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Bi-directional relay network employing signal space diversity

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In this work, signal space diversity (SSD) method is exploited for a simple two-way relay channel (TWRC) with two end users mutually exchanging information. In order to ensure simplicity, the same rotation angle and interleavers are used under SSD framework. Also the well known decode-and-forward (DF) and amplify-and-forward (AF) protocols are adapted for the proposed TWRC. The analytical symbol error rate (SER) performance of the proposed scheme is evaluated for DF and AF protocols individually and evaluated through simulation results. The proposed scheme is shown to improve the performance of TWRC in terms of SER for both two protocols.

K e y w o r d s: two-way relay channel, decode and forward, amplify and forward, signal space diversity

1 Introduction

Communication networks connect the sources of information with each other through a common channel. Although the data is transmitted through the same link, they are detected individually at the receiving end. In this context, two-way communication channel (TWCC) can be defined as a transmission medium through which the sources at the end nodes exchanges information. In the pioneering work [1] in which TWCC channel is introduced by Shannon, the achievable information rates are investigated. One of the factors that impacts the quality of this type of channel is the distance between sources. Whenever a secure communication between the two sources could not be established due to distance or if the security is to be increased, one or more relays can be used in order to support the communication. This type of bi-directional communication channel assisted with relay(s) is named as two-way relay channel (TWRC) [2] and may be assumed as a general extension of Shannons TWCC.

In parallel with the cooperative communication [3] research in the literature, there exist considerable works on TWRC [4-10]. Those proposed systems either employ a single relay or multiple relays between the end users. The transmission protocols are implemented in either three phase or two phase. In the former case, the sources transmit their signals to the relay(s) in the first two time slots and the relay(s) broadcast a joint information signal to the sources back in the final third time slot. Alternatively, the two phase protocol consists of multiple access and broadcast phases in which the relay receives a superimposed signal initially and then generates and transmits a broadcast signal in the second time slot. When the forwarding protocols at the relay(s) are investigated, we observe that the signals are relayed after either being decoded [4-5], amplified [6,7,12], or denoised [8]. These protocols are named as amplify-and-forward (AF), decode-and-forward (DF) and denoise-and-forward (DNF), respectively. There also exists hybrid type of DF-AF protocols adapted for TWRC [9]. An extension of TWRC, named as physical layer network coding (PLNC) [10-12], is the counterpart of networking coding at the physical layer. In PLNC, the relay either combines the source symbols by a simple XOR operation or via the linear combination. For all the different systems designed for TWRC, performance criteria is either error probability, outage probability or achievable rates. The early works on TWRC consider half duplex schemes in which a single node cannot transmit and receive a signal at the same time. But lately, the need for high data rates and bandwidth efficiency in 5G systems requires the full duplex operation for TWRC [13] in which any node can simultaneously transmit and receive signals. In a recent work on TWRC, the non-orthogonal multiple access technique is applied for two-way DF multiple relay network [14].

Signal space diversity (SSD) or also known as modulation diversity technique enables higher throughput in fading scenarios without any additional bandwidth and power requirements [15]. Provided that n-dimensional constellation is applied, its possible to attain nth order diversity by rotating the constellation and interleaving the identical coordinates within themselves separately. In this way individual coordinates of a single signal are enabled to fade independently. Specifically if the widely applied two-dimensional modulation schemes like Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) are considered, SSD can be implemented by rotating each element in the signal block by a certain constant phase and interleaving the in-phase and quadrature components separately, so that the two components can experience independent fading. The performance enhancement via SSD in fading channels is presented in

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[16]. SSD is exploited for cooperative diversity as well. In [17] a technique named signal space cooperation is introduced and SSD is implemented distributively through an expanded constellation. Specifically the source message is partitioned to be transmitted directly from source at the broadcast phase and from the relay at the relay phase. SSD enables different components of each member of original signal to be affected by independent channel fading. Hence, SSD is shown to outperform its counterparts like the distributed turbo coded cooperative schemes and trans-modulation scheme. Bit error probability (BEP) analysis is evaluated in [18] for a cooperative system combined with signal space diversity. The theoretical analysis is validated with simulation results and a significant BEP gain over traditional cooperative scheme without the need for additional power or bandwidth. There exist a few works that propose the application of SSD for TWRC. In a recent work [19], the authors extended the SSD system designed for cooperative communication for TWRC. The proposed scheme is shown to enhance both the spectral efficiency and reliability. In another work [20], following the scheme in [17], a multi-relay TWRC employing DF based SSD is presented. Again it has been shown that performance and spectral efficiency are enhanced. In this work, a communication system employing SSD together with network coding is proposed for TWRC. For a three node bi-directional communication system consisting of two single antenna sources and an assisting relay node, employing SSD results with out-performance which may be regarded as the main contribution of this work. For DF and AF protocols, a time division multiplexed transmission scheme is assumed and the corresponding mathematical definitions of the signals are expressed. The overall symbol error rate (SER) is obtained both analytically and through simulations. Furthermore, the simulations are repeated in order to determine the best rotation angle for SSD.

We provide the general framework of two way relay network and introduce the system model together with the applied transmisson protocols. Further, we present the proposed TWRC employing SSD using the signalling schemes within. The details of the analytical performance bound on symbol error rate are given and the performance of proposed system is evaluated through simulations results.

2 System model and transmission protocols

The considered communication system consists of two sources (A and B) at each end of the link and a single relay (R), all equipped with single antenna (Figure 1). It is assumed that no direct link exists between the sources and the relay is at equal distances to both sources. The multiplicative fading coefficient between the nodes k and l is denoted by h_{kl} where $\{k,l\} \in \{A,B,R\}$ and assumed as Rayleigh distributed random variable with zero mean and unit variance. The probability density function (pdf) of fading coefficient is given as $f(h) = 2he^{-h^2}$. All channel coefficients are assumed to be known at respective receiver nodes but not at the transmitters. Similarly additive white Gaussian noise (AWGN) between the nodes k and l is represented by $n_{k,l}$ and assumed to be a Gaussian distributed random variable with zero mean and variance N_0.

