Journal of ELECTRICAL ENGINEERING, VOL. 58, NO. 4, 2007, 214–219

OPTIMAL SHORT TERM HYDRO SCHEDULING OF LARGE POWER SYSTEMS WITH DISCRETIZED HORIZON

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In this paper, we present a new algorithm based on the discrete maximum principle for determining the optimal short-term operating policy of hydroelectric power systems consisting of multi-reservoirs, where the objective is to maximize the potential energy while satisfying all operating constraints over a short-term planning horizon. In order to improve the performance of the proposed algorithm, we have suggested subdividing the short-term planning horizon to shorter study horizons so shorter study periods are embedded in a longer one. Afterward, the objective becomes maximizing the value of potential energy stored at the end of the shorter horizon. The final state of the shorter horizon will be regarded as the initial state of the next shorter horizon and so on. Hence, a reduced size problem is solved in each shorter horizon. Consequently, the calculation effort is decreased considerably and moreover the adjustment of the parameters of the methods used is facilitated.

K e y w o r d s: short-term scheduling, potential energy, discrete maximum principle, augmented Lagrangian method, discretized horizon

1 INTRODUCTION

The short-term optimal operating policy of hydroelectric power systems is a deterministic problem [1, 2] which consists of choosing the quantity of water preliminary selected to release from each reservoir of the system over the short term planning horizon in order to meet an hourly electric power demand assigned in advance. The prime objective here is to perform the operation policy with the lowest use of water. This is achieved by avoiding spilling and by maximizing the hydropower generation, besides satisfying all operating constraints. Maximization of electrical power production is achieved by maximizing the heads. Consequently, this allows maximizing the reservoirs content.

In order to improve the performances of the proposed algorithm, we have suggested subdividing the short term planning horizon into shorter study horizons so that shorter study periods are embedded in a longer one.

Afterward, the objective becomes maximizing the value of potential energy stored at the end of the shorter horizon. The final state of a shorter horizon will be regarded as the initial state of the next shorter horizon and so on. Hence, a reduced size problem is solved in each short period.

When modelling the problem, and for more accuracy, the following factors which make the problem more complex are taken into consideration:

– Significant water travel time between reservoirs.
– The multiplicity of the input-output curve of hydroelectric reservoirs that have variable heads.
– The maximum generation of the hydro power plant (HPP) varies with the hydraulic head. In fact, the quantity of water required for a given power output decreases as the hydraulic head increases.
– The water stored in the upstream reservoir is more valuable than that stored in the downstream reservoir.
– Whether the reservoirs have very different storage capacity.
– Whether the system has quite complex topology with many cascaded reservoirs.

To solve the short-term operating policy problem, we use the discrete maximum principle [3 ,4]. While solving equations related to the discrete maximum principle, we use the gradient method [3]. However, to treat the equality constraint we use Lagrange's multiplier method. To treat the inequality constraint we use the augmented Lagrangian method [5].

Furthermore, the present paper is particularly concerned with treating the state variable constraints, which are of two-sided inequalities. The augmented Lagrangian method is proposed to deal with this type of inequalities.

The hydroelectric power system considered in this paper consists of ten reservoirs hydraulically coupled, *ie*, the release of an upstream reservoir contributes to the inflow of downstream reservoirs. All reservoirs are located on the same river. The time taken by water to travel from one reservoir to the downstream reservoir [8–10] and the water head variation are taken into account. The natural

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ISSN 1335-3632

inflow and the demand for electrical energy are known beforehand. The scheduling is stretched over one week divided into days; the days are subdivided into hours.

The decision variables in the optimization problem are the amounts of water to be released from each reservoir to its direct downstream reservoirs in a given period. The state variables are the contents of the reservoirs.

