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MODELLING OF NON–LINEAR DISTORTION IN VACUUM TRIODES USING TRANS–CHARACTERISTICS INVERSE AND NEWTON'S METHOD

Martin Dadić *

The increased interest in vacuum tube audio amplifiers led to an increased interest in mathematical modelling of such kind of amplifiers. The main purpose of this paper is to develop a novel global numerical approach in calculation of the harmonic distortion (HD) and intermodulation distortion (IM) of vacuum-triode audio amplifiers, suitable for applications using brute-force of modern computers. Since the 3/2 power law gives only the transcharacteristic inverse of a vacuum triode amplifier, unknown plate currents are determined in this paper iteratively using Newton's method. Using the resulting input/output pairs, harmonic distortions and intermodulations are calculated using discrete Fourier transform and three different analytical methods.

Keywords: vacuum triodes, audio amplifiers, harmonic distortion, intermodulation, Newton's method, discrete Fourier transform

1 INTRODUCTION

Vacuum-tube amplifiers are still widely used as electric guitar amplifiers. Moreover, they are recently reintroduced in "high-end" audio reproduction instead of the transistors. The main reason for this revival is supposed difference in sound produced by those two amplifier categories. Mathematical modelling is widely applied in different areas of audio amplification [1]. The increased interest in vacuum tube amplifiers led to an increased interest in mathematical modelling of such kind of amplifiers [2, 3]. Some of recognised differences between transistorised and "tube-sound" originate from different loudspeaker damping [4], intermodulation with the mains frequency and consequential "buzzing sound" in tube amplifiers [4], or influence of the exaggerated negative feedback in transistorised amplifiers [5]. The highly overdriven behaviour is also different in tubes and transistors [2].

It is often claimed that some of the preferable "tube-sound" originates from the specific type of the non-linear distortion [7, 2]. The 3/2 power law gives only the transcharacteristic inverse of a vaccum-triode amplifier, instead of the needed transcharacteristic. In other words, the input variable is in the analytical expression for the plate current given only as a function of the output variable. For a triode amplifier with nonreactive load and negative grid voltage (Fig. 1), a general form for the calculation of the higher harmonics traditionally is based on the Taylor series expansion of the plate current in terms of the gate-to-cathode voltage [7]. Taylor series expansion is used still today in evaluating non-linear distortions of solid-state circuits [8]. In [9] the inverse transcharacteristic of a vacuum triode is approximated by a number of straight-line segments, and represented by the Fourier series model using slopes of these segments. This method allows for prediction of harmonic distortion and intermodulation distortion factors.

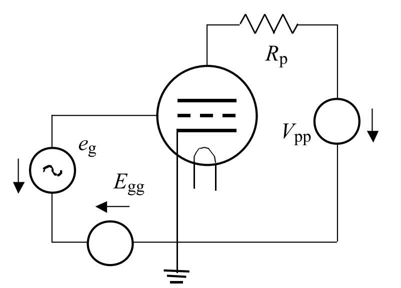

Fig. 1. Simple vacuum-triode amplifier

The attractive convergence properties of the Newton-based algorithms have recently been applied in the calculation of the intermodulation predictions of the transistor amplifiers in wireless communications [10] and transient analysis of analog circuits [11]. The main purpose of this paper is to develop a novel numerical approach in calculation of the harmonic distortion (HD) and intermodulation distortion (IM) of vacuum-triode amplifiers, suitable for applications using brute-force of modern computers. The unknown plate currents are determined iteratively using the Newton's method in solving the governing non-linear inverse transcharacteristic equation. Using the resulting input/output pairs, as well as calculated analytical derivative of the transcharacteristic inverse, a method primarily intended for semiconductor devices is applied in modelling of the harmonic distortion of a vacuum-triode. Also, two other analytical methods are applied, which are based on the polynomial fit. The results are compared with the calculations of the harmonic distortion and intermodulation based on the discrete Fourier trans-

* Faculty of Electrical Engineering and Computing University of Zagreb Unska 3, 10000 Zagreb, Croatia, martin.dadic@fer.hr

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form (DFT). The proposed approach is applicable both for large-signal and small-signal analysis.

