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MCNP5 DELAYED NEUTRON FRACTION ($\beta_{\rm eff}$) CALCULATION IN TRAINING REACTOR VR–1

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In this paper, possible determination methods of one main parameters of the reactor dynamics — the effective delayed neutron fraction $\beta_{\rm eff}$ are summarized and a calculation is made for VR-1 training reactor using the stochastic transport Monte Carlo method based code MCNP5.

K e y w o r d s: delayed neutron fraction $\beta_{\rm eff}$, reactor dynamics, MCNP transport code, nuclear reactor

1 INTRODUCTION

The influence of the delayed neutrons on the reactor dynamics can be understood through their impact on the reactor power change rate, in spite of that they constitute only a very small fraction of the total number of neutrons generated from fission, they play a dominant role in the fission chain reaction control. If only the prompt neutrons existed, the reactor operation would become impossible due to the fast reactor power changes. The uncertainties in determination of the basic delayed neutron parameters lead to the unwished conservatism in the reactor control system design and operation. Therefore, exact determination of the $\beta_{\rm eff}$ value is the main requirement in the field of reactor physics.

The interest in the delayed neutron data accuracy improvement started to increase at the end of the 80-ties and the beginning of the 90-ties, after discrepancies among the results of calculations and experiments. Because of difficulties in $\beta_{\rm eff}$ experimental measurement, this value in the exact state is determined by calculations. Subsequently, its reliability depends on the calculation method and the delayed neutron data used.

An accurate estimate of $\beta_{\rm eff}$ is essential for converting reactivity, as measured in dollars, to an absolute reactivity and to an absolute $k_{\rm eff}$. In the past, $k_{\rm eff}$ used to be traditionally calculated by taking the ratio of the adjoint- and spectrum-weighted delayed neutron production rate to the adjoint- and spectrum- weighted total neutron production rate [1]. An alternative method has also been used in which $\beta_{\rm eff}$ is calculated from simple k-eigenvalue solutions. The summary of the possible $\beta_{\rm eff}$ determination methods can be found in this work and also a calculation of $\beta_{\rm eff}$ for VR-1 training reactor at CVUT Prague in one operation state is made using the prompt method, by MCNP5 code.

2 TRAINING REACTOR VR–1

The VR-1 training reactor is a pool-type light-water reactor based on enriched uranium as fuel. Its core contains fuel assemblies which are submerged in water. There are 15 to 20 such fuel assemblies in the VR-1 reactor, depending on the geometric arrangement and kind of experiments to be performed in the reactor. UR-70 absorption rods serve as the reactor control and safe shutdown. All of the rods are identical in design and only differ in their function in the control system [2]. The core is accommodated in a cylindrical stainless steel vessel – pool – that is filled with demineralized light water.

In this reactor, water fulfils four basic functions. It slows down neutrons, returns a fraction of neutrons leaking from the core, removes the forming heat, and acts as biological shielding. Owing to the low power produced by the reactor, heat can be removed by natural water convention in the pool, no forced cooling by means of pumps being necessary.

The reactor has the shape of an octahedral body manufactured from special heavy concrete containing cast iron which possesses outstanding shielding properties. Two vessels of virtually identical design are accommodated in the shielding. One is designed for the core; the other one is a handling vessel. They are interconnected by a gate owing to which all materials, including fuel, can be transported under the water level. This contributes appreciably to the radiation safety during reactor operation. The reactor is fitted with a water circuit, including a standby tank and a demineralization station for preparing high-purity water and maintaining this high purity.

During the first six years of operation, the fuel of the IRT-2M type with the enrichment 36 %, imported from the former USSR, later replaced by IRT-3M type was used in the VR-1 reactor. Since 2005 the IRT-4M fuel with 19.7 weight percent of ^{235}U has been used at VR-1 reactor. The fuel replacement was accomplished within the US–Russian Reduced Enrichment for Research and Test Reactors Program. It is aimed at reducing a potential threat of using radioactive materials for terrorist purposes.

