considered to be constant or quasi-stationary. Analogically, for the q-axis current control law the adaptive law is given by

$$\dot{\boldsymbol{\Theta}}_q = -e_q\boldsymbol{\Gamma}_q\chi_q\,. \tag{23}$$

With these results it can be shown that all of the signals in the closed-loop systems are bounded, and the tracking error goes to zero as time $t \to \infty$.

The torque produced by the motor can be expressed in the form [4, 5]:

$$\begin{aligned} M_m = {} & \frac{3}{2}p\left((L_d - L_q)\, i_d i_q\right) - \\ & \frac{3}{2}p\left((\Psi_{q6}\sin(6\vartheta_e) + \Psi_{q12}\sin(12\vartheta_e))\, i_d\right) + \\ & \frac{3}{2}p\left((\Psi_{d0} + \Psi_{d6}\cos(6\vartheta_e) + \Psi_{d12}\cos(12\vartheta_e))\, i_q\right) \end{aligned} \tag{24}$$

where the first term has no contribution to the motor torque since $L_d = L_q$. The second term has no contribution to the DC component of the motor torque. Therefore, the zero value of d-axis current is desired, thus $r_d = 0$. The last term in (24) is used to derive the block for the conversion of commanded torque to the q-axis current reference signal. Let a *Current Factor* (CF) be the name of this block. The commanded (desired) torque $M_{\rm W}$ multiplied by the Current Factor gives the q-axis current reference signal: $r_q = CF \cdot M_{\rm W}$ where

$$CF = \frac{1}{\frac{3}{2}p(\Psi_{d0} + \Psi_{d6}\cos(6\vartheta_e) + \Psi_{d12}\cos(12\vartheta_e))} \tag{25}$$

Notice that the estimates of CF parameters Ψ_{d0}, Ψ_{d6} and Ψ_{d12} are already contained in vector $\boldsymbol{\Theta}_q$. Therefore, CF can be implemented with adaptive system derived above.

In the case when the perfect model matching is achieved, so currents tracks the reference models outputs, the following simplification can be assumed: The transfer function from the commanded torque $M_{\rm W}$ to the motor torque M_m is equal to the q-axis current reference model transfer function.

$$\frac{M_m(s)}{M_{\rm W}(s)} = \frac{b_{qm}}{s + a_{qm}}\,. \tag{26}$$

This fast dynamics of electromagnetical subsystem can be then considered in the adaptive speed controller design process instead of considering the infinitely fast dynamics.

4 MRAC OF THE SPEED SUBSYSTEM

In a speed control point of view the controlled system is described by equations in the form

$$\dot{\omega} = -\frac{B_f}{J}\omega + \frac{1}{J}M_m\,, \tag{27}$$

$$\dot{M}_m = -a_{qm}M_m + b_{qm}M_{\rm W} \tag{28}$$

where the load torque M_z is omitted for convenience. Notice that the load torque acts as a disturbance in input of the mechanical subsystem. The controlled system can be expressed in the transfer function form

$$\frac{\omega(s)}{M_{\rm W}(s)} = \frac{b_{qm}}{s + a_{qm}}\frac{1/J}{s + B_f/J} = k_p\frac{1}{s^2 + a_1 s + a_0} \tag{29}$$

where k_p, a_1 and a_0 are time-varying unknown coefficients. The k_p is referred to as the *high frequency gain* and in this case it has a positive sign because the moment of inertia can not be negative.

The reference model of the speed dynamics is chosen to have the same relative degree as (29) in the form

$$\frac{\omega_m(s)}{\omega_r(s)} = W_m(s) = k_m\frac{1}{s^2 + a_{m1}s + a_{m0}} \tag{30}$$

where k_m, a_{m1} and a_{m0} are constants chosen by the reference model designer and ω_r is the speed reference signal.

The Model Reference Control (MRC) problem for SISO (Single Input, Single Output) plants discussed in [15] leads to the use of control law which in this case is in the form

$$\begin{aligned} M_{\rm W}(s) = {} & \Theta_1^\star\frac{1}{(s+\lambda)}M_{\rm W}(s) + \\ & \Theta_2^\star\frac{1}{(s+\lambda)}\omega(s) + \Theta_3^\star\omega(s) + \Theta_4^\star\omega_r(s) \end{aligned} \tag{31}$$

where control law parameters $\Theta_1^\star$, $\Theta_2^\star$, $\Theta_3^\star$ and $\Theta_4^\star$ are given by a solution of the MRC problem and λ is an arbitrary constant which meets the requirement that $(s+$