2 SINGLE–ENDED TRIODE AMPLIFIERS

If grid-to-cathode voltage v_{GK} is negative (and grid current can be neglected), the total instantaneous plate current of a triode is approximately equal to

$$i_p = \begin{cases} K(\mu v_{GK} + v_{PK})^{3/2}, & \mu v_{GK} + v_{PK} \geq 0\,, \\ 0\,, & \mu v_{GK} + v_{PK} < 0 \end{cases} \tag{1}$$

where v_{PK} denotes plate-to-cathode voltage. Here, μ and K are constants for a particular tube [3]. The equation (1) we may rewrite for the resistive load R_p using

$$i_p = \frac{V_{pp} - v_{PK}}{R_p}\,, \tag{2}$$

which yields

$$v_{GK} = \frac{1}{\mu}\Big\{\Big(\frac{V_{pp} - v_{PK}}{KR_P}\Big)^{2/3} - v_{PK}\Big\}. \tag{3}$$

If we consider v_{GK} as the input variable, and v_{PK} as the output variable in the transcharacteristic of the device, the equation (3) gives the input variable as the function of the output variable.

3 NEWTON'S SOLUTION OF THE GOVERNING NON–LINEAR EQUATION

For a continuously differentiable non-linear function $g(x)$, the equation

$$G(x) = 0 \tag{4}$$

can be solved numerically using a iterative procedure, namely the Newton's method. The algorithm can be expressed [12] as

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For n=0,1,2,..., until satisfied, do:
    Calculate x_{n+1} = x_n - g(x_n)/g'(x_n)
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where $g'(x)$ denotes the derivative of g with respect to variable x. For any v_{GK} , the corresponding v_{PK} could be found by iteratively solving the equation

$$g\big(v_{PK}\big) = \frac{1}{\mu}\Big\{\Big(\frac{V_{pp} - v_{PK}}{KR_P}\Big)^{2/3} - v_{PK}\Big\} - v_{GK} = 0 \tag{5}$$

using Newton's algorithm, where the needed derivative equals to

$$g'(v_{PK}) = \frac{1}{\mu}\Big\{\frac{-2}{3(KR_P)^{2/3}(V_{pp} - v_{PK})^{1/3}} - 1\Big\}. \tag{6}$$

4 METHOD 1

The reference [13] gives an overview of the methods for analytical evaluation the harmonic distortion in class-AB output stages. The *Method* 1, which was presented initially in [14], assumes the third order expansion of the transcharacteristic of the non-linear block around the quiescent point. This method was originally developed for semiconductors. It does not provide accurate results if the original transcharacteristic significantly deviates from the third-order polynomial. Since the harmonic distortion of the triode is consisted mainly of second and third harmonic, the method can be applied for the analysis of the triode amplifiers.

If we assume the alternating component of the grid voltage to be

$$e_g(t) = E_g \sin(\tau)\,, \tag{7}$$

where E_g denotes signal amplitude and τ is normalized time, the output signal of the non-linear block will be

$$v_{PK}(\tau) = f\,\lfloor E_g \sin(\tau)\rfloor]. \tag{8}$$

Here, $f(\cdot)$ defines non-linear relationship between input and output signals. If x_q denotes quiescent (zero excitation) value of input signal to the non-linear block (here, it is quiescent grid voltage), and x denotes instantaneous total grid voltage

$$x = x_q + e_g\,, \tag{9}$$

its maximum and minimum values will be

$$x_{MAX} = x_q + E_g\,, \tag{10}$$

$$x_{MIN} = x_q - E_g\,. \tag{11}$$

If we define

$$f'_0 \equiv \frac{\mathrm{d}}{\mathrm{d}x} f(x_q)\,, \tag{12}$$

$$f'_{PM} \equiv \frac{\mathrm{d}}{\mathrm{d}x} f(x_{MAX})\,, \tag{13}$$

$$f'_{NM} \equiv \frac{\mathrm{d}}{\mathrm{d}x} f(x_{MIN})\,, \tag{14}$$

the second and third harmonic distortion component will be

$$HD_2 \approx \frac{1}{8}\left|\frac{f'_{PM} - f'_{NM}}{f'_0}\right|, \tag{15}$$

$$HD_3 \approx \frac{1}{12}\left|\frac{f'_0 - \frac{1}{2}(f'_{PM} + f'_{NM})}{f'_0}\right|. \tag{16}$$

