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COMMUNICATIONS

ON INDUCTION HEATING — CONDUCTOR EXCITED BY EXTERNAL FIELD

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Electromagnetic field in a banded strip conductor excited by external AC voltage driven coil is analyzed. Inhomogeneous wave equation describing this axis-symmetrical configuration is deduced and solved to find the induced current density and the directional energy flux density (Poynting vector) in the conductor.

Keywords: induction heating, equivalent induction, Poynting vector

1 INTRODUCTION

Different configurations to create heat using electricity and particularly those based on induction heating [1] are know in practice. Some of cases are theoretically analyzed and well understood. In this contribution a less known, nevertheless theoretically interesting induction-heated configuration with a conducting ring of rectangle cross-section ($a \times b$) placed in a slot of transformer-sheet stacked core was investigated, Fig. 1.

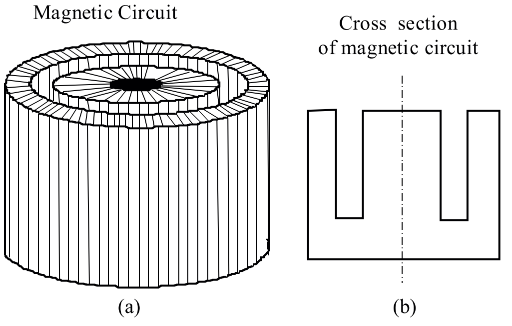

Fig. 1. Magnetic core to contain a ring-conductor: (a) – stacked magnetic core, and (b) – its cross-section

The conductor is primarily excited by magnetic field of flat-coils placed along its circumference and driven by a harmonic voltage source at technical frequencies (tens to hundreds of Hz), Fig. 2.

As a consequence of ac current in the driving coils there is a change of magnetic field that is practically perpendicular to the conductor circumferential walls Fig. 2(a). The magnetic flux radial trough the conductor is outside of it forming paths closed in the magnetic circuit, Fig. 2b. Inhomogeneous distribution of the eddy current-flow in conductor, depending on the depth, is due to the so called "skin effect" likely to push more of the current closer to the upper surface of the ring. It should be noted, that this is quite different from a well known skin-effect of in conductor placed inside a magnetic groove and excited by a current flowing through it endwise, that will be here referred to as "classical case" (see further).

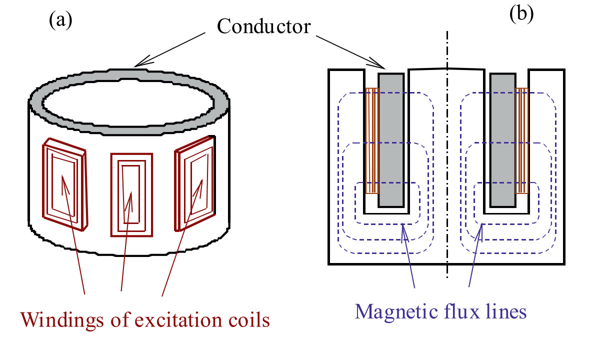

Fig. 2. Investigated configuration: (a) – conducting ring and the excitation coils, (b) – cross section of the configuration

The skin-effect is negligible if the depth of the conductor b is small compared with the electromagnetic waves penetration depth (that is a constant for given material and excitation frequency). As far as the conducting ring is high enough (filling a deep grove) there are layers where the electromagnetic power flows downwards (the real part of the Poynting vector is positive) but at the same time there are places with just opposite direction of the energy transport (the real part of the Poynting vector is negative). It is interesting (however should be expected) that the layers with the positive and negative energy flow are periodically repeated. As will be shown below, the real part of the Poynting vector is significantly lowered with the increased depth of the ring layer (*cf* Fig. 5).

2 MATHEMATICAL MODEL

To simplify the solution an one-dimensional model will be used providing all variables are depending on the depth (direction $+x$) only, and the in time harmonic (sine) excitation by the circumferential coils is homogeneous in space with $H_y(x)$, Fig. 3.

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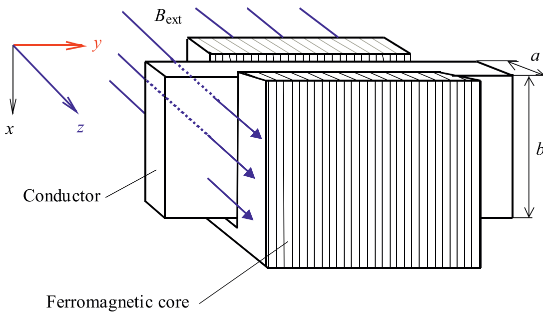


Fig. 3. Simplified configuration to solve. The "classical case" would be the same configuration with forced current flowing in y direction, hence wit the same B orientation.