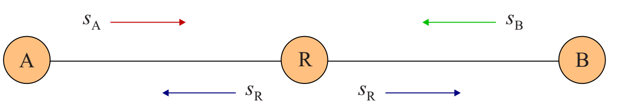

Fig. 1. Two-way relay channel

2.1 Decode-and-forward

Let the symbols for the sources A and B be s_A and s_B respectively, where $s_A = s_{AI} + js_{AQ}$ and $s_B = s_{BI} + js_{BQ}$ are the in-phase/quadrature components. Both two sources use the same constellation, i.e. $\{s_A, s_B\} \in \mathcal{S}_M$, where $\mathcal{S}_M$ is the set of all possible transmitted symbols of sources and M is size of the signal constellation. Assuming orthogonal source-relay channels, the signals received at the relay can be defined as

$$y_{\mathrm{AR}} = h_{\mathrm{AR}} s_A \sqrt{P_{\mathrm{A}}} + n_{\mathrm{AR}}, \tag{1}$$

$$y_{BR} = h_{\mathrm{BR}} s_B \sqrt{P_B} + n_{\mathrm{BR}}, \tag{2}$$

where P_A and P_B are the transmitted power for sources A and B respectively. Using abbreviations

$$\alpha_{\mathrm{AR}} = h_{\mathrm{AR}}\sqrt{P_{\mathrm{A}}}, \quad \alpha_{\mathrm{BR}} = h_{\mathrm{BR}}\sqrt{P_{\mathrm{B}}}.$$

The relay decodes the source signals individually using maximum likelihood method as

$$\hat{s}_{\mathrm{A}} = \arg\min_{\tilde{s}_{\mathrm{A}} \in \mathcal{S}} |y_{\mathrm{AR}} - \alpha_{\mathrm{AR}} \tilde{s}_{\mathrm{A}}|, \tag{3}$$

$$\hat{s}_{\mathrm{B}} = \arg\min_{\tilde{s}_{\mathrm{B}} \in \mathcal{S}} |y_{\mathrm{AB}} - \alpha_{\mathrm{BR}} \tilde{s}_{\mathrm{B}}|. \tag{4}$$

Based on a pre-defined mapping rule, the relay maps these two estimates of source symbols into a single symbol, s_R, and transmits this signal back to sources. Consequently, the signals received at the sources A and B can be defined, using again: $\alpha_{\mathrm{RA}} = h_{\mathrm{RA}}\sqrt{P_{\mathrm{R}}}$ and $\alpha_{\mathrm{RB}} = h_{\mathrm{RB}}\sqrt{P_{\mathrm{R}}}$ as,

$$y_{RA} = \alpha_{\mathrm{RA}} s_{\mathrm{R}} + n_{\mathrm{RA}}, \tag{5}$$

$$y_{RB} = \alpha_{\mathrm{RB}} s_{\mathrm{R}} + n_{\mathrm{RB}}. \tag{6}$$

The maximum likelihood (ML) detection rule at the sources requires finding the minimum metric defined as,

$$\hat{s}_{\mathrm{RA}} = \min_{\tilde{s}_{\mathrm{R}} \in \mathcal{R}} |y_{RA} - \alpha_{\mathrm{RA}} \tilde{s}_{\mathrm{R}}|, \tag{7}$$

$$\hat{s}_{\mathrm{RB}} = \min_{\tilde{s}_{\mathrm{R}} \in \mathcal{R}} |y_{RB} - \alpha_{\mathrm{RB}} \tilde{s}_{\mathrm{R}}|, \tag{8}$$

where $\mathcal{R}$ is the set of all symbols of QPSK. The sources can easily decode the message of other user from $\hat{s}_{RA}$ and $\hat{s}_{RB}$ already knowing their own message and the mapping rule applied at the relay.

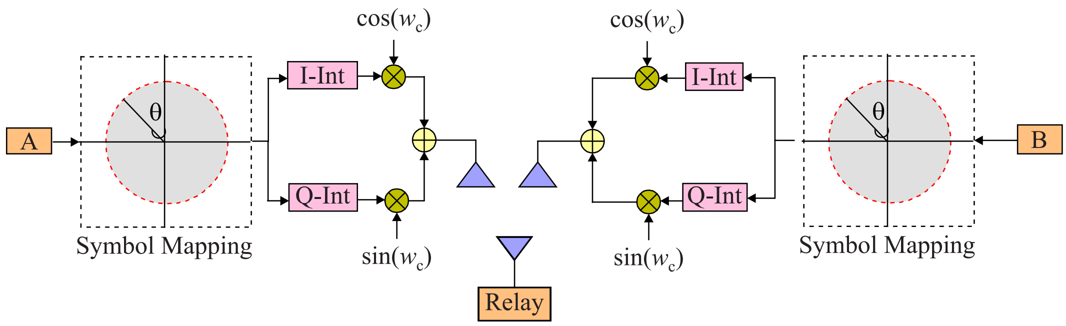

Fig. 2. Block diagram of transmission between the sources and relay

2.2 Amplify-and-forward

In AF, the relay only normalizes the received signal with a normalization factor that depends on the received power and retransmits back to sources. In this case, the transmitted signal from the relay to sources is defined as,

$$s_{\mathrm{R}} = \beta \left(\alpha_{\mathrm{AR}} s_{\mathrm{A}} + \alpha_{\mathrm{BR}} s_{\mathrm{B}} + n_{\mathrm{AR}} + n_{\mathrm{BR}} \right), \tag{9}$$

where the normalization constant, β, is,

$$\beta = \sqrt{\frac{P_{\mathrm{R}}}{\alpha_{\mathrm{AR}}^2 + \alpha_{\mathrm{BR}}^2 + N_0}}. \tag{10}$$