The derivative of the transcharacteristic function can be calculated analytically using the relation between the derivative of a function and the derivative of its inverse function [15]

$$v'_{GK} = \frac{\mathrm{d}v_{GK}}{\mathrm{d}v_{PK}} = \frac{1}{\mu}\Big\{\frac{-2}{3(KR_P)^{2/3}(V_{pp} - v_{PK})^{1/3}} - 1\Big\}, \tag{17}$$

$$f' = v'_{PK} = \frac{\mathrm{d}v_{PK}}{\mathrm{d}v_{GK}} = \frac{1}{v'_{GK}}\,. \tag{18}$$

where the derivative of the inverse function is determined during the iterative procedure of the Newton's method, and its final value can be used in (17), (18) and (12)–(14).

5 METHOD 2

Method 2 originates from D. C. Espley and it is described in details in [16]. It is also presented as the *Method* 2 in [13]. It assumes modelling of the non-linear transcharacteristic with a polynomial of order $2n$ ($n \geq 1$). For $n = 2$ we define

$$f_{PM/2} \equiv f(x_q + E_g/2)\,, \tag{19}$$

$$f_{nM/2} \equiv f(x_q - E_g/2)\,, \tag{20}$$

$$f_{PM} \equiv f(x_q + E_g)\,, \tag{21}$$

$$f_{NM} \equiv f(x_q - E_g)\,, \tag{22}$$

$$f_0 \equiv f(x_q)\,. \tag{23}$$

The second and third harmonic distortion component will be

$$HD_2 \approx \frac{3}{4}\left|\frac{f_{PM} + f_{NM} - 2f_0}{f_{PM} + f_{PM/2} - f_{NM} - f_{NM/2}}\right|, \tag{24}$$

$$HD_3 \approx \frac{1}{2}\left|\frac{f_{PM} - 2f_{PM/2} - f_{NM} + 2f_{NM/2}}{f_{PM} + f_{PM/2} - f_{NM} - f_{NM/2}}\right|. \tag{25}$$

6 SIMPLIFIED FORMULA FOR HD_2

If the third and higher harmonics are negligible, it is usual to approximate the second harmonic distortion using simple formula [8, 16, 17]

$$HD_2 \approx \frac{1}{2}\left|\frac{f_{PM} + f_{NM} - 2f_0}{f_{PM} - f_{NM}}\right|. \tag{26}$$

7 RELATIONSHIP BETWEEN INTERMODULATION AND HARMONIC DISTORTION

Suppose that the non-linear transcharacteristic of the system can be expanded about the operating point by mean of power series

$$e_p = a_0 + a_1 e_g + a_2 e_g^2 + a_3 e_g^3 \tag{27}$$

where e_g and e_p are the alternating parts of the grid and plate voltage. If the input signal is composed of two cosines

$$e_g = P\cos(pt) + Q\cos(qt) \tag{28}$$

the harmonic distortion and intermodulation components are [17]

$$\begin{aligned} e_p = {} & \left(\tfrac{1}{2}a_2P^2 + \tfrac{1}{2}a_2Q^2\right) + \left(a_1P + \tfrac{3}{4}a_3P^3 + \tfrac{3}{2}a_3PQ^2\right)\cos(pt) \\ & + \left(a_1Q + \tfrac{3}{4}a_3Q^3 + \tfrac{3}{2}a_3P^2Q\right)\cos(qt) + \tfrac{1}{2}a_2P^2\cos(2pt) \\ & + \tfrac{1}{2}a_2Q^2\cos(2qt) + \tfrac{1}{4}a_3P^3\cos(3pt) + \tfrac{1}{4}a_3Q^3\cos(3qt) \\ & + a_2PQ[\cos(p+q)t + \cos(p-q)t] \\ & + \tfrac{3}{4}a_3P^2Q[\cos(2p+q)t + \cos(2p-q)t]\,. \end{aligned} \tag{29}$$