There are three kinds of rods in the reactor: Safety rods (B1, B2, B3), Experimental rods (E1, E2) and Control

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Table 1. Basic reactor parameters [3].

Vessel dimensions	2300 mm (diameter), 4720 mm height)
Reactor shielding	above the core: 3000 mm water layer
	side: 850 mm water layer +950 mm extremely heavy concrete layer
Reactor temperature	working approx. 20 °C, due core cooling by natural convection
Pressure	Atmospheric
Power measurement	four wide-range fission chambers RJ 1300
Independent protection	four pulse corona boron counters SNM-12
Control system (CS)	microprocessor CS with 5-7 absorbing rods UR-70
Neutron flux	thermal (average value) $1.8\times10^{13}\mathrm{m^{-2}s^{-1}}$, fast $7.5\times10^{12}\mathrm{m^{-2}s^{-1}}$
Neutron source	Am-Be, activity185 GBq, emission rate $1.1\times10^{7}\mathrm{s^{-1}}$

rods (R1, R2). The position of the control rods changes according to the specific experiment.

3 $\beta_{\rm eff}$ MONTE CARLO CALCULATION

In general, for the total neutron production rate by fission for a reactor near criticality, and without external source, one writes [4]

$$P=\int \nu(E)\Sigma_f(\boldsymbol{r},R)\varphi(\boldsymbol{r},E,\Omega)\mathrm{d}E\mathrm{d}\boldsymbol{r}\mathrm{d}\Omega \tag{1}$$

where E, Ω, and $\boldsymbol{r}$ are the energy, solid angle, and position of the neutrons, φ is the neutron flux, Σ_f is the macroscopic fission cross section of the material at position $\boldsymbol{r}$, ν is the average neutron multiplicity per fission. For the production P_d of delayed neutrons, one replaces the factor $\nu(E)$ by $\nu_d(E)$, the average delayed neutron multiplicity per fission. For the sake of simplicity, it is not distinguished between the several delayed neutron time groups all of which have their own energy spectrum. The ratio P_d/P is then the fundamental delayed neutron fraction $\beta_0=\frac{P_d}{P}$. So far, this is fairly straightforward. The problems start when trying to assess how effective this fraction is in terms of reactor dynamics. The effect that neutrons have on reactor behaviour is through their ability to generate power, *ie*, to induce the next fission. It follows that we should compute the number of fissions that are induced by delayed neutrons, as well as by all neutrons.

In transport theory one calculates the effectiveness in generating fission by multiplying by the energy spectrum of the generated neutrons, $\chi(E')$, and by the adjoint function $\psi(\boldsymbol{r},E',\Omega')$, often referred to as 'adjoint flux'. The adjoint function is defined as a fundamental mode eigenfunction of the equation adjoint to the time independent transport equation. Here $\vec{r}$, E', and Ω are the position, energy, and solid angle of neutrons generated by the fissions that were induced by the incident neutrons characterized by $\boldsymbol{r}$, E, Ω. Position $\boldsymbol{r}$ is obviously the same for both. Factor $\chi(E')$ is needed because the energy the neutrons start with has an impact on their effectiveness in inducing fission. The adjoint function $\psi(\boldsymbol{r},E',\Omega')$ is used because it is important to account for the significance of a neutron with properties $\boldsymbol{r}$, E', Ω' for producing fission and is proportional to the asymptotic power level resulting from the introduction of a neutron in a critical system at zero power [1], leading to the so-called spectrum and adjoint weighted neutron production

$$P_{\rm eff}=\int \psi(\boldsymbol{r},E',\Omega')\chi(E')\nu(E)\times \Sigma_f(\boldsymbol{r},E)\varphi(\boldsymbol{r},E,\Omega)\mathrm{d}E\mathrm{d}\Omega\mathrm{d}E'\mathrm{d}\Omega'\mathrm{d}\boldsymbol{r}\,. \tag{2}$$