Consequently, the received signals at the sources can be expressed same as (5) and (6). When decoding other users signal, the sources cancel the self-information from the received signal. In this case, the estimates of the original source symbols, $\hat{s}_A$ and $\hat{s}_B$, where $\{\hat{s}_A, \hat{s}_B\} \in \mathcal{S}_M$, are obtained at B and A nodes respectively as,

$$\hat{s}_{\mathrm{A}} = \underset{\tilde{s}_{\mathrm{A}} \in \mathcal{S}_{\mathrm{M}}}{\operatorname{argmin}} \left| y_{\mathrm{RB}} - \beta h_{\mathrm{RB}} \left(\alpha_{\mathrm{AR}} \tilde{s}_{\mathrm{A}}, + \alpha_{\mathrm{BR}} s_{\mathrm{B}} \right) \right|, \tag{11}$$

$$\hat{s}_{\mathrm{A}} = \underset{\tilde{s}_{\mathrm{A}} \in \mathcal{S}_{\mathrm{M}}}{\operatorname{argmin}} \left| y_{\mathrm{RA}} - \beta h_{\mathrm{RA}} \left(\alpha_{\mathrm{BR}} \tilde{s}_{\mathrm{B}} + \alpha_{\mathrm{AR}} s_{\mathrm{A}} \right) \right|. \tag{12}$$

It can be seen that in AF mode, the ML detection directly outputs the transmitted symbol .

3 TWRC with signal space diversity

Assuming a two-dimensional signal constellation, traditional SSD is achieved by rotating each element of the signal set by a constant angle and then interleaving each dimension separately. In this way, it is possible to obtain two orthogonal channels that fade independently. Let $\mathcal{S}_M^\theta$ define the rotated version of original signal set, $\mathcal{S}_M$, by a phase of θ. Then $\mathcal{S}_M^\theta$, is obtained mathematically by employing a transformation matrix

$$\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \tag{13}$$

to $\mathcal{S}_M$. As a result, the Euclidean distances between the in-phase and quadrature components of two different elements of the rotated constellation become,

$$d_I^2 = \left(\cos(\phi_1 + \theta) - \cos(\phi_2 + \theta) \right)^2, \tag{14}$$

$$d_Q^2 = \left(\sin(\phi_1 + \theta) - \sin(\phi_2 + \theta) \right)^2. \tag{15}$$

Here ϕ_1 and ϕ_2 are phase angles of the signals. These Euclidean distances directly impact on the bit error rate performance of the system, so it is of basic importance to figure out the best rotation angle. Considering the attainable gain of SSD at fading channels, it is reasonable to exploit SSD for TWRC. Consequently, we search for a general framework and transmission scheme for network coded TWRC that has AF and DF protocols together with SSD. For both protocols the sources apply SSD before transmission and use the same rotated constellation and interleaver. In this case, the overall system diagram will be as in Fig. 2.

If the rotated signals transmitted from the sources are defined as, $s_i^\theta = s_{iI}^\theta + j s_{iQ}^\theta$, assuming perfect channel state information, the received signal at the relay for DF protocol is,

$$r_{AR} = \sqrt{P_{\mathrm{A}}} \left[h_{\mathrm{AR}}^I \left(s_{\mathrm{AI}}^\theta \right)_{\mathrm{int}} + j h_{\mathrm{AR}}^Q \left(s_{\mathrm{AQ}}^\theta \right)_{\mathrm{int}} + n_{\mathrm{AR}} \right], \tag{16}$$

$$r_{BR} = \sqrt{P_{\mathrm{B}}} \left[h_{\mathrm{BR}}^I \left(s_{\mathrm{BI}}^\theta \right)_{\mathrm{int}} + j h_{\mathrm{BR}}^Q \left(s_{\mathrm{BQ}}^\theta \right)_{\mathrm{int}} + n_{\mathrm{BR}} \right]. \tag{17}$$

Here $(.)_{\mathrm{int}}$ represents the interleaving operation. The diversity gain can be observed from (12) since the in-phase and the quadrature components of the transmitted signal are in effect of independent fading. The received signal at the relay is deinterleaved initially, and then decoded using the rotated constellation. The estimated signals are mapped and then modulated. Before transmitting back to the sources, the relay applies SSD. Hence, the received signals at the sources are deinterleaved and decoded.

For the other case of AF, the superposition signal does not undergo a detection process at the relay but only amplified and send back to sources. Therefore SSD is not applied at the relay for AF protocol. In this case the received signal at the sources can be mathematically defined as,

$$r_{\mathrm{AR}} = h_{\mathrm{RA}}\beta\Big[\sqrt{P_{\mathrm{A}}}\left(h^{\mathrm{I}}_{\mathrm{AR}}\left(s^{\theta}_{\mathrm{AI}}\right)_{\mathrm{int}} + jh^{\mathrm{Q}}_{\mathrm{AR}}\left(s^{\theta}_{\mathrm{AQ}}\right)_{\mathrm{int}}\right) + \sqrt{P_{\mathrm{B}}}\left(h^{\mathrm{I}}_{\mathrm{BR}}\left(s^{\theta}_{\mathrm{BI}}\right)_{\mathrm{int}} + jh^{\mathrm{Q}}_{\mathrm{BR}}\left(s^{\theta}_{\mathrm{BQ}}\right)_{\mathrm{int}}\right) + n_{\mathrm{AR}} + n_{\mathrm{BR}}\Big] + n_{\mathrm{RA}}, \tag{18}$$

