

# COST-EFFICIENT PHASE NOISE MEASUREMENT

Ana Perić<sup>\* \*\*</sup> — Milan Bjelica<sup>\*</sup>

In this paper, an automated system for oscillator phase noise measurement is described. The system is primarily intended for use in academic institutions, such as smaller university or research laboratories, as it deploys standard spectrum analyzer and free software. A method to compensate the effect of instrument intrinsic noise is proposed. Through series of experimental tests, good performances of our system are verified and compliance to theoretical expectations is demonstrated.

**Keywords:** phase noise, spectrum analyzer, python

## 1 INTRODUCTION

The impact of phase noise on signal corruption has been fully recognized only recently. Since then, characterization of receiver local oscillator in terms of intrinsic phase noise has become an important step in providing optimal operating conditions for advanced communication systems based on the OFDM [1–3]. Even small research and academic laboratories need to follow this trend by providing sufficient capabilities for phase noise measurements. As specialized measurement systems due to their high price may not be easily available to them, additional funding might be needed, which is often a time consuming process; a preferred alternative would be to reconfigure the existing instruments to perform basic phase noise measurements with adequate accuracy.

In this paper, we describe an automated phase noise measurement system which is based on a spectrum analyzer — a standard instrument present in virtually all communication laboratories, and open-source software. We start by reviewing the basics of phase noise theory and measurement. We continue by describing our proposal. We analyze in depth the systematic error induced by instrument intrinsic noise and propose a method to improve measurement accuracy. Finally, we present experimental results which verify the performances of our proposal.

## 2 FUNDAMENTALS OF PHASE NOISE MEASUREMENT

The output of a real oscillator is given with

$$V(t) = (V_0 + \epsilon(t)) \sin(2\pi f_0 t + \phi(t)), \quad (1)$$

where  $\epsilon(t)$  is deviation from the nominal amplitude  $V_0$ , and  $\phi(t)$  is phase deviation from the nominal phase  $2\pi f_0 t$ . In the frequency domain, oscillator phase instability is

characterized by one-sided spectral density of phase fluctuations [4],

$$S_\phi(f) = \frac{\Phi^2(f)}{B}, \quad (2)$$

where  $B$  is measurement system bandwidth. Let us note that although  $S_\phi(f)$  is one-sided, it includes fluctuations from both the upper and the lower sideband of the carrier.

The prevailing measure of phase noise among manufacturers and users of frequency standards is one half of (2), expressed in dBc/Hz units

$$\mathcal{L}(\Delta f) = 10 \log \frac{S_\phi(f_0 + \Delta f)}{2}. \quad (3)$$

Here,  $\Delta f$  is frequency offset from the nominal carrier frequency  $f_0$ .

A survey of various phase noise measurement techniques is given in [5]. The proposed circuits deploy components like low-noise mixers, correlators, or phase shifters, which may not be available in budget-restricted laboratories where the use of standard equipment is preferred. Luckily, in most practical applications oscillator amplitude fluctuations are small enough so that the only source of signal corruption is phase instability [6]; this is easily achieved by *eg* negative feedback or output amplitude-limiting circuit. In this case, (1) describes a narrowband phase modulated signal, so that (3) can be rewritten as

$$\mathcal{L}(\Delta f) = 10 \log \frac{P_{sb}(f_0 + \Delta f)|_{B=1 \text{ Hz}}}{P_c}, \quad (4)$$

where,  $P_{sb}(f_0 + \Delta f)$  at  $B = 1$  Hz is the single-sideband power at frequency offset  $\Delta f$ , with a measurement bandwidth of 1 Hz, while  $P_c$  is the carrier power. This definition leads us to a following simple method for phase noise measurement: If carrier level  $L_c$  is known and signal level within a 1 Hz bandwidth at frequency offset  $\Delta f$  is  $L(\Delta f)$  at  $B = 1$  Hz, then phase noise level (in dBc/Hz) at that frequency offset is

$$\mathcal{L}(\Delta f) = L(\Delta f)|_{B=1 \text{ Hz}} - L_c, \quad (5)$$

<sup>\*</sup> University of Belgrade, School of Electrical Engineering, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia, <sup>\*\*</sup> Cisco Systems GmbH Hallbergmoos, Germany, anaperic@gmail.com, milan@etf.rs