We may define n-th order intermodulation as the ratio of the respective intermodulation component and the fundamental. If the amplitudes P and Q are equal, for weakly non-linear transcharacteristic the harmonic distortion and intermodulation factor are

$$HD_2 \approx \frac{1}{2}\frac{a_2}{a_1}P\,, \tag{30}$$

$$HD_3 \approx \frac{1}{4}\frac{a_3}{a_1}P^2\,, \tag{31}$$

$$IM_2 \approx \frac{a_2}{a_1}P^2 = 2HD_2\,, \tag{32}$$

$$IM_3 \approx \frac{3}{4}\frac{a_3}{a_1}P^2 = 3HD_3\,. \tag{33}$$

In such a way, the intermodulation can be approximated using calculated harmonic distortions.

8 NUMERICAL FOURIER ANALYSIS

The harmonic distortion and intermodulation can be calculated using Fourier analysis of the calculated output signal, if the input signal consists of a pure sine (for harmonic distortion) or a pair of sine functions with equal amplitudes (for intermodulation).

Let the continuous-time function $f(t)$ exist over the some time interval T_0. This continuous-time function can be, applying the Shannon's sampling theorem, represented by a sequence of discrete-time samples. For a given sequence of samples represented by

$$\{f(nT)\} = f(0), f(T), f(2T), \ldots f[(N-1)T]\,. \tag{34}$$

The discrete Fourier-transform (DFT) is a sequence of complex samples in the frequency domain

$$F_D(k\Omega) = \sum_{n=0}^{N-1} f(nT)e^{-j\Omega Tnk}, \quad k = 0, 1, \ldots, N-1 \tag{35}$$

where N denotes the number of samples, $\Omega = 2\pi/NT$ denotes the separation of the components in the frequency domain and T denotes sampling period. The discrete Fourier transformation F_D is an approximation of the continuous Fourier transform F

$$F(\omega)\big|_{\omega=\frac{2\pi k}{NT}} \approx TF_D\{f(nT)\} \tag{36}$$

where the error of the approximation due to aliasing is [18]

$$\varepsilon = \sum_{n=-\infty}^{\infty} F\Big(\frac{2\pi k}{NT} - \frac{2\pi n}{T}\Big), \quad n \neq 0\,. \tag{37}$$

Hence, the sampling period T has to be sufficiently small to obtain a sufficiently small approximation error. Also,

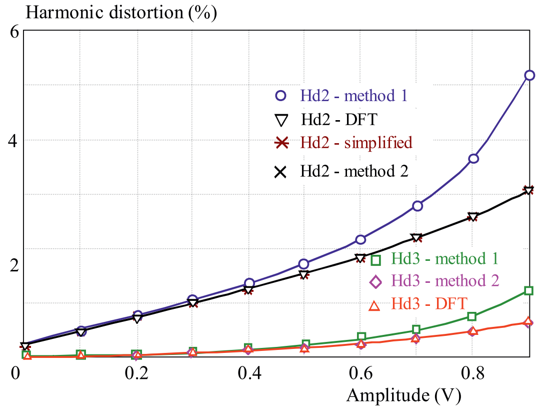

Fig. 2. Distortions for 12AX7 with quiescent point -1 V, $R_p = 150\,\mathrm{k}\Omega$, $V_{pp} = 180$ V

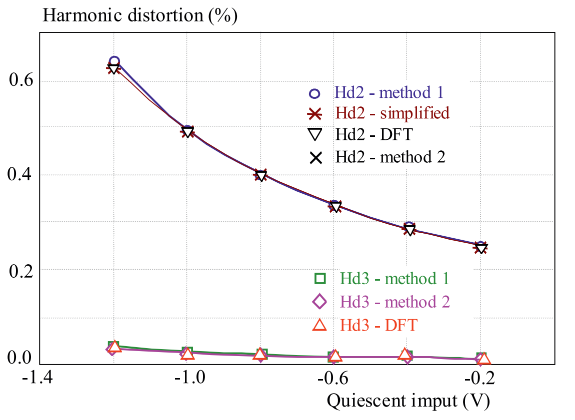

Fig. 3. Distortions for 12AX7 with alternating input amplitude 0.2 V, $R_p = 150\,\mathrm{k}\Omega$, $V_{pp} = 180$ V