$$r_{\mathrm{BR}} = h_{\mathrm{RB}}\beta\Big[\sqrt{P_{\mathrm{A}}}\left(h^{\mathrm{I}}_{\mathrm{AR}}\left(s^{\theta}_{\mathrm{AI}}\right)_{\mathrm{int}} + + jh^{\mathrm{Q}}_{\mathrm{AR}}\left(s^{\theta}_{\mathrm{AQ}}\right)_{\mathrm{int}}\right) + \sqrt{P_{\mathrm{B}}}\left(h^{\mathrm{I}}_{\mathrm{BR}}\left(s^{\theta}_{\mathrm{BI}}\right)_{\mathrm{int}} + jh^{\mathrm{Q}}_{\mathrm{BR}}\left(s^{\theta}_{\mathrm{BQ}}\right)_{\mathrm{int}}\right) + n_{\mathrm{AR}} + n_{\mathrm{BR}}\Big] + n_{\mathrm{RB}}. \tag{19}$$

4 Performance analysis

In order to evaluate the performance of the proposed scheme, the closed form expression for SER for a single user is obtained for both DF and AF protocols. For DF protocol, since the relay employs a detection process, there exists a possibility of an error propagation to the sources. Consequently, the overall error rate is defined as combination of events depending on whether an erroneous detection has occurred at relay or not. On the other hand, the since the function of relay is only to forward the source signals, noise amplification is in effect and the end-to-end link between users can be handled as cascaded fading channel.

4.1 Average symbol-error-rate for DF

The overall error rate for a single user in DF protocol depends on a possible erroneous detection at relay. The only case for end-to-end correct transmission requires no errorneous decoding at both the relay node and receiving source node. Hence, the overall symbol error probability, defined in [11], can be expressed as,

$$\begin{aligned} P^{\mathrm{DF}}_{\mathrm{e}} &= \\ &P(\text{relay error}) \times P(\text{source correct}) + \\ &+ P(\text{relay correct}) \times P(\text{source error}) + \\ &P(\text{relay error}) \times P(\text{source error}). \end{aligned} \tag{20}$$

where "correct" is related to "correct decoding". The last term in (20) can be ignored when compared with the first two error terms. In the following analysis, the error probability for the link $A \to R \to B$ will be given but it can be easily revised for the link $B \to R \to A$, by only changing the indices. If we refer the error probabilities at relay and source B as ${P_{\mathrm{e}}}^{\mathrm{R}}$ and ${P_{\mathrm{e}}}^{\mathrm{B}}$ respectively, the overall symbol error probability is upper bounded as,

$$\begin{aligned} P^{\mathrm{DF}}_{\mathrm{e}} &= P^{\mathrm{R}}_{\mathrm{e}}\left(1 - P^{\mathrm{B}}_{\mathrm{e}}\right) + \left(1 - P^{\mathrm{R}}_{\mathrm{e}}\right)P^{\mathrm{B}}_{\mathrm{e}} = \\ &= P^{\mathrm{R}}_{\mathrm{e}} + P^{\mathrm{B}}_{\mathrm{e}} - 2P^{\mathrm{R}}_{\mathrm{e}}P^{\mathrm{B}}_{\mathrm{e}}. \end{aligned} \tag{21}$$

Since SSD is employed at both sources and relay in DF protocol, the ${P_e}^R$ and ${P_e}^B$ can be expressed simply with the predefined point-to-point error rate for SSD links. Following the procedure given in [15], ${P_e}^R$ can be upper bounded as,

$$P^{\mathrm{R}}_{\mathrm{e}} \leq \frac{1}{M}\sum_{s_{\mathrm{A}} \in S^{\theta}_{\mathrm{M}}}\ \sum_{\substack{\hat{s}_{\mathrm{A}} \in S^{\theta}_{\mathrm{M}} \\ s_{\mathrm{A}} \neq \hat{s}_{\mathrm{A}}}} P\left(s_{\mathrm{A}} \to \hat{s}_{\mathrm{A}}\right), \tag{22}$$

where $P\left(s_A \to \hat{s}_A\right)$ is the pairwise error probability (PEP) that the relay decides $\hat{s}_A$ although s_A is transmitted and calculated as,

$$P(s_{\mathrm{A}} \to \hat{s}_{\mathrm{A}}) = \frac{1}{\pi}\int_0^{\infty} \frac{\sin^2(\psi)}{\left(\sin^2(\psi) + \frac{\bar{\gamma}_{\mathrm{AR}} d^2_{\mathrm{AI}}}{2}\right)} \times \frac{\sin^2(\psi)}{\left(\sin^2(\psi) + \frac{\bar{\gamma}_{\mathrm{AR}} d^2_{\mathrm{AQ}}}{2}\right)} \mathrm{d}\psi, \tag{23}$$

where d^2_{AI} and d^2_{AQ} are Euclidean distances of rotated constellation of source A which are already defined in (14) and (15). Here $\gamma_{\mathrm{AR}} = \frac{P_{\mathrm{A}}|h_{\mathrm{AR}}|^2}{N_0}$ is the instantaneous SNR for the $A \to R$ link and $\bar{\gamma}_{\mathrm{AR}}$ is the average value of γ_{AR}. Since h_{AR} follows a Rayleigh distribution, ??? follows an exponential distribution.