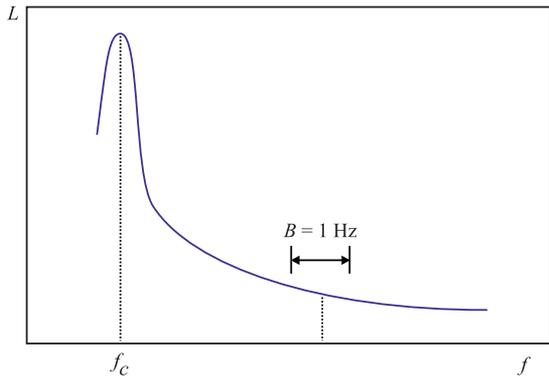


Fig. 1. Definition of oscillator phase noise

which is illustrated in Fig. 1. Let us note that phase noise is defined in [7, Eq. 6-3] with permuted terms on the right side of this equation.

In practical applications, it would be more convenient to measure signal level within bandwidth  $B > 1$  Hz. In this case, phase noise level would be

$$\mathcal{L}(\Delta f) = L(\Delta f, B) - L_c - 10 \log \frac{B}{1 \text{ Hz}}. \quad (6)$$

### 3 PHASE NOISE MEASUREMENT WITH SPECTRUM ANALYZER

When requirements on the dynamic range are not stringent, the aforescribed idea can be straightforwardly implemented by using the spectrum analyzer. As these instruments are easily found in most research facilities, this is certainly a cheaper and more attractive alternative to the use of specialized noise measuring systems.

When spectrum analyzer is used for phase noise measurement, measurement bandwidth  $B$  in (6) is determined by the noise bandwidths of its resolution and video filters. The noise bandwidth of a filter is usually given in the analyzer technical specifications and can be related to the 3 dB filter bandwidth. As in most applications the resolution bandwidth does not exceed the video bandwidth, the following will apply

$$\mathcal{L}(\Delta f) = L(\Delta f, B) - L_c - 10 \log \frac{B_{RBW}}{1 \text{ Hz}} + \beta, \quad (7)$$

where  $B_{RBW}$  is 3 dB resolution bandwidth and  $\beta$  factor dependent on both filter and detector type. To simplify the measurement procedure, most spectrum analyzers available nowadays feature noise marker functions which allow direct readout of the phase noise.

There are two fundamental prerequisites to obtain valid measurement results by spectrum analyzer [7]:

a) the intrinsic phase noise of the analyzer must be lower than the phase noise originated from the oscillator being tested, and

b) the frequency drift of the oscillator must be smaller than the sweep time of the analyzer.

Let us now discuss these two conditions in more detail. Until recently, the first condition has been the limiting factor for the application of spectrum analyzers in phase noise measurements, especially in the vicinity of the carrier frequency, as the first local oscillator was a predominant source of phase noise within an analyzer. To investigate the impact of analyzer intrinsic phase noise on the measured value, let us assume that a signal given with (1) is fed to an analyzer with instantaneous phase instability  $\phi_{sa}(t)$ . Assuming that the amplitude of the input signal is small enough not to overload the input of the analyzer, its first mixer will operate as a linear frequency converter, feeding the IF stage with signal

$$V_{IF}(t) = V_{IF,0} \cos(2\pi f_{IF}t + \phi_{sa} - \phi(t)). \quad (8)$$

If we further assume that  $\phi_{sa}(t)$  and  $\phi(t)$  are uncorrelated random processes, their variances will add up, so the effect of phase noise will be additive by nature. The total spectral density of phase fluctuations will then be

$$S_{\phi, tot}(f) = S_{\phi, sa}(f) + S_{\phi}(f). \quad (9)$$

This expression enables us to improve the measurement accuracy by correcting the readout for the noise introduced by the instrument. If analyzer intrinsic phase noise  $\mathcal{L}_{sa}(\Delta f)$  is known, then phase noise of the tested signal can be determined from the total (*ie* measured) value  $\mathcal{L}_{tot}(\Delta f)$  as

$$\mathcal{L}(\Delta f) = 10 \log(10^{\mathcal{L}_{tot}(\Delta f)/10} - 10^{\mathcal{L}_{sa}(\Delta f)/10}). \quad (10)$$

A convenient way of obtaining the data on analyzer intrinsic phase noise could be during the instrument calibration in a reference laboratory.