2 MATHEMATICAL FORMULATIONS

2.1 Notation

x_i^k Contents of reservoir i in period k, in Mm^3.
u_i^k Discharge from reservoir i during period k, in Mm^3.
v_i^k Spillage from reservoir i during period k, in Mm^3.
q_i^k Total inflow to reservoir i during period k, in Mm^3.
y_i^k Natural inflow to reservoir i during period k, in Mm^3.
x_i^0 Initial content of reservoir i.
$x_i^{k_f}$ Contents of the reservoir i in the end of period k_f, in Mm^3.
k_f The last hour of the planning horizon, in hours.
$\underline{x}_i, \overline{x}_i$ Lower and upper bounds on reservoir storage capacity, respectively, for reservoir i, in Mm^3.
$\underline{u}_i, \overline{u}_i$ Lower and upper bounds on water discharge, respectively, of the HPP i, in Mm^3.
$u_{mi}^{k-s_{mi}}; v_{mi}^{k-s_{mi}}$ The discharge and the spilled outflows, respectively, from the upstream reservoir m incoming later to the downstream reservoir i during period k, in Mm^3.
S_{mi} Time required for the water discharged from reservoir m to reach its direct downstream reservoir i, in hours.
m The reservoir immediately preceding reservoir i.
n The number of reservoirs in the system.
e The extreme upstream reservoirs.
D^k System load demand in each period k, in MW.
P_i^k Electric power generated by the HPP i in period k, in MW.
$h_i(x_i^k)$ Effective water head of the HPP i in period k.

2.2 The objective function

The main objective is to maximize the total potential energy of water stored in all reservoirs. The formulation has to take into account the fact that the water stored in one reservoir will be used in all its downstream reservoirs, hence, the water stored in the upstream reservoir is more valuable than that stored in the downstream reservoir, hence:

$$\max \sum_{i=1}^{n} E_p\big(x_i^{k_f}, h_i^{k_f}\big) + \sum_{k=k_f-S_{mi}}^{k_f} E_p(u_{mi}^k, v_{mi}^k).$$

Here $E_p(u_{mi}^k, v_{mi}^k)$ is the potential energy of water stored in reservoir i at the end of the planning horizon k_f. This energy depends on the amount of water stored in reservoir i, on its effective water head $h_i^{k_f}$ and on the effective water head of the downstream reservoirs. $\sum E_p^k\big(u_{mi}^k, v_{mi}^k\big)$ is the total potential energy of the outflow from reservoir m, which will later reach the downstream reservoir i after the last hour of the planning horizon k_f.

2.3 Operational constraints

The operational constraints are [1 ,2] and [8–15]:

– Hydraulic continuity constraint:

The flow balance equation of each reservoir i of the system, for every period k is represented by the following hydraulic continuity equation:

$$x_i^k = x_i^{k-1} + q_i^k - u_i^k - v_i^k.$$

Here

$$q_i^k = \begin{cases} y_i^k & \text{if } i \le e\,, \\ y_i^k + \sum_m \big(u_{mi}^{k-s_{mi}} + v_{mi}^{k-s_{mi}}\big) & \text{otherwise.} \end{cases}$$

– Limits on the storage capacity of each reservoir i:

$$\underline{x}_i \le x_i^k \le \overline{x}_i\,.$$

– Limits on the discharged outflow of the HPP i:

$$\underline{u}_i \le u_i^k \le \overline{u}_i\,.$$

– Load constraints:

The total power generated by all HPP must satisfy the system load demand at each period of the planning horizon. In mathematical terms, this has the following form:

$$\sum_{i=1}^{n} P_i^k = D^k.$$

Here the generation P_i^k is a function of the water discharge u_i^k and of the effective water head h_i^k.

2.4 Modelling the short-term operating policy problem

The mathematical model proposed for the short-term scheduling problem of a hydroelectric power system is as follows:

$$\max \sum_{i=1}^{n} E_p\big(x_i^{k_f}\big) + \sum_{k=k_f-S_{mi}}^{k_f} E_p\big(u_{mi}^k\big) \tag{1}$$

subject to the following constraints:

$$x_i^k = x_i^{k-1} + q_i^k - u_i^k, \tag{2}$$

$$\sum_{i=1}^{n} P_i^k = D^k, \tag{3}$$

$$0 \le u_i^k \le \overline{u}_i, \tag{4}$$

$$\underline{x}_i \le x_i^k \le \overline{x}_i. \tag{5}$$

To avoid the spillage, we force v_i^k to vanish.