Table 1. Calculated distortions for 12AX7 with quiescent point -1 V, $R_p = 150\,\mathrm{k}\Omega$, $V_{pp} = 180$ V

Ampl. (V)	HD_2 (%) Method 1	HD_2 (%) Method 2	HD_2 (%) (simplified)	HD_2 (%) DFT	HD_3 (%) Method 1	HD_3 (%) Method 2	HD_3 (%) DFT
0.1	0.2461	0.2455	0.2455	0.2455	0.0050	0.0042	0.0042
0.2	0.4987	0.4935	0.4936	0.4935	0.0205	0.0171	0.0171
0.3	0.7651	0.7469	0.7472	0.7469	0.0473	0.0390	0.0390
0.4	1.0545	1.0087	1.0094	1.0086	0.0876	0.0711	0.0711
0.5	1.3793	1.2828	1.2843	1.2825	0.1445	0.1147	0.1147
0.6	1.7586	1.5741	1.5768	1.5731	0.2238	0.1723	0.1723
0.7	2.2241	1.8893	1.8940	1.8870	0.3355	0.2474	0.2474
0.8	2.8355	2.2388	2.2465	2.2334	0.4989	0.3459	0.3457
0.9	3.7257	2.6395	2.6522	2.6272	0.7582	0.4780	0.4773
1.0	5.2878	3.1249	3.1458	3.0957	1.2439	0.6643	0.6618

Table 2. Calculated distortions for 12AX7 with alternating input amplitude 0.2 V, $R_p = 150\,\mathrm{k}\Omega$, $V_{pp} = 180$ V

Quiescent input (V)	HD_2 (%) Method 1	HD_2 (%) Method 2	HD_2 (%) (simplified)	HD_2 (%) DFT	HD_3 (%) Method 1	HD_3 (%) Method 2	HD_3 (%) DFT
-0.2	0.2478	0.2469	0.2469	0.2469	0.0060	0.0051	0.0051
-0.4	0.2858	0.2846	0.2846	0.2846	0.0077	0.0066	0.0066
-0.6	0.3356	0.3337	0.3337	0.3337	0.0102	0.0086	0.0086
-0.8	0.4030	0.3999	0.4000	0.3999	0.0141	0.0118	0.0118
-1	0.4987	0.4935	0.4936	0.4935	0.0205	0.0171	0.0171
-1.2	0.6441	0.6341	0.6343	0.6341	0.0321	0.0264	0.0264

the length of the sequence N divided by the time-period of the each analysed frequency component in spectrum must be an integer number, to avoid spurious components and influence of the picket-fence effect [19, 20]. The values of the time-domain sequence have to be calculated iteratively, using Newton's method for each point $f(nT)$. Finally, IM and HD factors are calculated using the ratio of the respective components in DFT. Since algorithm converges very rapidly, for all further presented numerical DFT calculations the execution time was within few seconds even on an Intel Celeron machine with clock frequency 1.7 GHz.

9 RESULTS — HARMONIC DISTORTION

For a 12AX7 triode, the parameters in (3) were taken as $K = 1.73 \times 10^{-6}$ and $\mu = 83.5$, which had been previously extracted and presented in [3]. The Newton's iteration was repeated until the difference between two successive approximations of the function value was below 10^{-6}. The DFT analysis was performed in MATLAB using function FFT [19]. For DFT, sampling frequency was 100 kHz, input signal frequency was 1 kHz and number of samples was 100. This way, the second component in the discrete spectrum was the fundamental (1 kHz). HD_2 was calculated as the ratio of the amplitudes of the third and the first line, and HD_3 was calculated as the ratio of the amplitudes of the fourth and the second line.