λ) is a Hurwitz polynomial. By substituting (31) to (29) the closed loop transfer function is obtained in the form

$$\frac{\omega(s)}{\omega_r(s)} = G_c(s) = \frac{k_p\Theta_4^\star(s+\lambda)}{R_p(s)\big((s+\lambda)-\Theta_1^\star\big)-k_p\big(\Theta_2^\star+\Theta_3^\star(s+\lambda)\big)} \tag{32}$$

where $R_p(s) = (s^2 + a_1 s + a_0)$. The MRC objective is the equality of responses obtained from the closed loop system and the reference model. Therefore, the matching equation is $G_c(s) = W_m(s)$. In the known plant parameters case the ideal control law parameters can be calculated from the matching equation, then $\Theta_4^\star = \frac{k_m}{k_p}$; $\Theta_1^\star = a_1 - a_{m1}$; $\Theta_3^\star = \frac{1}{k_p}\big(\lambda(a_1-a_{m1})+(a_0-a_{m0})-a_1\Theta_1^\star\big)$; $\Theta_2^\star = \frac{1}{k_p}\big(\lambda(a_0-a_{m0})+a_0\Theta_1^\star - k_p\lambda\Theta_3^\star\big)$.

The plant (29) can be represented in the state space in the form

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \boldsymbol{b}M_{\rm W}\,, \tag{33}$$
$$\omega = \boldsymbol{c}^\top\boldsymbol{x} \tag{34}$$

where $\boldsymbol{x}$ is the state vector of plant and $\mathbf{A}$; $\boldsymbol{b}$; $\boldsymbol{c}^\top$ are the matrices of coefficients with corresponding dimensions. The control law (31) can be represented in the state space in the form

$$\dot{\nu}_1 = -\lambda\nu_1 + M_{\rm W}\,, \tag{35}$$
$$\dot{\nu}_2 = -\lambda\nu_2 + \boldsymbol{c}^\top\boldsymbol{x}\,, \tag{36}$$
$$M_{\rm W} = \boldsymbol{\Theta}^\top\boldsymbol{D}\boldsymbol{X} + \Theta_4\omega_r \tag{37}$$

where $\mathbf{D} = \operatorname{diag}([\,\boldsymbol{c}^\top \quad 1 \quad 1\,])$ is introduced, $\mathbf{X} = [\,\boldsymbol{x} \quad \nu_1 \quad \nu_2\,]^\top$ is an augmented plant state vector and $\boldsymbol{\Theta} = [\,\Theta_3 \quad \Theta_1 \quad \Theta_2\,]^\top$; Θ_4 are the estimates of ideal control law parameters. Including of the auxiliary states ν_1 and ν_2 to the plant states space representation leads to following equations

$$\dot{\mathbf{X}} = \mathbf{A}_o\mathbf{X} + \mathbf{B}_c M_{\rm W}\,, \tag{38}$$
$$\omega = \mathbf{C}_c^\top\mathbf{X} \tag{39}$$

where

$$\mathbf{A}_o = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ 0 & -\lambda & 0 \\ \boldsymbol{c}^\top & 0 & -\lambda \end{bmatrix}; \quad \mathbf{B}_c = \begin{bmatrix} \boldsymbol{b} \\ 1 \\ 0 \end{bmatrix}; \quad \mathbf{C}_c = \begin{bmatrix} \boldsymbol{c} \\ 0 \\ 0 \end{bmatrix}.$$

The equations (38) and (39) will be called an augmented plant in this paper. Substituting the control law with ideal parameters to equation (38), the ideal closed loop transfer function is obtained in the form

$$\dot{\mathbf{X}} = \mathbf{A}_c\mathbf{X} + \mathbf{B}_c\Theta_4^\star\omega_r\,, \tag{40}$$
$$\omega = \mathbf{C}_c^\top\mathbf{X} \tag{41}$$

where $\mathbf{A}_c = \mathbf{A}_o + \mathbf{B}_c\boldsymbol{\Theta}^{\star\top}\mathbf{D}$, which implies

$$G_c(s) = \mathbf{C}_c^\top(s\mathbf{I}-\mathbf{A}_c)^{-1}\mathbf{B}_c\Theta_4^\star = W_m(s)\,. \tag{42}$$

This also means, that the reference model can be described by the nonminimal state space representation in the form

$$\dot{\mathbf{X}}_m = \mathbf{A}_c\mathbf{X}_m + \mathbf{B}_c\Theta_4^\star\omega_r\,, \tag{43}$$
$$\omega_m = \mathbf{C}_c^\top\mathbf{X}_m \tag{44}$$

where $\mathbf{X}_m$ is the state vector of the reference model nonminimal state space representation.