Considering the broadcast links from relay to sources, the probability of an erroneous detection at the individual source can be again expressed using the equations (22) and (23). Considering the $R \to B$ link, the probability for source B to make an incorrect decision for relay signal can be expressed as,

$$P^{\mathrm{R}}_{\mathrm{e}} \leq \frac{1}{M}\sum_{s_{\mathrm{R}} \in R^{\theta}}\ \sum_{\substack{\hat{s}_{\mathrm{R}} \in R^{\theta} \\ s_{\mathrm{R}} \neq \hat{s}_{\mathrm{R}}}} P\left(s_{\mathrm{R}} \to \hat{s}_{\mathrm{R}}\right), \tag{24}$$

where

$$P(s_{\mathrm{R}} \to \hat{s}_{\mathrm{R}}) = \frac{1}{\pi}\int_0^{\infty} \frac{\sin^2(\psi)}{\left(\sin^2(\psi) + \frac{\bar{\gamma}_{\mathrm{RB}} d^2_{\mathrm{RI}}}{2}\right)} \times \frac{\sin^2(\psi)}{\left(\sin^2(\psi) + \frac{\bar{\gamma}_{\mathrm{RB}} d^2_{\mathrm{RQ}}}{2}\right)} \mathrm{d}\psi. \tag{25}$$

As discussed in section 2, source B decodes the data of source A jointly using $\hat{s}_{\mathrm{R}}$ and self information s_{B}. Since all the nodes are assumed to use the same constellation and the same rotation angle, all the Euclidean distances will be equal, *ie* $d^2_{\mathrm{AI}} = d^2_{\mathrm{BI}} = d^2_{\mathrm{RI}}$ and $d^2_{\mathrm{AQ}} = d^2_{\mathrm{BQ}} = d^2_{\mathrm{RQ}}$.

4.2 Average symbol error rate for AF

The link between the end users of TWRC in AF protocol can be viewed as cascaded two hop fading channel. The end-to-end performance of Rayleigh fading AF TWRC channels has been investigated in various works. Either an average error rate or outage probability analysis is carried out in these works. Since the relay forwards the superimposed signals from end users in AF mode, the first task in obtaining either the SER or the outage performance is to determine the SNR values of end-to-end links. For the proposed scheme, let us investigate the SER performance from a single source point of view in AF mode, specifically for $A \rightarrow R \rightarrow B$ link. Let be

$$\gamma^{I}_{\mathrm{AB}} = \frac{P_{\mathrm{A}} d^2_{\mathrm{I}} \gamma^{\mathrm{I}}_{\mathrm{AR}} P_{\mathrm{R}} \gamma_{\mathrm{RB}}}{(P_{\mathrm{A}}+P_{\mathrm{R}})\gamma_{\mathrm{RB}} + P_{\mathrm{A}} d^2_{\mathrm{I}} \gamma^{\mathrm{I}}_{\mathrm{AR}}},$$

and

$$\gamma^{Q}_{\mathrm{AB}} = \frac{P_{\mathrm{A}} d^2_{\mathrm{Q}} \gamma^{\mathrm{Q}}_{\mathrm{AR}} P_{\mathrm{R}} \gamma_{\mathrm{RB}}}{(P_{\mathrm{A}}+P_{\mathrm{R}})\gamma_{\mathrm{RB}} + P_{\mathrm{A}} d^2_{\mathrm{Q}} \gamma^{\mathrm{Q}}_{\mathrm{AR}}}.$$

The overall SNR of the cascaded link, $A \rightarrow R \rightarrow B$, can be defined as [6],

$$\gamma_{\mathrm{AB}} = \gamma^{I}_{\mathrm{AB}} + \gamma^{Q}_{\mathrm{AB}}. \tag{26}$$

It can be stated the independent SNRs for in-phase and quadrature components appear as γ^{I}_{AR} and $\gamma^{\mathrm{Q}}_{\mathrm{AR}}$ respectively as a result of SSD in $A \rightarrow R$ link. The probability density function (PDF) of $\gamma^{\mathrm{I}}_{\mathrm{AB}}$ can be found with the help of [6, (17)] denoting: $a = \dfrac{P_{\mathrm{A}}+P_{\mathrm{R}}}{d^2_{\mathrm{I}} P_{\mathrm{A}} P_{\mathrm{R}}}$, $b = 2\sqrt{\dfrac{a}{\bar{\gamma}_1 \bar{\gamma}_2}}$ and $c = (\dfrac{1}{\bar{\gamma}_1} + \dfrac{a}{\bar{\gamma}_2})$, where $\gamma_1 = P_{\mathrm{R}} \gamma_{\mathrm{RB}}$, $\gamma_2 = \gamma^{I}_{\mathrm{AR}}$

$$f^{I}_{\mathrm{AB}}(\gamma) = e^{-c\gamma} \gamma \left(b K_0(b\gamma) c K_1(b\gamma) \right), \tag{27}$$

where, $K_\nu(.)$ is the ν-th order modified Bessel function of the second kind. In order to obtain $f^{Q}_{\mathrm{AB}}(\gamma)$, all the $\gamma^{\mathrm{I}}_{\mathrm{AR}}$ and d^2_{I} values are simply replaced with $\gamma^{\mathrm{Q}}_{\mathrm{AR}}$ and d^2_{Q}.

For the reverse link, *ie* $A \rightarrow R \rightarrow B$, the overall SNR, γ_{BA}, can be expressed as

$$\gamma_{\mathrm{BA}} = \gamma^{I}_{\mathrm{BA}} + \gamma^{Q}_{\mathrm{BA}}, \tag{28}$$

$$\gamma^{I}_{\mathrm{BA}} = \frac{P_{\mathrm{B}} d^2_{\mathrm{I}} \gamma^{\mathrm{I}}_{\mathrm{BR}} P_{\mathrm{R}} \gamma_{\mathrm{RA}}}{(P_{\mathrm{A}}+P_{\mathrm{R}})\gamma_{\mathrm{RA}} + P_{\mathrm{B}} d^2_{\mathrm{I}} \gamma^{\mathrm{I}}_{\mathrm{BR}}},$$