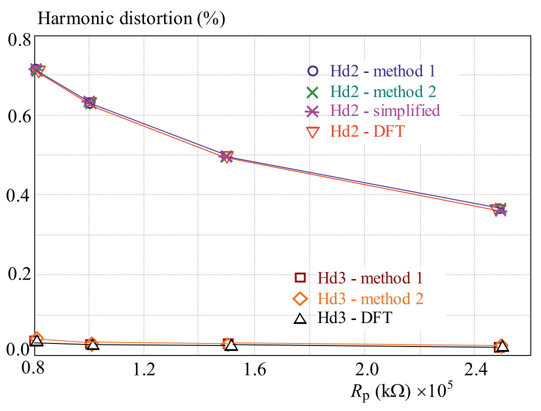

Fig. 4. Distortions for 12AX7 with alternating input amplitude 0.2 V, quiescent point −1 V and $V_{pp} = 180$ V

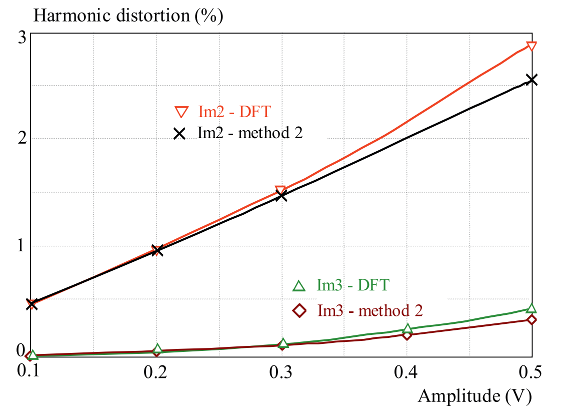

Fig. 5. Calculated intermodulations for 12AX7 with quiescent point −1 V, $R_p = 150\,\text{k}\Omega$, $V_{pp} = 180$ V

Table 3. Calculated distortions for 12AX7 with alternating input amplitude 0.2 V, quiescent point −1 V and $V_{pp} = 180$ V

R_p (kΩ)	HD_2 (%) Method 1	HD_2 (%) Method 2	HD_2 (%) (simplified)	HD_2 (%) DFT	HD_3 (%) Method 1	HD_3 (%) Method 2	HD_3 (%) DFT
80	0.7204	0.7122	0.7124	0.7122	0.0301	0.0230	0.0230
100	0.6347	0.6277	0.6278	0.6277	0.0263	0.0208	0.0208
150	0.4987	0.4935	0.4936	0.4935	0.0205	0.0171	0.0171
250	0.3628	0.3593	0.3593	0.3593	0.0147	0.0129	0.0129

Table 4. Calculated intermodulations for 12AX7 with quiescent point −1 V, $R_p = 150\,\text{k}\Omega$, $V_{pp} = 180$ V

Ampl. (V)	IM_2 (%) Method 2	IM_2 (%) DFT	IM_3 (%) Method 2	IM_3 (% DFT
0.1	0.4909	0.4927	0.0127	0.0128
0.2	0.9870	1.0013	0.0512	0.0528
0.3	1.4938	1.5458	0.1171	0.1263
0.4	2.0174	2.1564	0.2133	0.2475
0.5	2.5656	2.8942	0.3442	0.4529

Table 1 gives HD_2 and HD_3 calculated using all methods presented for different amplitudes of alternating grid voltage. The quiescent grid voltage was set to −1 V, R_p was 150 kΩ and V_{pp} was 180 V. Since the single-ended triode amplifier mainly suffers from second and third-order harmonic distortion, it is reasonable to expect that *Method* 1 produce accurate enough results. DFT based calculations can be used as the reference in evaluating the results. It can be seen that for moderate amplitudes all three analytical methods give reliable results, and that for large amplitudes *Method* 1 predicts greater HD_2 distortion. Simplified method gives almost identical results as *Method* 2. All methods show general trend of the increase in distortion when increasing the input signal amplitude. The results are comparable with those presented in [3].