3 SOLUTION METHOD

The problem (1)–(3) is solved using the discrete maximum principle as follows [3–7]:
Associate the constraint (2) to the criterion (1) with the dual variable λ_i^k. Furthermore, to satisfy the balance between electric power demand and generation, we associate the constraint (3) to the criterion (1) with the Lagrange multiplier β^k, and then we define the function H^k called the Hamiltonian function, which has the following form:

$$H^k = \sum_{i=1}^{n} \left[\lambda_i^k \left(x_i^k + q_i^k - u_i^k\right)\right] + \beta^k \Big(\sum P_i^k - D^k\Big), \quad (6)$$

where u_i^k and x_i^k represent, respectively, the control and state variables.

To take into account the possible violation of constraint (5) we proceed as follows:
The two-sided inequality constraint (5) can be broken into two inequalities constraints and rewritten, following the substitution of equation (2) for x_i^k:

$$\left(x_i^{k-1} + q_i^k - u_i^k\right) - \overline{x}_i \le 0\,, \quad (7)$$

$$\left(x_i^{k-1} + q_i^k - u_i^k\right) - \underline{x}_i \ge 0\,. \quad (8)$$

To treat these inequalities constraints we use the augmented Lagrangian method [6,7], which consists of adding functions R_i^k and Q_i^k to the Hamiltonian H^k that penalize, respectively, the violations of inequalities constraints (7) and (8), *ie*, the violation of lower and upper limits of the original constraint (5). Then Hamiltonian H^k becomes as follows:

$$H^k = \sum i = 1^n \left[\lambda_i^k \left(x_i^k + y_i - u_i^k\right)\right] + \beta^k \Big(\sum P_i^k - D^k\Big) + R_i^k + Q_i^k\,. \quad (9)$$

The penalty function R_i^k is defined as follows [6,7]:

$$R_i^k = \rho_i^k \Psi_i^k + r\left(\Psi_i^k\right)^2, \quad (10)$$

where is the penalty weight, and ρ_i^k are Lagrange multipliers being updated as follows:

$$\rho_{i\,\mathrm{new}}^k = \rho_{i\,\mathrm{old}}^k + 2r \max\Big(x_i^k - \overline{x}_i\,, -\frac{\rho_i^k}{2r}\Big). \quad (11)$$

The function Ψ_i^k is determined as follows:

$$\Psi_i^k = \max\Big(x_i^k - \overline{x}_i\,, -\frac{\rho_i^k}{2r}\Big). \quad (12)$$

The penalty function is calculated in the same manner as . The problem (1)-(5) becomes:

$$\max H^k\,. \quad (13)$$

The necessary conditions for the optimum are:

$$\frac{\partial H^k}{\partial u_i^k} = 0\,. \quad (14)$$

To find the optimal water discharge trajectory u_i^k from Eq. (14), we must solve the difference equations (2) and the following one called the adjoined equation [6]:

$$\lambda_i^k = -\frac{\partial H^k}{\partial x_i^k}\,. \quad (15)$$

The boundary conditions for Eqs. (2) and (15) are:

- The first one is the initial state, which is specified, *ie*, the initial content of all reservoirs is known, thus:

$$x_i^0 = a_i\,. \quad (16)$$

- The second one is the terminal condition for the adjoined equation:

$$\lambda_i^{k_f} = \frac{\partial E_p\left(x_i^{k_f}\right)}{\partial x_i^{k_f}}\,. \quad (17)$$

The necessary conditions for the optimality constitute a two-point boundary value problem whose solution determines the optimal state and control variables. This problem is solved iteratively using the gradient method [3].