Next, an adaptive law to generate the speed control law parameter estimates on-line has to be determined. Let $\boldsymbol{e} = \mathbf{X} - \mathbf{X}_m$ and $e_1 = \omega - \omega_m$ to be the speed tracking error. Parametrization of the augmented plant equation (38) in terms of the ideal control law parameters can be done by adding and subtracting the ideal control law in the form $\mathbf{B}_c\boldsymbol{\Theta}^{\star\top}\mathbf{D}\mathbf{X} + \mathbf{B}_c\Theta_4^\star\omega_r$. Then the plant equation is in the form

$$\dot{\mathbf{X}} = \mathbf{A}_c\mathbf{X} + \mathbf{B}_c\Theta_4^\star\omega_r + \mathbf{B}_c(M_{\rm W} - \boldsymbol{\Theta}^{\star\top}\mathbf{D}\mathbf{X} - \Theta_4^\star\omega_r), \tag{45}$$
$$\omega = \mathbf{C}_c^\top\mathbf{X}\,. \tag{46}$$

Subtracting (43) from (45) and (44) from (46) yields to the error equations in the form

$$\dot{\boldsymbol{e}} = \mathbf{A}_c\boldsymbol{e} + \overline{\mathbf{B}}_c\frac{1}{\Theta_4^\star}(M_{\rm W} - \boldsymbol{\Theta}^{\star\top}\mathbf{D}\mathbf{X} - \Theta_4^\star\omega_r)\,, \tag{47}$$
$$e_1 = \mathbf{C}_c^\top\boldsymbol{e} \tag{48}$$

where $\overline{\mathbf{B}}_c = \mathbf{B}_c\Theta_4^\star$ is introduced, so it can be clearly seen that if the control law (37) is substituted to (48) then the control law parameters errors are related to the tracking error through the transfer function $W_m(s)$. For the use of the SPR-Lyapunov design method to determine the adaptive law, the transfer function which relates the control law parameter error to the estimation error has to be SPR. However, $W_m(s)$ can not be made SPR because of its relative degree $n^\star = 2$. Using the identity $(s+\rho)(s+\rho)^{-1} = 1$, for some $\rho > 0$, the equation (48) can be expressed in the form

$$\dot{\boldsymbol{e}} = \mathbf{A}_c\boldsymbol{e} + \overline{\mathbf{B}}_c\frac{1}{\Theta_4^\star}(s+\rho)\big(M_{{\rm W}f} - \boldsymbol{\Theta}^{\star\top}\mathbf{D}\mathbf{X}_f - \Theta_4^\star\omega_{rf}\big) \tag{49}$$

where $M_{{\rm W}f} = (s+\rho)^{-1}M_{\rm W}$, $\mathbf{X}_f = (s+\rho)^{-1}\mathbf{X}$ and $\omega_{rf} = (s+\rho)^{-1}\omega_r$ are a filtered signals. Let $W_m(s)$ and ρ to be chosen so that $W_m(s)(s+\rho)$ is the SPR transfer function and consider the following control law instead of (37) in the form

$$M_{{\rm W}f} = \boldsymbol{\Theta}^\top\mathbf{D}\mathbf{X}_f + \Theta_4\omega_{rf}\,. \tag{50}$$

Substituting the (50) to (49) and introducing $\boldsymbol{\theta} = \boldsymbol{\Theta} - \boldsymbol{\Theta}^\star$ and $\theta_4 = \Theta_4 - \Theta_4^\star$ leads to the error equations in the form

$$\dot{\boldsymbol{e}} = \mathbf{A}_c \boldsymbol{e} + \overline{\mathbf{B}}_c \frac{1}{\Theta_4}^\star (s+\rho)\big(\boldsymbol{\theta}^\top \mathbf{DX}_f + \theta_4 \omega_{rf}\big)\,, \tag{51}$$

$$e_1 = \mathbf{C}_c^\top \boldsymbol{e} \tag{52}$$

or

$$e_1 = W_m(s)(s+\rho)\frac{1}{\Theta_4^\star}\big(\boldsymbol{\theta}^\top \mathbf{DX}_f + \theta_4 \omega_{rf}\big) \tag{53}$$

which means that the SPR-Lyapunov design approach can be used.

The originally considered control law has to be changed because $M_{\mathrm{W}f} = (s+\rho)^{-1} M_{\mathrm{W}}$ which implies $M_{\mathrm{W}} = (s+\rho) M_{\mathrm{W}f}$ which can be written in the form

$$M_{\mathrm{W}} = \dot{\boldsymbol{\Theta}}^\top \mathbf{DX}_f + \boldsymbol{\Theta}^\top \mathbf{DX} + \dot{\Theta}_4 \omega_{rf} + \Theta_4 \omega_r\,. \tag{54}$$