In described configuration, of course, there are places with more intense and weaker excitation due to final dimensions of the driving coils. However, homogenization of the exciting field will not substantially change the model. Further assumption is that the permeability of the magnetic environment of the ring-conductor (the transformer-sheet stacked core) is high enough to concentrate the whole *emf* across the grove - filled with conductor of width $a << b$.

Now it is easy to find the wave equation for electric and magnetic field respectively. As already mentioned, the time changes of the field quantities are harmonic (sine-waveform) hence we shall use their complex representation (phasors): $E \longleftrightarrow \mathcal{E}$, $B \longleftrightarrow \mathcal{B}$, $H \longleftrightarrow \mathcal{H}$, *etc.*

The magnetic field will be treated as composed of external B_{ext} (exciting) and the "other" part B. In the conductor volume, where the solution is to be found the external field B_{ext} is solenoidal and in the technical frequency range the displacement current can be neglected (quasi-static approach), hence we can write

$$\operatorname{rot}\vec{\mathcal{E}} = -j\omega(\vec{\mathcal{B}} + \vec{\mathcal{B}}_{ext}) = -j\omega\mu_0(\vec{\mathcal{H}} + \vec{\mathcal{H}}_{ext}) \tag{1}$$

$$\operatorname{rot}\vec{\mathcal{H}} = \kappa\vec{\mathcal{E}} \tag{2}$$

$$\operatorname{rot}\operatorname{rot}\vec{\mathcal{H}} = \kappa\operatorname{rot}\vec{\mathcal{E}} = -\gamma^2\vec{\mathcal{H}} + \vec{\mathcal{H}}_{ext} \tag{3}$$

$$\nabla^2\vec{\mathcal{H}} - \gamma^2\vec{\mathcal{H}} = \gamma^2\vec{\mathcal{H}}_{ext} \tag{4}$$

where $\gamma = \alpha + j\beta = \sqrt{j\omega\mu_0\kappa}$, and $\delta = 1/\beta = \sqrt{2/(\omega\mu_0\kappa)}$ - is the so called *penetration depth.*

Equation (4) is a nonhomogeneous wave equation with solution given by a sum of general solution of homogeneous equation (with zero on the rigth side) and a particular solution of nonhomogeneous equation. For electric component of the electromagnetic field in a similar manner the following can be obtained

$$\operatorname{rot}\operatorname{rot}\vec{\mathcal{E}} = -j\omega\mu_0\operatorname{rot}\vec{\mathcal{H}} = -\gamma^2\vec{\mathcal{E}} \tag{5}$$

$$\nabla^2\vec{E} - \gamma^2\vec{E} = 0 \tag{6}$$

Deriving (6) we have taken into account that the rotation of the external (driving) field, which is solenoidal, must be zero. The general solution of (4) for vector $\vec{\mathcal{H}}(x) = \mathcal{H}_z(x)\vec{u}_z$ and that of homogeneous (6) for vector $\vec{\mathcal{E}}(x) = \mathcal{E}_y(x)\vec{u}_y$ with $\vec{u}_z, \vec{u}_y$ being unit vectors to respective directions (see Fig. 3) are

$$\mathcal{H}_y(x) = \mathcal{K}_1 e^{\gamma x} + \mathcal{K}_2 e^{-\gamma x} - \mathcal{H}_{ext} \tag{7}$$

$$\mathcal{E}_z(x) = \mathcal{Z}_0(\mathcal{K}_2 e^{-\gamma x} - \mathcal{K}_1 e^{\gamma x}). \tag{8}$$

Here $\mathcal{Z}_0 = \gamma/\kappa$ is the so called wave impedance, and unknown constants $\mathcal{K}_1, \mathcal{K}_2$ in (7) and (8) can be determined from two boundary conditions:

- as a consequence of Ampére law the intensity H at the bottom of ring conductor $x = b$ must be zero,
- Intensity of electric field ant the upper conducting ring surface $x = 0$ equals to time change of the driving magnetic field flux through the side wall of the ring per unit length.

$$\mathcal{H}|_{x=b} = 0 \tag{9}$$

$$\mathcal{E}|_{x=0} = -j\omega\mu_0 b\mathcal{H}_{ext} \tag{10}$$

using (9) and (10) we have

$$\mathcal{K}_1 = \mathcal{H}_{ext}\frac{\exp\gamma b + \gamma b}{\exp 2\gamma b + 1} \tag{11}$$

$$\mathcal{K}_2 = \mathcal{K}_1 - \gamma b\mathcal{H}_{ext} \tag{12}$$

Now we are ready to evaluate any of relevant field quantities in the conducting ring, including the thermal field (not in the scope of this contribution).