The proposed algorithm for the solution proceeds as follows:

Step 1. Initialize the first shorter period (first day) $j = 1$.

Step 2. Fix α, r and initialize the multipliers β^k and ρ^k for $i = 1,\dots,n$ and $k = 1,\dots,k_f$.

Step 3. Select an admissible control trajectory u_i^k for $i = 1,\dots,n$ and $k = 1,\dots,k_f$.

Step 4. Using the known initial contents x_i^0 and the selected control trajectory u_i^k; solve Eq. (2) forward in time to obtain x_i^k.

Step 5. Using the known u_i^k, x_i^k and the terminal condition (17), solve equation (15) backward in time to obtain λ_i^k.

Step 6. Adjust the multiplier β^k so as to have equilibrium between the demand and the production.

Step 7. Using $u_i^k, x_i^k, \lambda_i^k$ and the adjusted β^k, compute for all i and k the gradient G_i^k from Eq. (14).

Step 8. Compute the new trajectory $\hat{u}_i^k$ using the following expression:

$$\hat{u}_i^k = u_i^k + \alpha G_i^k\,, \quad (18)$$

where α is a pre-selected step size.

Step 9. If some values of the new control variable $\hat{u}_i^k$ that satisfy the optimality condition (14) violate the inequality constraint (4), we fix only the values of the control variable $\hat{u}_i^k$ that violate the boundary values

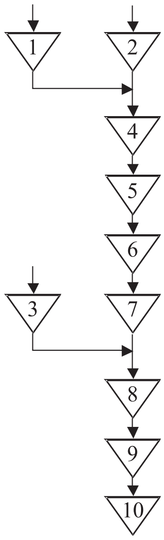

Fig. 1. The reservoir network

Table 1. Characteristics of the installations

i	$\overline{x}_i$ (M.m^3)	$\overline{u}_i$ (M.m^3)	h_i (m)	S_{mi} (m)
1	8777.2	1.1232	0.00	55
2	986.4	0.5272	0.00	70
3	998.0	0.5054	0.00	42
4	504.9	2.5531	66.61	5
5	8.5	2.4181	114.1	7
6	4.2	2.5650	92.41	2
7	4.8	2.5240	83.28	22
8	26.9	2.7648	55.72	3
9	4.54	3.0476	107.66	2
10	3.4	3.4686	40.81	0

Table 2. Natural inflows and initial contents of reservoirs

i	x_i^0 (M.m^3)	y_i^k(M.m^3/h)
1	4386.6	0.1476
2	986.4	0.5272
3	998.0	0.5054
4	504.9	0.5531
5	8.5	0.4181
6	4.2	0.5650
7	4.8	0.5240
8	26.9	0.7648
9	4.5	0.0476
10	3.4	0.4686

to their limit. The others are left free. Then calculate the new value of the multiplier β^k in order to satisfy constraint (3).

Step 10. Compute for all i and k the new value of the gradient G_i^k as done in step 6 using the new value of the multiplier β^k calculated in step 9.

Step 11. Compute the new trajectory $\hat{u}_i^k$ using the following expression:

$$\hat{u}_i^k = \mathrm{Max}\left[0\,,\, \min\{\overline{u}_i\,,\, u_i^k + \alpha G_i^k\}\right] \,.$$

If Max $\left[\hat{u}_i^k,\, u_i^k\right]$ is greater than the desired accuracy limit, set $u_i^k = \hat{u}_i^k$ and go back to step 3.

Step 12. Verify whether the state constraints (5) are satisfied within the desired accuracy limits. If they are not, update the value of the Lagrange multipliers ρ_i^k and go back to step 3.

Step 13. Verify whether the algorithm is performed for all the periods. If they are not, update the period ($j = j+1$) and go back to step 4.

Step 14. Print the results.

4 CASE STUDY

The algorithm just described is implemented in FORTRAN. In order to test its efficiency, we apply it to the system composed of ten reservoirs located on the same river as shown in Fig. 1.