and

$$\gamma^{Q}_{\mathrm{BA}} = \frac{P_{\mathrm{B}} d^2_{\mathrm{Q}} \gamma^{\mathrm{Q}}_{\mathrm{BR}} P_{\mathrm{R}} \gamma_{\mathrm{RA}}}{(P_{\mathrm{A}}+P_{\mathrm{R}})\gamma_{\mathrm{RB}} + P_{\mathrm{B}} d^2_{\mathrm{Q}} \gamma^{\mathrm{Q}}_{\mathrm{BR}}},$$

and the PDF of γ^{I}_{BA} can again be calculated using (27) with modifications $a = \dfrac{P_{\mathrm{A}}+P_{\mathrm{R}}}{d^2_{\mathrm{I}} P_{\mathrm{B}} P_{\mathrm{R}}}$, $\gamma_1 = P_R \gamma_{RA}$ and $\gamma_2 = \gamma^{I}_{BR}$. Likewise the previous case, all the γ^{I}_{BR} and d^2_{I} values should be simply replaced with γ^{Q}_{BR} and d^2_{Q} in order to calculate the PDF of γ^{Q}_{BA}.

The overall SER value for the $A \rightarrow B$ link can simply be calculated through the moment generation function (MGF) approach in which the probability of error is calculated as,

$$P_{\mathrm{e}} = \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\gamma_{\mathrm{AB}}} \left(-\frac{k^2}{2\sin^2\theta} \right) \mathrm{d}\theta, \tag{29}$$

where the constant k is a specific value that depends on the type and order of modulation and equals to 1 for QPSK type of modulation. $\Phi_{\gamma_{\mathrm{AB}}}(s)$ is the MGF of γ_{AB}. Remembering that MGF is simply the expected value of exponential of the random variable, $\Phi_{\gamma_{\mathrm{AB}}}(s)$ can be written as,

$$\begin{aligned} \Phi_{\gamma_{\mathrm{AB}}}(s) = \mathrm{E}_{\gamma_{\mathrm{AB}}}\left[e^{-s\gamma_{\mathrm{AB}}} \right] = \\ = \int_0^\infty e^{-s\gamma^{\mathrm{I}}_{\mathrm{AB}}} f^{I}_{\mathrm{AB}}(\gamma)\mathrm{d}\gamma \int_0^\infty e^{-s\gamma^{\mathrm{Q}}_{\mathrm{AB}}} f^{Q}_{\mathrm{AB}}(\gamma)\mathrm{d}\gamma \\ = \Phi_{\gamma^{I}_{\mathrm{AB}}}(s) \Phi_{\gamma^{Q}_{AB}}(s). \end{aligned} \tag{30}$$

In order to derive a closed form expression of $\Phi_{\gamma_{\mathrm{AB}}}(s)$, the result of the definite integral of special functions generated by Bessel functions, see (6.621.3) in [21], is given as

$$\begin{aligned} \int_0^\infty x^{\mu-1} e^{-cx} K_\nu(bx)\mathrm{d}x = \\ = \frac{\sqrt{\pi}(2b)^\nu}{(c+b)^{\mu+\nu}} \frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma\left(\mu+\frac{1}{2}\right)} \times \\ \times {}_2F_1\left(\mu+\nu, \nu+\frac{1}{2}; \mu+\frac{1}{2}; \frac{c-b}{c+b}\right), \end{aligned} \tag{31}$$

where ${}_2F_1(a,b;c;d)$ is the Gauss hypergeometric function. Using predefined $a, \bar{\gamma}_1$ and $\bar{\gamma}_2$ values in (27), we set $c = \dfrac{1}{\bar{\gamma}_1} + \dfrac{a}{\bar{\gamma}_2}$ and $b = 2\sqrt{\dfrac{a}{\bar{\gamma}_1 \bar{\gamma}_2}}$ and obtain $\Phi^{I)}_{\gamma_{\mathrm{AB}}}(s)$ as,

$$\begin{aligned} \Phi_{\gamma^{I}_{\mathrm{AB}}}(s) = \frac{4b^2}{3(c+b+s)^2}\, {}_2F_1\left(2, \frac{1}{2}; \frac{5}{2}; \frac{c-b+s}{c+b+s}\right) + \\ + \frac{32b^2c}{3(c+b+s)^3}\, {}_2F_1\left(3, \frac{3}{2}; \frac{5}{2}; \frac{c-b+s}{c+b+s}\right). \end{aligned} \tag{32}$$

The derivation of the expression for $\Phi_{\gamma^{\mathrm{Q}}_{\mathrm{AB}}(s)}$ is straightforward using the same approach. As mentioned before, the c and b values in $\Phi_{\gamma^{\mathrm{Q}}_{\mathrm{AB}}(s)}$ will use updated c, γ_1 and γ_2 values of $c = \dfrac{P_{\mathrm{A}}+P_{\mathrm{R}}}{d^2_{\mathrm{I}} P_{\mathrm{B}} P_{\mathrm{R}}}$, $\gamma_1 = P_{\mathrm{R}} \gamma_{\mathrm{RA}}$ and $\gamma_2 = \gamma^{\mathrm{I}}_{\mathrm{BR}}$. Having obtained the MGF of received SNR value, the final step in order to obtain the SER is simply to insert (32) in the definite integral defined in (29) and evaluate it. To the best of our knowledge, there exists no closed form solution for the integral in (29), so we use numerical integration in order to obtain SER values.

5 Simulation Results

In order to evaluate the performance of the proposed scheme, simulations are performed for fading channels. The end-to-end SER performance of individual user is obtained with respect to E_s/N_0, where E_s is the symbol energy. QPSK modulation is assumed to be used at both the source nodes and at the relay. The Rayleigh fading channel coefficients are assumed to be reciprocal, *ie* $h_{kl} = h_{lk}$ where $\{k, l\} \in \{A, B, R\}$. All the transmit powers are assumed to be equal and chosen as unity, *ie* $P_A = P_B = P_R = 1$. Frequency flat, Rayleigh fast fading channel is assumed and the block size is chosen as 512. It assumed that the receivers have perfect channel state information while transmitters do not. Additionally, the noise power is assumed to be equal at all the links in the system. In order to determine the best rotation angle used in SSD block, the simulations are iterated for different rotation angles.

