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# THE METHODOLOGY FOR OPTIMIZATION OF ENERGY DISTRIBUTION NETWORK

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In the article, the authors depicted the methodology for optimization of energy distribution network based on the criterion of minimum costs of energy distribution. The presented methodology has been implemented as a tool for evaluation of the functioning of the Polish energy network. It has also been used in the model analysis of the Slovakian energy network. As a supplement to the network optimization, one uses also software based on this methodology, which enables the analysis of investment outlays for different network types. All in all, the presented methodology leads to increased efficiency of energy distribution, also in the aspect of balanced development.

K e y w o r d s: optimization, costs of distribution, efficiency, investment outlays

## 1 INTRODUCTION

The theoretical basics for network optimization form the basis for estimation of development trends based on minimum costs of energy distribution. There are certain problems with estimation of some coefficients, especially cost coefficients that should reflect current conditions of energy distribution in the network. [11]

The optimization of the 110 kV and medium voltage (MV) network is based on the substitution network of surface area character. For a low voltage network (LV), the best solution is based on the linear structure, including MV/LV transformers.

Reflecting the network as a set of elements allows its optimization as a whole, but one can include only the number of stations supplying the network and the average section of lines that form this network. The possibility of optimization results from the fact that it is possible to calculate load losses stemming from the flow of power and energy. Losses constitute an element of costs and there are no possibilities of network optimization if calculation of losses is not possible. Optimization of the network must refer to the quantity of energy that will flow through the analyzed network. So, the element cost forms only the part of losses that are connected with quantity of the flowing energy. Other losses *ie* voltage and trade ones can be omitted in calculations because as fixed costs have no impact on optimization. [13]

The energy network can be broken into:

a) network that can be attributed to the particular area, where they supply all receivers *ie* networks of surface are structure. This network includes: low voltage city network, all medium and 110 kV networks.

b) network that cannot be attributed to the particular area. This is the network of linear structure *ie* rural low voltage network.

Costs elements from the optimization of network development point of view are: 1 – depreciation of the costs of construction of the supplying station, 2 – depreciation of the costs of construction of the station lines, but only in part that is connected to the size of section of lines and 3 – costs of energy losses.

## 2 OPTIMIZATION FORMULAS

The basic cost formula is as follows

$$K_r = F\big(k_{FS} + \overline{S}k_{FZ}\big)r_{gF} + L\big(k_{LS} + s_{hr}k_{LZ}\big)r_{gL} + \Delta E_o k_\Delta \tag{1}$$

where

$F$ – number of stations supplying the network,
$k_{FS}$ – fixed part, *ie* independent of power, costs of construction, cu/sub; (currency unit/substation)
$\overline{S}$ – average power of the supplying station, MVA,
$k_{FZ}$ – variable part *ie* dependent on power, costs of construction, cu/sub MVA,
$r_{gF}$ – annual depreciation of the costs of construction of the supplying station,
$L$ – length of the distribution network, km
$K_{LS}$ – fixed part, *ie* independent of section, costs of construction, cu/ km,
$S_{hr}$ – average section of network, $\mathrm{mm}^2$,
$K_{LZ}$ – variable part *ie* dependent on section, costs of construction, cu/ km$\mathrm{mm}^2$,
$R_{gL}$ – annual depreciation of the costs of construction of the station lines,

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$\Delta E_o$ – energy losses in the network, MWh/a; a=for a year,
$k_\Delta$ – costs of energy losses, cu/MWh.a

Based on the formula, optimal values of parameters are set. These are: optimal section, optimal number of stations supplying the network and optimal level of load losses in the network.

The 110 kV and medium voltage network constitute network of surface area structure. In such a case, the following formula is used:

- optimal number of stations supplying the network

$$F_{o(t)} = \sqrt[3]{\frac{a_s E^2_{(t)} W_s k_\Delta k_{Lz} r_{gL}}{100(k_{Fs} r_{gF})^2}} \tag{2}$$

where: $W_s$ – network coefficient,

- optimal section

$$s_{hro} = \frac{1}{L}\sqrt[3]{\frac{a_s E^2 W_s k_\Delta k_{Fs} r_{gF}}{100(k_{Lz} g_{Lz})^2}} + b_s \tag{3}$$

where: $a_s$, $b_s$ – coefficients for section levels; for calculations values: $a_s = 0.305$, $b_s = 24.3$ have been assumed,

- optimal level of load losses is then

$$\Delta E_{oo(t)\%} = \sqrt[3]{\frac{a_s W_s k_{Fs} r_{gF} k_{Lz} r_{gL} \times 10^4}{E_{(t)} k^2_\Delta}} \tag{4}$$

where: $W_s = \dfrac{A_{\chi^2}(2t_s+1)}{67 t_s U_r^2 \cos^2\varphi_r k_{rL}\delta_s}$ – network coefficient.