The characteristics of the reservoirs and water time travel are shown in table 1.

The natural inflows are assumed constant throughout the week in all reservoirs. Their values are depicted in table 2 as well as the initial contents of each reservoir.

The hourly demand D^k for each day of the planning horizon is depicted in Fig. 2.

The electrical power in MW produced by the i-th HPP during a period k is given by the following expression:

$$P_i(h_i^k,\, u_i^k) = h_i^k(x_i^k)u_i^k. \tag{19}$$

The length of the planning horizon is one week divided into seven days. The length of one day is 24 hours.

5 IMPLEMENTATION RESULTS

In this section, we present the results obtained from the implementation of the proposed algorithm described in the preceding section. The algorithm is implemented in Fortran.

The solution is achieved after a few numbers of iterations with all constraints being satisfied.

The daily optimal scheduling, *ie*, optimal water discharges from each hydropower plant obtained are depicted in Fig. 3.

We observe that the discharge of the hydropower plants 4–10 follows the demand because it is proportional to the production on one hand. On the other hand, this production must be equal to the demand. Furthermore, the discharge from the downstream reservoir is greater than in the upstream one as shown in Fig. 3, for the reason that the water stored in the upstream reservoir is more valuable than that stored in the downstream one, *ie*, the water of the upstream reservoir will be used again in all the downstream ones.