As the analyzer sweep time is inversely proportional to squared resolution bandwidth, *ie*  $T_{sweep} \sim (B_{RBW})^{-2}$ , condition b) could be easily met by setting the resolution bandwidth small enough. However, longer sweep times mean that more time will be needed to conduct a measurement, which will then be prone to environmental noise disturbances.

## 4 SYSTEM DESCRIPTION

As it is often of interest to make phase noise measurements within a wide frequency range, manual measurements comprising of marker positioning, data readout and correction would be time consuming. A logical alternative would be to automate the measurement procedure by coupling a spectrum analyzer to a PC.

Our measurement system is based on Rohde & Schwarz FSC6 spectrum analyzer. As this device belongs to a lower price class, it is a good candidate for budget-restricted institutions.

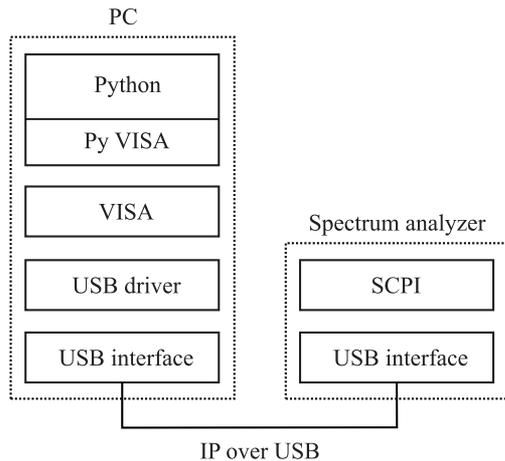


Fig. 2. Logical components of measurement system

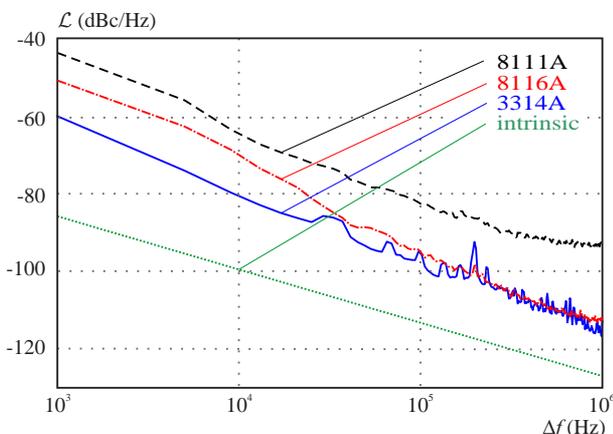


Fig. 3. Measured phase noise levels for tested function generators

We automated the measurement procedure by using Python 2.7.2 programming language, with its modules PyLab and PyVISA. This free software tool offers not only the functionalities of controlling the equipment of different types and vendors, but also provides fast and reliable data acquisition and display of the measured values, which could then be saved for future processing. The separation of data processing — which is now performed by a PC — from data logging (left to the instrument) further enables the creation of higher-level virtual instrumentation using the existing pieces of hardware [8] thus adding new value and capabilities to it and avoiding the need (and costs) of buying new equipment.

Logical organization of our system is shown in Fig. 2. PyVISA module acts as an interface between Python and VISA (Virtual Instrument Software Architecture), a widely used software interface for communicating with the instruments connected to a PC. Once an IP emulated over USB connection has been established, the instrument is communicated to by SCPI (Standard Commands for Programmable Instruments) command set.