Figure 2 presents graphically the results from Table 1. Table 2 presents HD_2 and HD_3 calculated using all presented methods for different quiescent grid voltages. The amplitude of alternating grid voltage was set to 0.2 V, R_p was 150 kΩ, and V_{pp} was 180 V. It can be seen that the increase of the negative grid bias increases the non-linear distortion. This is in accordance with the classical literature [8]. Figure 3 presents graphically the results from Table 2.

In Table 3 the distortions were calculated for different load resistances. The amplitude of alternating grid voltage was set to 0.2 V, R_p was 150 kΩ, and V_{pp} was 180 V. It can be seen that the increase of the load resistance decreases the non-linear distortion, which is also in accordance with the classical literature [8]. The results from Table 3 are depicted in Figure 4.

10 RESULTS — INTERMODULATION

The Newton's iteration was repeated until the difference between two successive approximations of the function value in each time instant was below 10^{-6}. The DFT analysis was performed in MATLAB using function FFT [21]. For DFT, sampling frequency was 100 kHz and number of samples was 1000. The frequencies of the two-tone, equal-amplitude input signal were 4 kHz and 5 kHz. This way, the 41$^{\text{st}}$ component in the discrete spectrum represented 4 kHz, and 11$^{\text{st}}$ component is the second order difference signal (1 kHz). As well, the 31$^{\text{st}}$ frequency line represents the third order difference signal (3 kHz). IM_2 was calculated as the ratio of the amplitudes of the 11$^{\text{st}}$ and the 41$^{\text{st}}$ line , and IM_3 was calculated as the ratio of the amplitudes of the 31$^{\text{st}}$ and the 41$^{\text{st}}$ line. The maximum input signal amplitude considered was 0, 5 V, since the magnitude of the composite signal must satisfy the negative grid voltage condition. The results of the calculations are given in Table 4 and Fig. 5. Also, the predicted IM_2 and IM_3 values are calculated using (32) and (33), based on the analytically calculated HD_2 and HD_3 using *Method* 2.

Comparing the DFT and HD-based calculations, it may be concluded that the IM factors can be calculated accurately using simple HD formulae for small and moderate amplitudes. When the amplitude of the composite signal approaches maximum allowed, the HD-based calculations tend to show lower values of intermodulation.

11 CONCLUSIONS

In this paper a novel numerical approach in calculation of the harmonic distortion (HD) and intermodulation distortion (IM) of vacuum-triode audio amplifiers was developed, suitable for applications using brute-force of modern computers. Since the 3/2 power law gives only the transcharacteristic inverse of a vacuum triode amplifier, unknown plate currents are determined iteratively using Newton's method. Using the resulting input/output pairs, harmonic distortions and intermodulations of a vacuum triode amplifier were calculated using discrete Fourier transform and three different analytical methods. The fast and accurate determination of the harmonic distortion and intermodulation is always possible using DFT and Newton's method, since the algorithm converges very rapidly. For DFT, the sampling frequency and sequence length must be carefully selected to avoid aliasing and erroneous results due to picket fence effect and spurious frequency content. All presented analytical methods produce comparable results for small and moderate magnitudes of the excitation. Also, they are computationally simpler than DFT. *Method* 1 is computationally less demanding than *Method* 2, but for the magnitudes greater than approximately 50 % of the maximum allowed method fails to give accurate results. *Method* 2, which is computationally more demanding, gives reliable results both for HD_2 and HD_3. From the presented calculations, significant difference in HD_2 does not occur even with large input amplitudes. The third (simplified) method gives practically the same results as *Method* 2. Although the second harmonic distortion dominates, any single-ended vacuum triode amplifier exhibits a moderate third-order harmonic distortion too. The simple relations between harmonic distortion and intermodulation give reliable approximations of IM2 and IM3 for moderate excitation, and thus may be used as computationally simpler alternative instead of the DFT. The presented approach should also be potentially applied in modelling of transmitter triodes.

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Martin Dadić was born in 1970 in Split, Croatia. He received Dipl Ing, MSc and PhD degrees in electrical engineering from the Faculty of Electrical Engineering and Computing, University of Zagreb in 1994, 1997 and 2000, respectively. Currently, he is an associate professor of electrical engineering at his alma mater. His main research interests lie in the union of electromagnetic devices modelling and signal processing.