One can calculate the same quantity for delayed neutrons only $P_{\rm d,eff}$, by replacing $\chi(E')$ by $\chi_{\rm d}(E')$ and $\nu(E)$ by $\nu_{\rm d}(E)$. One arrives at the Keepin definition taking the ratio $\beta_{\rm eff}=P_{\rm d,eff}/P_{\rm eff}$:

$$\beta_{\rm eff}=\frac{\sum_i\sum_m\int\psi\chi_{di}^m\nu_{di}^m\Sigma_f'^m\varphi'\mathrm{d}\Omega'\mathrm{d}E'\mathrm{d}\Omega\mathrm{d}E\mathrm{d}r}{\sum_m\psi\chi_t^m\nu_t^m\Sigma_f'^m\varphi'\mathrm{d}\Omega'\mathrm{d}E'\mathrm{d}\Omega\mathrm{d}E\mathrm{d}r} \tag{3}$$

where m means the $m^{\rm th}$ isotope and i is the $i^{\rm th}$ delayed neutron group, other symbols are mentioned earlier.

It is instructive to interpret P as the neutron source (*the number of neutrons produced per unit of time*), and $P_{\rm eff}$ as the number of fissions produced by this source per unit of time.

In Monte Carlo calculations the physical processes are simulated as realistically as possible. Consider the introduction of a neutron with properties $\boldsymbol{r}$, E', Ω' in a critical system with zero power. This neutron will produce other neutrons by inducing fission and these neutrons will in turn cause fission and thereby lead to further neutrons, etc. The number of fissions produced in this way will approach a limit which is given by the iterated fission probability that which is proportional to the adjoint function. Using the number of fissions counted and the knowledge of whether the neutron was prompt or delayed at its start, one can easily calculate $\beta_{\rm eff}$.

3.1 Prompt Method

Denoting the integral in (2) as $\langle\chi\nu\rangle$, making use of that the integrals are linear and introducing $\nu_p=\nu-\nu_d$ one can rewrite the expression for $\beta_{\rm eff}$ as follows

$$\beta_{\rm eff}=\frac{\langle\chi_d\nu_d\rangle}{\langle\chi\nu\rangle}=1-\frac{\langle\chi\nu-\chi_d\nu_d\rangle}{\langle\chi\nu\rangle}\cong1-\frac{\chi_p\nu_p}{\langle\chi_\nu\rangle}. \tag{4}$$

The approximation in the last step is based on the following arguments. The term $(\chi_d-\chi)\nu_d$ is two orders of magnitude smaller than the one with $\chi\nu_p$, because ν_d is two orders of magnitude smaller than ν_p. For the same reason, the shape of χ is almost equal to that of χ_p.

At this point a crucial step is taken. Often it is simply stated that

$$\frac{\langle \chi_p \nu_p \rangle}{\chi\nu} = \frac{k_p}{k} \to \beta_{\text{eff}} \cong 1 - \frac{k_p}{k} \,. \tag{5}$$

In fact, this is an approximation. It is true that the k-eigenvalue is the ratio of production P and loss L, and that this also holds for the ratio of $P_{\text{d,eff}}$ and $L_{\text{d,eff}}$. But the difficulty lies in the definition of k_p. Since this parameter is supposed to be calculated by means of a transport theory code, it should be defined as the eigenvalue pertaining to a reactor with $\chi = \chi_p$, and $\nu = \nu_p$. The difference is that in such a calculation the shapes of φ and ψ will not be the same as for the original system with χ and ν, for which the eigenvalue is k. This subtlety is generally ignored in papers dealing with the 'prompt' method of calculating β_{eff} based on equation (5) [5], where it is assumed that, since the neutron multiplication factor for a reactor is proportional to the total fission yield (ν), the multiplication factor for prompt neutrons only is proportional to $\big(\nu - \overline{E}v_d\big) = \nu\big(1 - \beta_{\text{eff}}\big)$. Here ν_d is the total delayed fission neutron yield and $\overline{E}$ is the average effectiveness of these delayed neutrons. The total effective delayed neutron fraction is therefore

$$\beta_{\text{eff}} = \frac{k - k_p}{k} = \sum_i \beta_{\text{eff},i} \tag{6}$$

where k is the eigenvalue for all neutrons and k_p is the eigenvalue for prompt neutrons only.