Measurement procedure comprises of the following steps:

- 1 Open the connection,
- 2 Initialize the instrument,

- 3 Invoke two markers,
- 4 Position marker #1 onto a carrier frequency,
- 5 Position marker #2 (delta) onto a given frequency offset from carrier,
- 6 Query the delta marker for phase noise level,
- 7 Modify the readout using (10),
- 8 Increase the frequency offset,
- 9 Repeat the steps 5–7 until the desired frequency range is swept,
- 10 Store the results,
- 11 Close the connection.

To increase the measurement accuracy and minimize the effects of other types of noise (*eg* thermal or impulse), a series of 32 consecutive sweeps is performed in step 2 and their results are averaged before the marker readout. A special care has been taken in programming the queries to the analyzer, so to reduce the time needed to conduct a measurement.

Technical specifications of the analyzer we used declare that its intrinsic phase noise typically ranges from  $-105$  dBc/Hz at frequency offset  $\Delta f = 30$  kHz to  $-127$  dBc/Hz at  $\Delta f = 1$  MHz, for carrier frequency  $f_c = 500$  MHz. One can expect these values to be even lower for lower carrier frequencies. Phase noise levels at other frequency offsets can be estimated by using log-linear interpolation [9], *ie*

$$\mathcal{L}(\Delta f) = a \log(\Delta f) + b, \quad (11)$$

with  $a = -14.45$  and  $b = -40.31$ .

Complete Python code is given in the Appendix.

## 5 EXPERIMENTAL RESULTS

To test our system, we measured phase noise of three function generators, Hewlett-Packard 8111A, 8116A, and 3314A. Each generator was connected to the analyzer directly, without any attenuating device. Output signal was set to sine, carrier frequency to 15 MHz, and amplitude to 160 mVpp, which was a maximum value that would not overload the input stage of the analyzer. To stabilize their operating conditions, the generators were left to run on room temperature for about an hour prior to the actual measurements. Analyzer resolution bandwidth was set to 1 kHz, which proved to be a good tradeoff between the precision and the speed, as the time needed to complete a sweep was 6 s; this should be fair enough to satisfy the condition b).

Experimental results are given in Fig. 3. For the reason of comparison, the extrapolated intrinsic phase noise level of the applied analyzer is plotted as a dotted line.

Let us begin the discussion by noting that the measured values fall well above the estimated noise level introduced by the analyzer. Of three tested generators, HP 3314A performed the best, starting with the phase

noise level of  $-60$  dBc/Hz at 1 kHz carrier offset. Spurious peaks visible at higher frequency offsets are probably due to generator internal clock instability and analyzer intrinsic noise.

The average slope of the obtained curves is about  $-20$  dB/dec, which is in accordance with phase noise theoretical model presented in [10]; this clearly validates our system.

## 6 CONCLUSION

We developed and experimentally verified a system for phase noise measurement. Thanks to the use of standard laboratory equipment (*ie* spectrum analyzer) and open-source software, our proposal is simple and cost efficient, but at the same time accurate as well; these features make it especially suitable for use in the budget-constrained laboratories. The effect of instrument intrinsic noise is compensated by applying the proposed method. We emphasize the fact that our system relies on standardized instructions set for communicating to the measuring device, so it is cross-platform compatible, which means that it could be easily adapted to any compliant analyzer.

Two issues remain for our future work. For the reasons of simplicity, we measured the phase noise only in the upper sideband, *ie* for positive frequency offsets  $\Delta f$ . A system could be easily modified to calculate the phase noise as average of the readouts in both upper and lower sideband. Secondly, we have observed that the particular analyzer we used measures the noise within bandwidth  $B$  positioned asymmetrically to the marker frequency. It might be useful to investigate the methods for compensation of this asymmetry, which could further improve the overall accuracy of the obtained results.

## Acknowledgements

The authors wish to thank Prof. Predrag Pejović from Belgrade School of Electrical Engineering and Mr Petar Pavasović from Rohde & Schwarz representative office Belgrade for their continuous support in conducting this research.

This work was partially supported by Serbian Ministry of Education and Science, through grant TR 32048.