The equations (51) and (52) can be after some rearrangement transformed by introducing $\overline{\boldsymbol{e}} = \boldsymbol{e} - \big(\mathbf{B}_c \frac{1}{\Theta}{}_4^\star \boldsymbol{\theta}^\top \mathbf{DX}_f + \mathbf{B}_c \frac{1}{\Theta_4^\star} \theta_4 \omega_{rf}\big)$ to the form

$$\dot{\overline{\boldsymbol{e}}} = \mathbf{A}_c \overline{\boldsymbol{e}} + \mathbf{B}_1 \frac{1}{\Theta_4^\star}\big(\boldsymbol{\theta}^\top \mathbf{DX}_f + \theta_4 \omega_{rf}\big)\,, \tag{55}$$

$$e_1 = \mathbf{C}_c^\top \overline{\boldsymbol{e}} \tag{56}$$

where $\mathbf{B}_1 = \mathbf{A}_c \overline{\mathbf{B}}_c + \rho \overline{\mathbf{B}}_c$ and the fact that $\mathbf{C}_c^\top \mathbf{B}_c = 0$ is used. Equation (55) is suitable for the Lyapunov-like function design. Consider the Lyaponov-like function in the form

$$V = \frac{1}{2}\overline{\boldsymbol{e}}^\top \boldsymbol{P} \overline{\boldsymbol{e}} + \frac{1}{2}\Big|\frac{1}{\Theta_4^\star}\Big| \boldsymbol{\theta}^\top \boldsymbol{\Gamma}^{-1} \boldsymbol{\theta} + \frac{1}{2}\Big|\frac{1}{\Theta_4^\star}\Big| \gamma^{-1} \theta_4^2 \tag{57}$$

where $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^\top > 0$, $\gamma > 0$ are the design parameters of adaptive law and $\boldsymbol{P} = \boldsymbol{P}^\top > 0$ satisfies the algebraic equation implied by Mayer-Kalman-Yakubovich (MKY) lemma If $W_m(s) = \mathbf{C}_c^\top (s\mathbf{I} - \mathbf{A}_c)^{-1} \mathbf{B}_1$ is SPR, then we can write

$$\mathbf{A}_c^\top \mathbf{P} + \mathbf{P}\mathbf{A}_c = -\boldsymbol{Q}\,, \tag{58}$$

$$\mathbf{P}\mathbf{B}_1 = \mathbf{C}_c \tag{59}$$

where $\mathbf{Q} = \mathbf{Q}^\top > 0$. The time derivate $\dot{V}$ of V along the trajectory of (55) is given by

$$\dot{V} = \overline{\boldsymbol{e}}^\top(-\mathbf{Q})\overline{\boldsymbol{e}} + \frac{1}{\Theta_4^\star}\overline{\boldsymbol{e}}^\top \mathbf{P}\mathbf{B}_1 \boldsymbol{\theta}^\top \mathbf{DX}_f + \frac{1}{\Theta_4^\star}\overline{\boldsymbol{e}}^\top \mathbf{P}\mathbf{B}_1 \theta_4 \omega_{rf} + \Big|\frac{1}{\Theta_4^\star}\Big| \boldsymbol{\theta}^\top \boldsymbol{\Gamma}^{-1} \dot{\boldsymbol{\theta}} + \Big|\frac{1}{\Theta_4^\star}\Big| \gamma^{-1} \theta_4 \dot{\theta}_4\,. \tag{60}$$

The adaptive law results from the requirement that $\dot{V} \le 0$. Choosing $\dot{\boldsymbol{\theta}} = -\operatorname{sgn}(1/\Theta_4^\star) e_1 \boldsymbol{\Gamma} \mathbf{DX}_f$ and $\dot{\theta}_4 = -\operatorname{sgn}(1/\Theta_4^\star) e_1 \gamma \omega_{rf}$ leads to $\dot{V} = \overline{\boldsymbol{e}}^\top(-\mathbf{Q})\overline{\boldsymbol{e}} \le 0$. Therefore, the adaptive law which generates the control law parameters estimates on-line is given by

$$\dot{\boldsymbol{\Theta}} = -\operatorname{sgn}\Big(\frac{1}{\Theta_4^\star}\Big) e_1 \boldsymbol{\Gamma} \mathbf{DX}_f\,, \tag{61}$$

$$\dot{\Theta}_4 = -\operatorname{sgn}\Big(\frac{1}{\Theta_4^\star}\Big) e_1 \gamma \omega_{rf} \tag{62}$$

where $\overline{\boldsymbol{e}}^\top \mathbf{P}\mathbf{B}_1 = e_1$ and $\frac{1}{\Theta_4^\star} = \Big|\frac{1}{\Theta_4^\star}\Big| \operatorname{sgn}\Big(\frac{1}{\Theta_4^\star}\Big)$ is used. The further analysis shows, see [15], that all of the closed-loop system signals are bounded, and the tracking error goes to zero as $t \to \infty$.

5 SIMULATION EXPERIMENTS

The used PMSM model parameters are listed in Table 1. The PMSM rated torque is 0.5 Nm.