3.2 Spriggs Method

The delayed neutron fraction is traditionally counted from

$$\beta_{\text{eff}} = \frac{\sum_i \sum_m \int \psi \chi_{di}^m \nu_{di}^m \Sigma_f'^m \Phi' \mathrm{d}\Omega' \mathrm{d}E' \mathrm{d}\Omega \mathrm{d}E \mathrm{d}r}{\sum_m \psi \chi_t^m \nu_t^m \Sigma_f'^m \Phi' \mathrm{d}\Omega' \mathrm{d}E' \mathrm{d}\Omega \mathrm{d}E \mathrm{d}r} \tag{7}$$

Alternatively, Spriggs et al. [6] rewrite β_{eff} as

$$\beta_{\text{eff}} = \frac{\langle \chi_d \nu_d \rangle}{\langle \chi\nu \rangle} = 1 - \frac{\langle \chi_d \nu_d \rangle}{\langle \chi_d \nu \rangle} \frac{\langle \chi_d \nu \rangle}{\langle \chi\nu \rangle} = \beta_0' \frac{\langle \chi_d \nu \rangle}{\langle \chi\nu \rangle} \tag{8}$$

where we have introduced, after Spriggs *et al*, yet another delayed neutron fraction β_0'. For the present purposes we restrict ourselves to the approximation that $\beta_n' \cong \beta_n$ because we still need to perform adjoint weighting to calculate β_0'. By approximating $\beta_n' \cong \beta_n$ we can simplify the calculation to something that can easily be implemented in a Monte Carlo code. As remarked by Spriggs et al., this approximation works well for homogeneous cases. Also with the introduction of a ratio of k-values

$$\frac{\langle \chi_d v \rangle}{\langle \chi\nu \rangle} = \frac{k_d}{k} \to \beta_{\text{eff}} = \beta_0' \frac{k_d}{k} \,. \tag{9}$$

As in the case of the 'prompt' method, this is also an approximation. Here the problem lies in the definition of k_d. Since this parameter is supposed to be calculated by means of a transport theory code, it should be defined as the eigenvalue pertaining to a reactor with $\chi = \chi_d$ and $\nu = \nu_d$. Again, the shapes of φ and ψ will not be the same as for the original system with χ and ν, for which the eigenvalue is k. This subtlety is explained by Spriggs *et al* for their method of calculating β_{eff}.

3.3 THE HOLLAND MC METHOD

This method was proposed by van der Mark and K. Meulekamp from Petten NRG, Holland [4]. Therefore, the neutron production and the spectrum weighting, which can be provided by counting the number of fissions generated per history. One can then calculate the average number of fissions generated by all neutrons. This is β_{eff} as defined in equation (3), where $\beta_{\text{eff}} = P_{\text{d,eff}}/P_{\text{eff}}$, [4].

On the other hand, when we apply the generation definition of history to β_{eff} calculation, an approximation is introduced because history now only runs until the next fission. Therefore our counting of the number of induced fissions also counts only until the next fission, *ie*, we are not calculating the iterated fission probability but the next fission probability. This is no longer exactly proportional to the adjoint function, nevertheless, it is a useful approximation.

For the calculation of β_{eff} it is not required to know the adjoint function very precisely. The value for β_{eff} is largely determined by the value of β_0, the fraction of delayed neutrons produced. For most critical systems, the value for β_{eff} differs $< 20\,\%$ from β_0. Since the effectiveness of neutrons, *ie*, the adjoint function only influences this small difference between β_{eff} and β_0, one only needs to know the adjoint function with $10\,\%$ accuracy to get $2\,\%$ accurate results for β_{eff}.