## REFERENCES

- [1] ARMADA, A. G.: Understanding the Effects of Phase Noise in Orthogonal Frequency Division Multiplexing (OFDM), IEEE Trans. Broadcasting **47** No. 2 (June 2001), 153–159.
- [2] ZHANG, Y—LIU, H.: MIMO-OFDM Systems in the Presence of Phase Noise and Doubly Selective Fading, IEEE Trans. Vehicular Tech. **56** No. 4, Part 2 (July 2007), 2277–2285.
- [3] BARAN, O.—KASAL, M.: Modeling of the Phase Noise in Space Communication Systems, Radioengineering **19** No. 1 (Apr 2010), 141–148.
- [4] IEEE Std 1139-2008 — IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology — Random Instabilities, IEEE, 2009.
- [5] WALLS, W. F.: Practical Problems Involving Phase Noise Measurements, Proc. 33rd Annual Precise Time and Time Interval (PTTI) Meeting 2001, pp. 407–416.
- [6] HAJIMIRI, A.—LEE, T. H.: A General Theory of Phase Noise in Electrical Oscillators, IEEE J. Solid-State Circuits **33** No. 2 (Feb 1998), 179–194.
- [7] RAUSCHER, C.—JANSSEN, V.—MINIHOLD, R.: Fundamentals of Spectrum Analysis, Rohde & Schwarz, 2008.
- [8] HUGHES, J. M.: Real World Instrumentation with Python, O'Reilly Media, 2011.
- [9] LEE, T. H.—HAJIMIRI, A.: Oscillator Phase Noise: A Tutorial, IEEE J. Solid-State Circuits **35** No. 3 (Mar 2000), 326–336.
- [10] LEESON, D. B.: A Simple Model of Feedback Oscillator Noise Spectrum, Proc. IEEE, **54** No. 2 (Feb 1966), 329–330.

## Appendix: Python code

The following code is applicable to FSC6 spectrum analyzer, but with minor changes could be adjusted to other SCPI-compatible analyzers as well.

```

from pylab import *
import visa

filename = 'results'

# establish a connection over a pre-defined IP address:
fsc = visa.instrument('TCPIP::172.16.10.10::INSTR')

n = 500                                # number of points
df = (1e6 - 1e3)/(n-1)                 # frequency increment

fsc.write('DISP:TRAC:Y:ADJ') # autoscale the vertical axis
fsc.write('SWE:COUN 34')     # 34 consecutive sweeps
fsc.write('INIT:CONT OFF')   # turn on the single sweep mode
fsc.write('DISP:TRAC:MODE AVER')

                                # turn on the trace averaging
# start the measurement and wait for the end of the sweep:
fsc.write('INIT;*WAI')

fsc.write('CALC:MARK1 ON')    # turn on marker 1
# position marker 1 on the current trace maximum:
fsc.write('CALC:MARK1:MAX')

# turn on the frequency counter at marker 1:
fsc.write('CALC:MARK1:COUN ON;*WAI')

fsc.write('CALC:DELT2 ON')   # turn on delta marker
# position delta marker 1 kHz relative to marker 1:
fsc.write('CALC:DELT2:X:REL 1kHz')

# turn on the phase noise measurement:
fsc.write('CALC:MARK2:FUNC:NOIS ON')

f=[]                                # frequency offset vector
nl=[]                                # phase noise vector

for i in range(n):
    f.append(1000 + df*i)          # new frequency offset
    # position delta marker on new frequency:
    fsc.write('CALC:DELT2:X:REL '+str(f[i])+';*WAI')
    # query delta marker for phase noise:
    pn = float(fsc.ask('CALC:MARK2:FUNC:NOIS:RES?'))
    # modify the readout for intrinsic noise:
    nl.append(10*log10(10**(pn/10)-10**(-1.445*log10(f[i])-4.031)))

save(filename +'_f.npy', f)        # frequency offsets
save(filename +'_nl.npy', nl)     # phase noise levels
fsc.close()                        # close the connection

```

Received 11 September 2012

Ana Perić, Milan Bjelica, biographies not supplied.