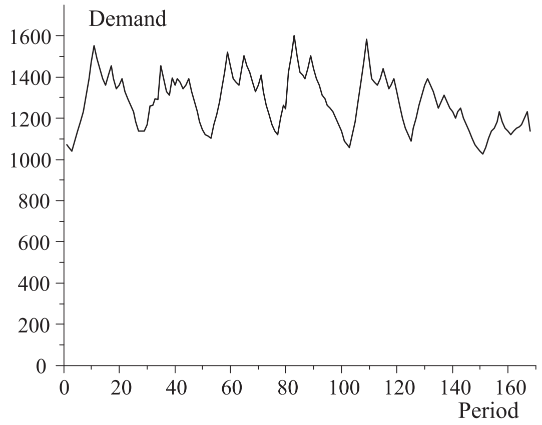

Fig. 2. Hourly demand profile during one week

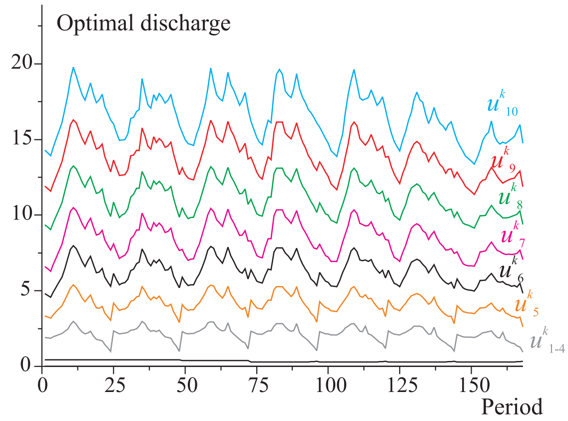

Fig. 3. Optimal discharge trajectories

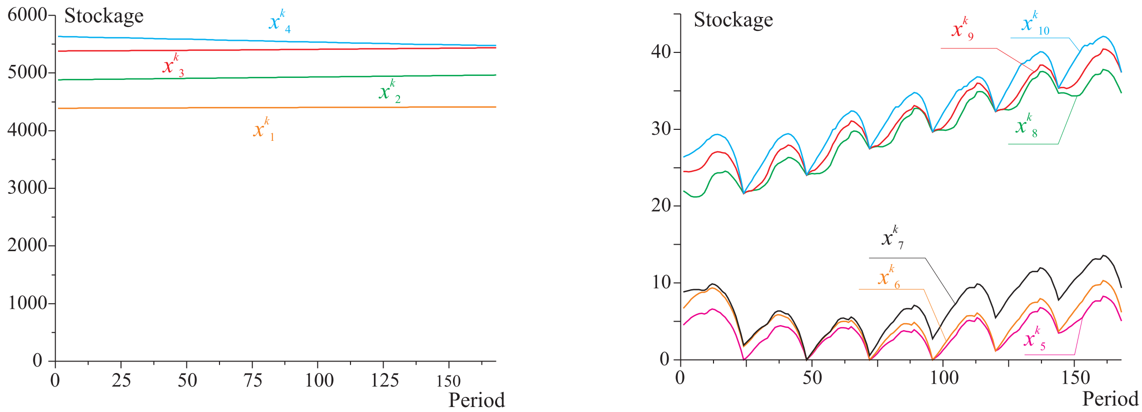

Fig. 4. Optimal evolution of the storage water: left — reservoirs 1-4, right — reservoirs 5-10

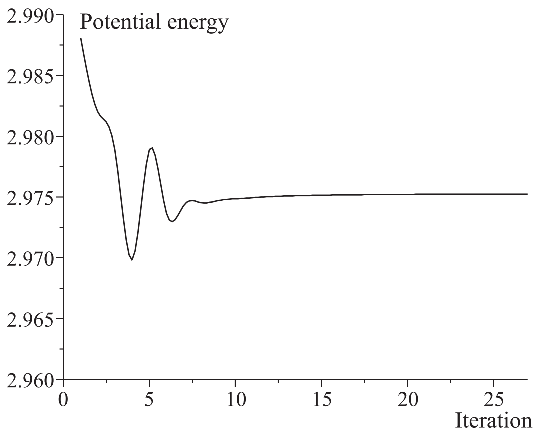

Fig. 5. Behaviour of the search trajectory (Objective function)

Consequently, in economic terms, water in the upstream reservoirs should be preserved as shown in Fig. 4. Thus, the consequences of the optimal scheduling of the water discharge are the filling of the upstream reservoirs as against the downstream ones.

On the other hand, by fixing the end of each day as the end of the shorter planning horizon, the reservoirs are shown to fill up till after mid day, before they start to drain up slowly until the end of the day while their volumes tend to increase as we get closer to the end of week. This represents an economic management of the reservoirs as shown in Fig. 4.

The behaviour of the objective function during the optimization process for one day is illustrated in Fig. 5. The convergence was achieved for each day in about ten iterations. This proves the effectiveness of the discretized technique and the methods used. We note also that the search for the optimum becomes slower when we approach the optimal solution; this is due to the gradient method itself.

We can reduce once more the number of iterations by updating the step size of the gradient method and by increasing penalty weight that will lead to a rapid escape from the violated zone.

6 CONCLUSIONS

A new method is presented in this paper for scheduling hydro power systems. In fact, we have subdivided the short term planning horizon to shorter study horizons, so shorter study periods are embedded in a longer one. Moreover, we have presented a new model for the short-term operating policy of hydroelectric power systems, which consists to maximize the potential energy of the whole system. With the discrete maximum principle, the optimal solution is obtained by solving simultaneous equations representing the optimality conditions. The principle turned out to be very efficient.

To deal with the inequalities constraints, we have introduced the augmented Lagrangian method. The results confirm the promising properties of the augmented Lagrangian. In fact, the proposed algorithm based on those methods requires moderate time and storage for its execution, thus allowing the solution of large-scale scheduling problems. With discretized planning horizon, the calculation effort is decreased considerably.

Moreover, it makes easy the adjustment of the parameters of the methods used and let more stable the process for searching the optimal operating policy. Consequently, it allows the solution of very large- scale scheduling problems.

More improvements can be made to the proposed algorithm in order to increase the convergence speed of the algorithm and its execution time by using an optimal step size rather than a fixed one.

The proposed algorithm take into account the time of water travel between upstream and downstream reservoirs.

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Received 24 March 2006

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