The performance of the above designed adaptive control system is studied in the several simulation experiments. In all of the experiments, the same step signal around 50 rad/s is used as a rotational speed reference signal ω_r. The whole experiment takes 0.5 s. The load torque is almost zero at the beginning of the simulation, then a step increase in the load torque to $M_z = 0.25$ Nm at time $t = 0.225$ s is simulated.

The d-axis and q-axis reference model parameters are chosen as follows: $a_{dm} = a_{qm} = 1000$ and $b_{dm} = b_{qm} = 1000$. Thus the desired time constant of the currents dynamics is 1 ms. The desired time constant of the PMSM mechanical subsystem is chosen to be 5 ms. The speed subsystem reference model consist both of desired time constants and its parameters are: $a_{m1} = 1200$, $a_{m0} = 200000$ and $k_m = a_{m0}$.

The auxiliary filters characterized with parameter $\lambda = 500$ and the filters for obtaining filtered signals, such as ω_{rf}, with parameter $\rho = 500$, are designed to be faster relative to the desired mechanical subsystem dynamics.

The ideal control law parameters can be calculated using values in Table 1. The initial values of the control law parameters are chosen to be close to these ideal values.

The used initial control law parameters are: $\boldsymbol{\Theta}_d = [5 \quad 0 \quad 0 \quad 0 \quad 28]$, $\boldsymbol{\Theta}_q = [5 \quad 0 \quad 0.2 \quad 0 \quad 0 \quad 30]$, $\boldsymbol{\Theta} = [0 \quad -200 \quad 0]$ and $\Theta_4 = 0$.

Table 1. Parameters of the PMSM

	Value (Unit)		Value (Wb)
p	2		
R	33.6 (Ω)	Ψ_{d0}	0.303
L_d	0.0284 (H)	Ψ_{d6}	0.0181
L_q	0.0284 (H)	Ψ_{d12}	0.0024
J	0.000016 (kg/m^2)	Ψ_{q6}	0.0036
B_f	8.2×10^{-6} (kg m^2/s)	Ψ_{q12}	0.0022