For a network of linear structure, the formula for costs of energy distribution, including the costs of losses in transformers is as follows [10]:

$$K_r = F\big(k_{FS} + \overline{S}k_{FZ}\big) r_{gF} + L\big(k_{LS} + s_{hr}k_{LZ}\big) r_{gL} + \frac{E^2 L W_{LN}}{100F^2 s_{hr}} k_\Delta + F^{\frac14}\Big(\frac{E}{T_S\cos\varphi_s\beta_s}\Big)^{\frac34} k_r\big(a_u\beta_s^2 k_T\tau k_{\Delta T} + a_j T_r k_{\Delta j}\big) \tag{5}$$

where:

$F$ – number of stations supplying the network, N;
$K_{FS}$ – fixed part, *ie* independent from power, costs of construction, cu/sub;
$\overline{S}$ – average power of the transformers in $F$ stations, cu/sub;
$k_{FZ}$ – variable part, *ie* dependent on power, costs of construction, cu/MVA substation;
$r_{gF}$ – annual depreciation of the costs of construction of the supplying station;
$L$ – length of the distribution network, km;
$K_{LS}$ – fixed part, *ie* independent from section, costs of construction, cu/km;
$S_{hr}$ – average section of network, mm$^2$;
$K_{LZ}$ – variable part, *ie* dependent on section, costs of construction, zł/mm$^2$, km;
$R_{gL}$ – annual depreciation of the costs of construction of the station lines;
$K_\Delta$ – costs of energy losses, cu/MWh,
$W_{LN}$ – linear coefficient based on the following formula: $W_{LN} = \frac{\Delta E_{0\%} F^2 s_{hr}}{EL}$;
$E$ – energy supply into the low voltage network.

After calculation of the first derivative with respect to the number of stations and sections as well as equating it to zero, one gets the following formula

$$k_{Fs} r_{gF} - \frac{2ELW_{LN}k_\Delta}{10F^2}\sqrt{\frac{k_{LZ} r_{g1}}{W_{LN}k_\Delta}} + \Big(\frac{E}{T_s\cos\varphi_s\beta_s}\Big)^{\frac34}\frac{k_r\big(a_u\beta_s^2 k_T\tau k_{\Delta T} + a_j T_r k_{\Delta j}\big)}{4F^{\frac34}} = 0\,. \tag{6}$$

This equation does not have an analytical solution due to the number of $F$ stations. It can be solved by Chebyshev formula.
– the optimal number of stations

$$F_o = \frac{1}{2k_{Fs}r_{gF} + 0.026\big(\frac{E}{T_s\cos\varphi_s\beta_s}\big)^{\frac34}\lambda}\Big(0.104\big(\frac{E}{T_s\cos\varphi_s\beta_s}\big)^{\frac64}x^2 + \big[4k_{Fs}r_{gF} + 0.052\big(\frac{E}{T_s\cos\varphi_s\beta_s}\big)^{\frac34}\lambda\big] \times \big[0.2EL\sqrt{W_{LN}k_\Delta k_{LZ} r_{gL}} + 0.071\big(\frac{E}{T_s\cos\varphi_s\beta_s}\big)^{\frac34}\big]\Big)^{1/2} - \frac{-0.323\big(\frac{E}{T_s\cos\varphi_s\beta_s}\big)^{\frac34}\lambda}{2k_{Fs}r_{gF} + 0.026\big(\frac{E}{T_s\cos\varphi_s\beta_s}\big)^{\frac34}\lambda}\,, \tag{7}$$

$$\lambda = k_r a_j T_r k_{\Delta j} + k_r a_u \beta_s^2 k_T \tau k_{\Delta T}\,,$$

– optimal section

$$s_{hro} = \sqrt{\frac{E^2 W_{LN} k_\Delta}{100F^2 k_{Lz} r_{gL}}}\,, \tag{8}$$

– optimal coefficient of losses

$$\Delta E_{o\%} = \frac{W_{LN}EL}{F_o^2 s_{hro}}\,. \tag{9}$$

**Table 1.** Optimal parameters for low voltage network

| No. | Region | $W_{LN}$ | $\beta_s$ | $F_o$ | $S_{hro}$ | $\Delta E_{o\%}$ |
|---|---|---|---|---|---|---|
| 1 | RE1 | 0.1984 | 0.81 | 716 | 70.15 | 5.08 |
| 2 | RE2 | 0.4655 | 0.81 | 951 | 31.16 | 8.97 |
| 3 | RE3 | 0.4370 | 0.80 | 1084 | 41.87 | 7.55 |
| 4 | RE4 | 0.5413 | 0.79 | 1038 | 27.95 | 9.45 |
| 5 | RE5 | 0.4422 | 0.79 | 1257 | 29.32 | 8.81 |
| 6 | RE6 | 0.3821 | 0.79 | 688 | 49.71 | 6.84 |

Equation (6) can be solved also with usage of numerical methods as there is $\beta_s$ optimal coefficient of transformers load in medium/low voltage network. Solving the equation, one shall set the value of $\beta_s$ which is unknown at this moment.