Finally, it should be noted that a calculation of β_{eff} in this way is done by means of some minor bookkeeping in the code, which will give a result for β_{eff} in the same run with which one calculates k_{eff}.

3.4 β_{eff} calculation

The effective delayed neutron fraction β_{eff} in training reactor VR-1 is calculated by MCNP5 transport code using prompt method, which requires two calculations. According to [5] it is defined:

$$\beta_{\text{eff}} \cong 1 - \frac{\langle \chi_p \nu_p \rangle}{\langle \chi\nu \rangle} \cong 1 - \frac{k_p}{k} \,. \tag{10}$$

The required value of the effective multiplication factor k_{eff} taking both prompt and delayed neutrons into account was acquired in the straight calculation mode of MCNP5 calculation, using the data card KCODE. In the

KCODE mode the mean values of both prompt and delayed neutrons (if these are included in the used cross-section libraries) are used in criticality calculations. To prevent the influence of the delayed neutrons, TOTNU data card with entry NO had to be used, to obtain the value of effective multiplication factor for prompt neutrons (k_p). A TOTNU card with NO as the entry causes ν_p to be used, and consequently k_p to be calculated, for all fissionable nuclides for which prompt values are available. If the TOTNU card is used and has no entry after it, the total average number of neutrons from fission (ν) using both prompt and delayed neutrons is used and the total effective multiplication factor (k) is calculated.

The calculation was realized for the complex MCNP5 model of training reactor VR-1, composed and provided by Jan Rataj, MSc and Dr Lubomir Sklenka, from the Department of Nuclear Reactors of the Czech Technical University in Prague, under international cooperation. In the model, several simplifications were considered:

a) The tool for dynamic studies is not included — model of fuel assembly is used instead
b) The fission chambers RJ 1300 for power measurement during operation are not considered in the model
c) The SNM detectors are not considered in the model

The positions of the rods considered in the model were as follows:

a) Safety rod: B1, B2, B3 in top level, which corresponds to a height of 680 mm
b) Experimental rods E1 at 350 mm and E2 at 340 mm
c) Control rods R1 in top level and R2 at the bottom level of the core

The spatial distribution of source neutrons was created using the KCODE data card. The source neutrons in each cycle are determined by the points generated in the previous cycle. For the initial cycle, the point neutron sources are necessary to specify. These are specified explicitly by KSRC data card. In this case 8 source neutron points were defined. The fuel rods are divided into 8 layers, and in each layer there is one source neutron point specified. 5000 active cycles and 250 neutron generations were used in the calculations.

Calculation results:

The first calculation in KCODE mode with TOTNU card with NO, brought following value of effective multiplication factor for prompt neutrons: $k_p = 0.99728 \pm 0.00085$. The second calculation result was the value of total effective multiplication factor: $k = 1.00519 \pm 0.00083$. The final effective delayed neutron fraction for the model of VR-1 training reactor, calculated using prompt method (10):

$$\beta_{\rm eff} = 1 - \frac{k_p}{k} = 1 - \frac{0.99728 \pm 0.00085}{1.00519 \pm 0.00083} = \\ = 0.007869 \pm 0.00168.$$

4 CONCLUSION

The final value of the effective delayed neutron fraction 0.00787 with the standard deviation of 0.168 % is 10.2 % greater than the theoretical value estimated and used by the VR-1 operators. The currently used value of $\beta_{\rm eff}$ is 0.00714 [7]. The discrepancies between these two values were caused by uncertainties of the selected calculation method, using the simplifications in the MCNP model of the reactor, so as the assumptions summarized above. Development of a method of $\beta_{\rm eff}$ measurement based on measuring the reactor response to periodically inserted reactivity is the main goal of our future work.

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