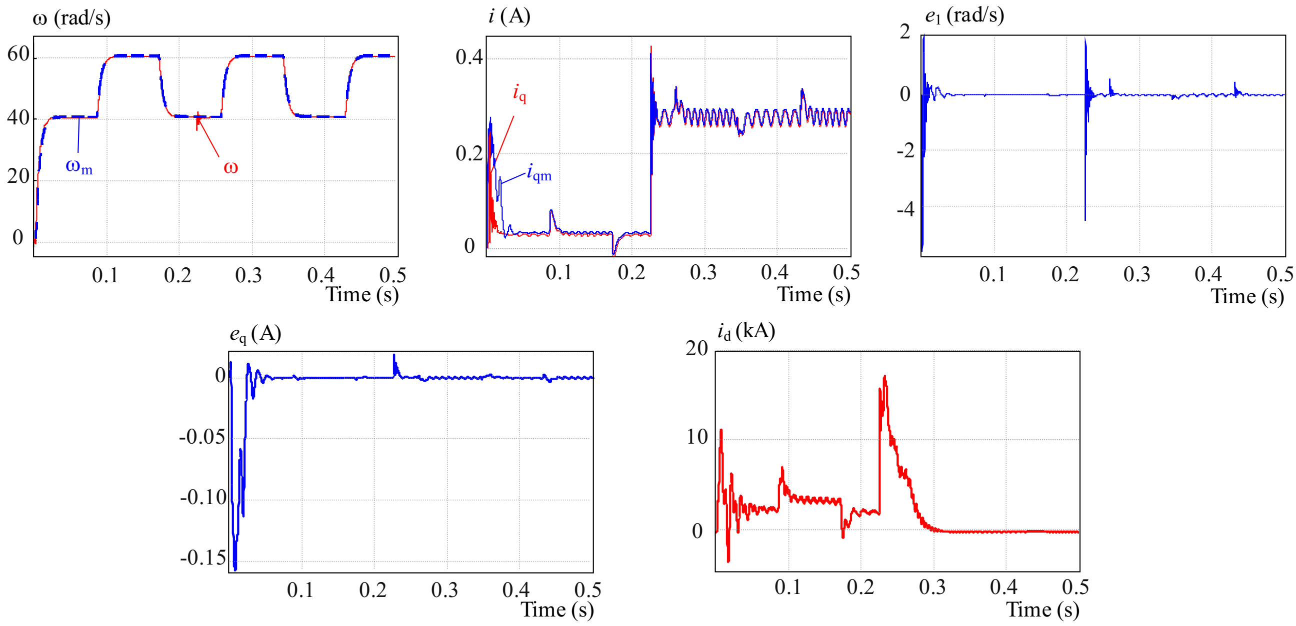

Fig. 2. Experiment 1: All of PMSM parameters remains unchanged

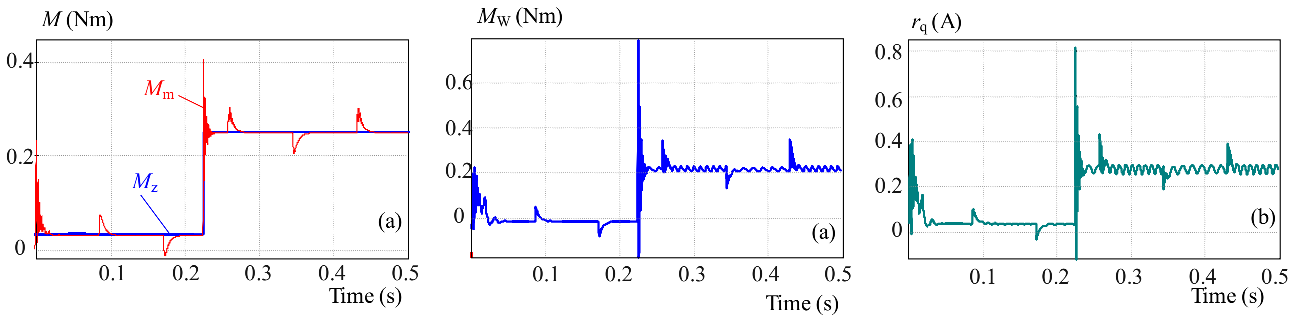

Fig. 3. Experiment 1: The commanded torque M_W, current reference r_q and resultant motor torque M_m

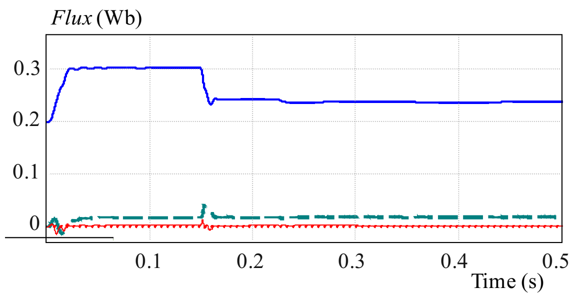

Fig. 4. Experiment 2: Parameters of CF: Ψ_{d0} - thick solid line; Ψ_{d6} - dashed line; Ψ_{d12} - solid line

The adaptation gain matrices has been determined using trial and error method. A good values was found to be:

$\mathbf{\Gamma}_d = \mathrm{diag}\left([2 \quad 2 \quad 0.8 \quad 0.8 \quad 2]\right)$,
$\mathbf{\Gamma}_q = \mathrm{diag}\left([2 \quad 2 \quad 0.8 \quad 0.8 \quad 0.8 \quad 2]\right)$,
$\mathbf{\Gamma} = \mathrm{diag}\left([12 \quad 12 \quad 12]\right)$ and $\gamma = 12$.

Convergence of the parameters is then fast enough with low oscillations.

In the first simulation experiment all of the PMSM model parameters remains unchanged at their nominal values. Therefore, the adaptation process due to the initial control law parameters deviation from the ideal values is expected only at the beginning of simulation. Results of the first simulation experiment is in the Fig. 2 and Fig. 3.

Because the ω is required to be smooth, or ripple-free, the motor torque M_m has to be smooth. However, the smooth motor current i_q leads to ripple in M_m, see (24). To cancel this ripple the CF block is used to generate appropriate r_q. In the ideal case the transfer from r_q to i_q is not infinitely fast, but given by dynamics of q-axis reference model. This causes that the M_W has to contains ripple too. Nevertheless, the result is smooth motor torque M_m. To illustrate this, the simulation results in Fig. 3 are provided.

In the second simulation experiment the parameters variation in the electromagnetical subsystem is simulated. Specifically, the 20% step decrease of parameter Ψ_{d0} at time $t = 0.15$ s and the 80% step decrease of parameter Ψ_{d12} at time $t = 0.3$ s are simulated. The control law