The simulation results of the proposed TWRC employing SSD under DF protocol framework are presented at Fig. 3. Since employing SSD for TWRC results with the same data rate and bandwidth with the original scheme, the performance of the proposed system is compared with the one that does not employ SSD, *ie* basic TWRC. It can be stated that the SSD achieves an additional gain of around 10 dB for TWRC at a SER value of $\times 10^{-3}$. It can be observed that the optimum rotation angle for DF protocol is between 5° and 25°, which is determined to be 14° with repeated simulations for different angles.

The sources extract the data of the opponent after de-interleaving and decoding the received signal following the method given in part 2.2.

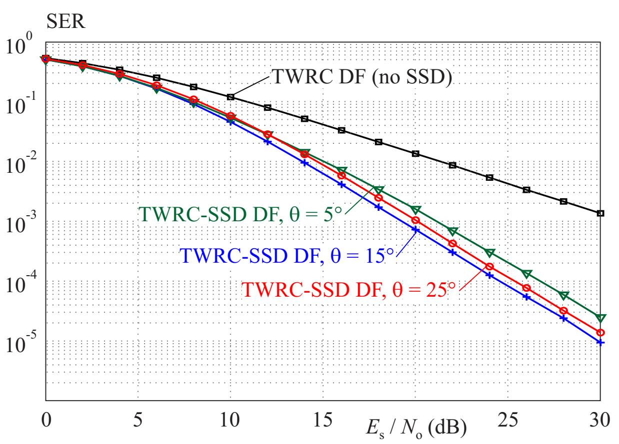

Fig. 3. SER performance for DF protocol

Considering the results for AF protocol given in Fig. 4, the gain attained by the proposed TWRC with SSD scheme is around 8 dB at a SER value of 4×10^{-3}. On the other hand, the optimum value of the rotation angle is found out to be 16° for AF. It can also be stated that DF protocol is superior to AF since the latter protocol might cause an error propagation from relay to the end sources nodes.

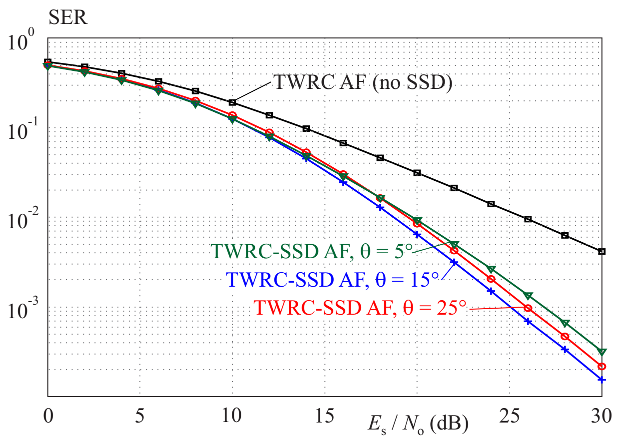

Fig. 4. SER performance for AF protocol.

The evaluation of the theoretical performance bounds obtained analytically for both DF and AF protocols are depicted in Fig. 5. The rotation angle is chosen as 15° and the SER values obtained through simulations are compared with theoretical upper bounds. It is observed that the analytical bounds have common behavior with simulation results for both DF and AF protocols and can be regarded as a consistent upper bound for both protocols.

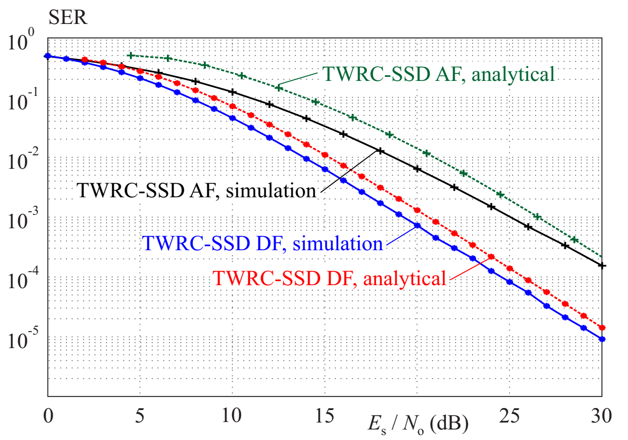

Fig. 5. Analytical performance of TWRC MD scheme

6 Conclusion

In this work, a possible implementation of signal space diversity method for two-way relay channel is investigated. The AF and DF transmission protocols are jointly applied under SSD framework for TWRC. The end-to-end SER performance is determined first analytically and then evaluated through simulations. The theoretical results are in accordance with simulation results. The rotation angle, an important parameter that determines the performance of SSD, is determined by repeated simulations. The results reveal that SSD enables diversity gain for fading TWRC, similar to other cooperative schemes.

The overall error performance is enhanced by incorporating SSD in TWRC. As a future work, the to the theoretical bound on SER can be optimized by incorporating the rotation angle parameter in the analysis. onal wadc in the face of communication uncertainties.

References

[1] C. E. Shannon, "Two-way communication channels, ", *Proc. 4th Berkeley Symp. Mathematical Statistics Probability*, Berkeley, CA, 1961, pp. 611644.

[2] S. J. Kim, N. Devroye, and V. Tarokh, "A class of bi-directional multi-relay protocols", *2009 IEEE International Symposium on Information Theory*, Seoul, 2009, pp. 349-353.DOI: 10.1109/ISIT .2009.5205684.