The formula for costs of energy losses in the group of transformers is as follows

$$K_{\Delta TB} = F^{1/4}\Big(\frac{E}{T_s\cos\varphi_s\beta_s}\Big)^{3/4} k_r\big(a_u\beta_s^2 p_{RT}\tau k_\Delta + a_j T_r k_{\Delta j}\big). \quad (10)$$

Equating the cost derivative on the load level of transformers $\beta_s$ to zero, equation (10) and solving it, one gets the formula for the optimal coefficient of transformers load

$$\beta_{\rm soptB} = \sqrt{\frac{3a_j T_r k_{\Delta j}}{5a_u p_{RT}\tau k_\Delta}}\,. \quad (11)$$

Taking into account the costs of construction of the station, equation (10) will take the following form:

$$K = F\Big(k_{Fs} + \Big(\frac{E}{FT_s\cos\varphi\beta}\Big)k_{Fz}\Big)r_{gF} + F^{1/4}\Big(\frac{E}{T_s\cos\varphi\beta}\Big)^{3/4} k_r\big(a_u\beta^2 k_T\tau k_\Delta + a_j T_r k_{\Delta j}\big)\,. \quad (12)$$

Calculus of equation (12) on the load level of transformers is as follows

$$\frac{\mathrm{d}K}{\mathrm{d}\beta} = -\Big(\frac{E}{T_s\cos\varphi}\Big)k_{Fs}r_{gF}\beta^{-2} + \frac{5}{4}F^{1/4}\Big(\frac{E}{T_s\cos\varphi}\Big)^{3/4}k_r a_u p_{RT}\tau k_\Delta\beta^{3/4} + -\frac{3}{4}F^{1/4}\Big(\frac{E}{T_s\cos\varphi}\Big)^{1/4}a_j k_r T_r k_{\Delta j}\beta^{-7/4}. \quad (13)$$

Setting $\beta^{-1/4} = x$, $a = \Big(\frac{E}{T_s\cos\varphi}\Big)k_{Fs}r_{gF}$, $b = \frac{5}{4}F^{1/4}\Big(\frac{E}{T_s\cos\varphi}\Big)^{3/4}k_r a_u p_{RT}\tau k_\Delta$, $c = \frac{3}{4}F^{1/4}\Big(\frac{E}{T_s\cos\varphi}\Big)^{1/4}a_j k_r T_r k_{\Delta j}$. makes the equation in the following form:

$$-ax^9 + b - cx^8 = 0$$

The equation does not have an analytical solution. It can be solved with usage of iterative methods *eg* the tangent Newton method. The approximation can be found with the following formula

$$x^8(ax + c) = b$$

as $\beta_{\rm opt}$ can be in the range of $0.6 \div 1$; $x$ is about $1$. So, $x^8(a+c) = b$. Then, the formula for optimal coefficient of the load of transformers is as follows

$$\beta_{\rm soptC} = \sqrt{\frac{3F^{1/4}k_r a_j T_r k_{\Delta j} + 4\big(\frac{E}{T_s\cos\varphi_s}\big)^{1/4}k_{Fz}r_{gF}}{5F^{1/4}k_r a_u p_{RT}\tau k_\Delta}}\,, \quad (14)$$

The results of calculations are depicted in Tab. 1. They referred to one of the distribution companies that is composed of 6 energy regions. The optimal loss ratio was calculated with usage of formula (14). The optimal parameters *ie* optimal number of stations, optimal section and optimal percentage loss ratio were calculated with usage of the following formulas respectively: tangent Newton method (6), formula (8), where $F$ was set with number of stations calculated previously and formula (9). [12]

The optimal losses depicted in the table constitute optimal losses in the network and losses in transformers on condition of optimal number of stations and optimal coefficient of load of transformers.

The results of calculations can be confronted with the real data from the network. This data shall not deviate much from the ones from Tab. 1. In the analyzed network, the optimal number of stations is a bit less than in reality, which is connected with little higher level of load of transformers set during optimization process. Such situation will guarantee maintenance of the lowest level of costs of energy distribution. This is of strategic importance from the competitive market of electrical energy point of view.

## 3 CONCLUSION

The methodology presented in the article has been used by the authors in the calculation of efficiency of energy distribution both in Poland and Slovakia. The conducted analysis form the basis for designing of harmonized development of the distribution network that assures appropriate quality parameters of energy and highest efficiency of energy distribution.

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