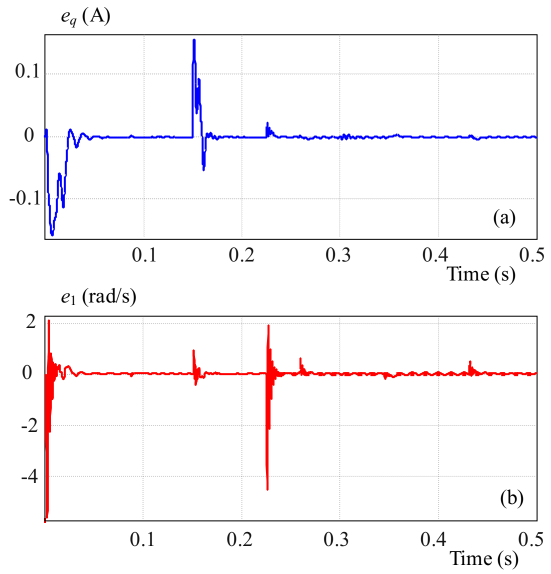

Fig. 5. Experiment 2: Behavior of tracking errors e_q and e_1

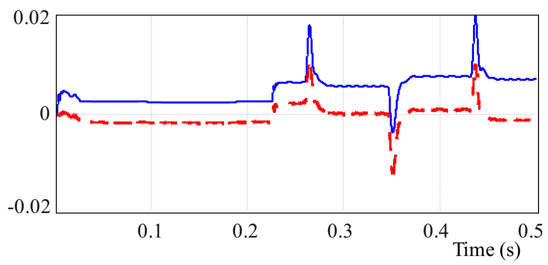

Fig. 6. Experiment 3: Θ_4 - solid line; Θ_3 - dashed line

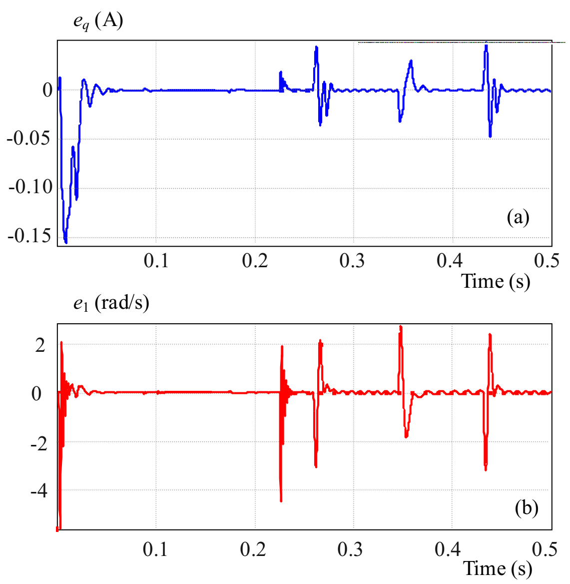

Fig. 7. Experiment 3: Behavior of tracking errors e_q and e_1

parameters which corresponds to these PMSM model parameters are also contained in the CF block.

The evolution of these parameters during the second simulation experiment is in the Fig. 4. The influence of adaptation process is illustrated by means of tracking errors in Fig. 5.

In the third experiment the parameters variation in the mechanical subsystem is simulated. The viscose friction coefficient B_f increases 10 times at $t = 0.1$ s and the moment of inertia J increases 20 times at $t = 0.25$ s. All other parameters of the PMSM model remains unchanged in nominal values. The third experiment simulation results are in Fig. 6 and in Fig. 7. In Fig. 6 only the two of speed control law parameters are shown to simplify the figure. To demonstrate that these adapted parameters are bounded the longer simulation was performed and result is in Fig. 8.

6 CONCLUSION

As evidenced by the simulation results, the theory of the Model Reference Adaptive Control has been successfully applied in the design process of the Permanent Magnet Synchronous Motor control system. The resulting adaptive control loops are able to adapt to changes in the any of PMSM parameters which has been also confirmed by means of simulation experiments.

The adaptive schemes can be also modified to reduce the effect of unmodelled dynamics and external disturbances, see [18, 19].

The designed current adaptive controller and the speed adaptive controller are independent and they can be combined with the other (even non-adaptive) controllers. For example, the conventional PI controller can be used in the speed control loop while the above designed adaptive current controller is used in the current control loop.

On the other hand, searching for the appropriate values of adaptive controllers design parameters, such as the adaptive gain or the auxiliary filters parameter, may be difficult and depends on the particular case.

Acknowledgments

This work has been supported by Slovak scientific grant agency through grant VEGA-1/0592/10.

References

[1] ØVREBØ, S.: Sensorless Control of Permanent Magnet Synchronous Machines, Norwegian University of Science and Technology, Trondheim, 2004.

[2] ANDERSSON, S.: Optimization of a Servo Motor for an Industrial Robot Application, Lund University: Universitetstryckeriet, 2000.

[3] KRAUSE, P. C.—WASYNCZUK, O.—SUDHOFF, S. D.: Analysis of Electic Machinery and Drive Systems, Second Edition, IEEE Press, 2002.

[4] PETROVIC, V.—STANKOVIC, A. M.: Modeling of PM synchronous motors for Control and Estimation Tasks, IEEE Proc. on Decision and Control **3** (2001), 2229–2234.

[5] WALLMARK, O.: Control of a Permanent Magnet Synchronous Motor with Non-Sinusoidal Flux Density Distribution, Diploma Thesis, Department of Electric Power Engineering Chalmers University of Technology, Gteborg, 2001.