[3] N. J. Laneman, D. N. C. Tse, and G. Wornell, "Cooperative Diversity Wireless Networks: Efficient Protocols and Outage Behaviour", *IEEE Trans. Information Theory*, vol. 50, no.12 pp. 3062 3080, 2004.DOI: 10.1109/TIT.2004.838089.

[4] S. J. Kim, N. Devroye, P. Mitran, and V. Tarokh, "Comparison of bi-directional relaying protocols", *2008 IEEE Sarnoff Symposium*, Princeton, NJ, 2008, pp. 1-5.DOI: 10.1109/SARNOF .2008.4520117.

[5] Q. F. Zhou, Y. Li, F. C. M. Lau, and B. Vucetic, "Decode-and-Forward Two-Way Relaying with Network Coding and Opportunistic Relay Selection", *inIEEE Transactions on Communications*, vol. 58, no.11, pp. 3070-3076, November 2010.DOI: 10.1109/TCOMM.2010.100510.090258.

[6] K. S. Hwang, Y.-C. Ko, and M. S. Alouini, "Performance analysis of two-way amplify-and-forward relaying with adaptive modulation over multiple relay network", *IEEE Trans. Communications*, 2011, vol. 59, no.2, pp. 402406, DOI:10.1109/TCOMM .2010.111710.090617.

[7] P. Popovski and H. Yomo, "Wireless network coding by amplify-and-forward for bidirectional traffic flows", *IEEE Communications Letters*, vol. 11, no.1, pp. 1618 2007.DOI: 10.1109/ LCOMM.2007.061436.

[8] T. Koike-Akino, P. Popovski, and V. Tarokh, "Denoising maps and constellations for wireless network coding twoway relaying systems", *IEEE Global Telecommunications Conference*, New Orleans, 2008.DOI: 10.1109/GLOCOM.2008.ECP.727.

[9] T. M. C. Chu, and H. Zepernick, "Performance Optimization for Hybrid Two-Way Cognitive Cooperative Radio Networks with Imperfect Spectrum Sensing", *IEEE Access*, vol. 6, pp. 70582-70596, 2018.DOI: 10.1109/ACCESS.2018.2879901.

[10] S. Zhang, S. C. Liew and P. P. Lam, "Physical-layer network coding", *Proceedings of the International Conference on Mobile Computing and Networking (MobiCom)*, Los Angeles, 2006, pp. 358-365.DOI: 10.1145/1161089.1161129.

[11] S. Sahin and U. Aygolu, "Physical-layer network coding with limited feedback two-way relay chan- nels", *IET Communications*, vol. 6, no.5, pp. 548-556, 2012.DOI: 10.1049/ iet-com .2011.0276.

[12] A. Farzamnia, N. Hlaing, L. Kong, M. Haldar, and T. Rezaii, "Investigation of error performance network coded MIMO-VBLAST wireless communication systems", *Journal of Electrical Engineering*, vol. 70, no:4, pp. 273-284, 2019.DOI: https://doi .org/10.2478/jee-2019-0057.

[13] Z. Zhang, Z. Ma and M. Xiao, "Two-time slot two-way full-duplex relaying for 5G wireless communication networks", *IEEE Transactions on Communications*, vol. 64.no.7, pp. 28732887, 2016.DOI: 10.1109/TCOMM.2016.2574845.

[14] X. Wang, M. Jia, I. W. Ho, Q. Guo, and F. C. M. Lau, "Exploiting Full-Duplex Two-Way Relay Cooperative Non-Orthogonal Multiple Access", *IEEE Transactions on Communications*, vol. 67, no.4, pp. 2716-2729, April 2019, DOI: 10.1109/TCOMM.2018. 2890264.

[15] J. Boutros and E. Viterbo, "Signal space diversity: a power and bandwidth efficient diversity technique for the Rayleigh fading channel", *IEEE Trans Information Theory*, vol. 44, no.4, pp. 14531467, 1998.DOI: 10.1109/18.681321.

[16] N. F. Kiyani, J. H. Weber, A. Zajic, and G. L. Stuber, "Performance Analysis of a System using Coordinate Interleaving and Constellation Rotation Rayleigh Fading Channels", *Proceedings of the IEEE Vehicular Technology Conference (VTC)*, Calgary, 2008.DOI: 10.1109/VETECF.2008.219.

[17] S. Ahmadzadeh, S. Motahari and A. Khandani, "Signal space cooperative communication", *IEEE Transactions on Wireless Communications*, vol. 9, no.4, pp. 12661271, 2010.DOI: 10.1109/ TWC.2010.04.090059.

[18] T. Lu, J. Ge, Y. Yang, and Y. Gao, "BEP analysis for DF cooperative systems combined with signal space diversity", *IEEE Communication Letters*, vol. 16, no.4 pp. 486489, 2012.DOI:10.1109/ LCOMM.2012.020212.112313.

[19] H. Sokun, M. Ilter, S. Ikki and H. Yanikomeroglu, "A Spectrally Efficient Signal Space Diversity-Based Two-Way Relaying System", *IEEE Trans Vehicular Technology*, vol. 66, no.7, pp. 6215-6230, 2017.DOI: 10.1109/TVT.2017.2647813.

[20] M. A. Khan, K. R. Raveendra, and X. Wang, "Two-way decode-and-forward cooperative systems with signal space diversity", *EURASIP Journal on Wireless Communications and Networking*, vol. 2016, no.1, pp. 1-14, 2016.DOI: 10.1186/s13638 -016-0640-2.

[21] I. S. Gradshteyn and I. M. Ryzhik, "Table of Integrals, Series, and Products", *6th ed. San Diego*, CA, Academic Press, 2000.DOI: 10.1016/C2010-0-64839-5.

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