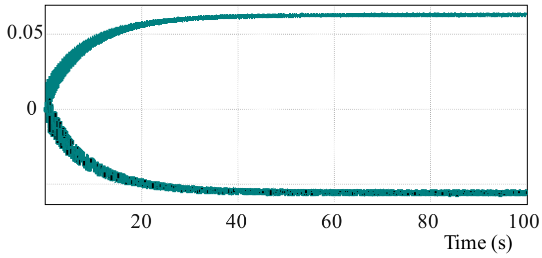

Fig. 8. Experiment 3: – Θ_4 - solid line; Θ_3 - dashed line

[6] KIM, K. H.: Model Reference Adaptive Control-Based Adaptive Current Control Scheme of a PM Synchronous Motor with an Improved Servo Performance, IET Electr. Power Appl. **3** No. 1 (2009), 8-18.

[7] MOHAMED, Y. A-R. I.—EL-SAADANY, E. F.: A Current Control Scheme with an Adaptive Internal Model for Torque Ripple Minimization and Robust Current Regulation in PMSM Drive Systems, IEEE Trans. on Energy Conversion **23** No. 1 (March 2008), 92–100.

[8] QIAN, W.—NONDHAL, A. T.: Mutual Torque Suppression of Surface-Mounted Permanent Magnet Synchronous Motor, Proceedings of the Eighth International Conference on Electrical Machines and Systems, vol. 1, 2005, pp. 315-320.

[9] PETROVIC, V.—ORTEGA, R.—STANKOVIC, A. M.—TADMOR, G.: Design and Implementation of an Adaptive Controller for Torque Ripple Minimization in PM Synchronous Motors, IEEE Trans. on Power Electronics **15** No. 5 (Sep 2000), 871–880.

[10] SHIXIONG, Z.—YOUGUO, P.:: Model Reference Adaptive Control Application Study in PM Synchronous Motor Servo System, 2nd International Conference on Power Electronics and Intelligent Transportation System (PEITS), vol. 1, 2009, pp. 428–432.

[11] KO, J.—YOUN, S.—KIM, Y.:: A Robust Adaptive Precision Position Control of PMSM, Industry Applications Conference, 37th IAS Annual Meeting, vol. 1, Pittsburgh, PA , USA, 2002, pp. 120–125.

[12] LIU, T.-H.—PU, H.-T.—LIN, C.-K.:: Implementation of an Adaptive Position Control System of a Permanent-Magnet Synchronous Motor and its Application, IET Electr. Power Appl. **4** No. 2 (2010), 121-130.

[13] YANG, M.—WANG, X.—ZHENG, K.: Adaptive Backstepping Controller Design for Permanent Magnet Synchronous Motor, Proceedings of the 8th World Congress on Intelligent Control and Automation, China, July 6-9 2010.

[14] GOLEA, N.—GOLEA, A.—KADJOUDJ, M.: Robust MRAC Adaptive Control of PMSM Drive under General Parameters Uncertainties, IEEE International Conference on Industrial Technology, 2006, pp. 1533–1537.

[15] IOANNOU, P.—SUN, J.: Robust Adaptive Control, Prentice Hall, Inc, 1996.

[16] IOANNOU, P.—FIDAN, B.: Adaptive Control Tutorial, Society for Industrial and Applied Mathematics, USA, 2006.

[17] TAO, G.: Adaptive Control Design and Analysis, John Wiley & Sons, Inc, 2003.

[18] MIKLOVIČOVÁ, E.—MURGAŠ, J.—GONOS, M.: Reducing the Effect of Unmodelled Dynamics by MRAC Control Law Modification, 16th IFAC World Congress, Czech Republic, 2005, pp. Tu-M02–TP/11.

[19] POLIAČIK, M.—MURGAŠ, J.—FARKAS, Ľ.—BLAHO, M.: A robust MRAC Modification and Performance Improvement in the Presence of Uncertainties, IFAC International Workshops on Adaptation and Learning in Control and Signal Processing, Turkey, 2010.

Received 13 November 2010

Marián Tárník (Ing) was born in Levice, Slovakia in 1986. He received Bachelor of Science degree from the Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava in 2008 and Master of Science degree in technical cybernetics in 2010 at the same place where he is from 2010 until now doing further studies to get his PhD title. He is interested in Adaptive Control, Lyapunov design approach and network control.

Ján Murgaš (Prof, Ing, PhD), born in 1951, graduated in control engineering in 1975 and received the PhD degree in 1980 form the Faculty of Electrical Engineering, Slovak University of Technology in Bratislava. Since 1996 he has been Full Professor for control engineering. His research interests include adaptive and non-linear control, large-scale systems. He is a member of the IEEE, of the American Mathematical Society and of the Slovak Society for Cybernetics and Informatics.