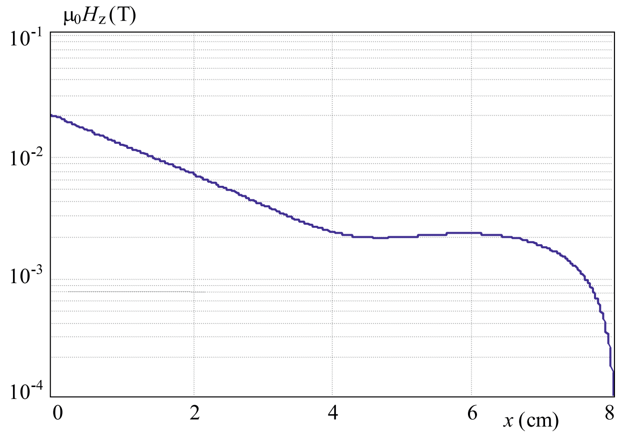
3 NUMERICAL RESULTS

Parameters used in calculations were: $a = 80$ mm, $b = 9$ mm, $f = 100$ Hz, $\mathcal{B}_{ext} = j2.5\times10^{-3}$ *ie* $B_{ext} = 2.5$ mT and its phase equals 90 deg, $I = 157$ A, conductivity $\kappa = 12.8\times10^6$ S/m (brass).

The distribution of magnetic field $H_z(x)$ or $B_z(x) = \mu_0 H_z(x)$ is in Fig. 4(a) and the distribution of electric field $E_y(x)$ to which the current density is proportional, is shown in Fig. 4(b).

It is well known that in the conductor profile in general the current density is higher in layers closer to the surface (skin-effect). In this case unlike in "classical case" magnetic and electric field are not monotonous but are possessing local minima. While the magnetic field towards the bottom $x \to b$ vanishis, the eletric fields tends to grow again. Figure 4 shows that magnetic field is in a quite wide interval practically of the same magnitude as is the external (driving) field. However the resulting magnetic field at the surface is substantially higer than $B_{ext} = 2.5$ mT.

Interesting is the dependence of the real part of complex Poynting vector (in conductor volume) as a function of the depth x, see Fig. 5. As far as the conductor is deep enough (relating to the penetration depth), the magnitude of real part of the complex Poynting vector



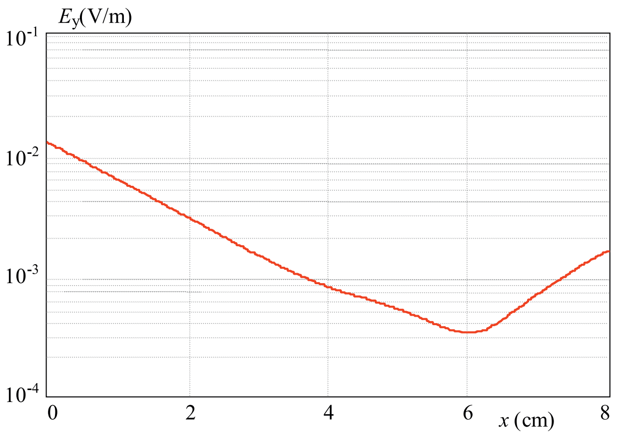


Fig. 4. Resulting fields: (a) – magnetic, (b)– electric

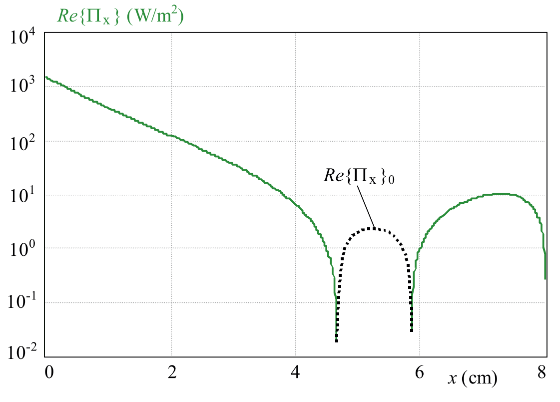


Fig. 5. Energy flux density (magnitude of Poynting vector)

falls to zero and with further growth of the depth (larger x) this vector changes its orientation to just opposite one, while its absolute value is growing. By other words there are layers with the mean value of power flow from top to bottom and layers with opposite (bottom to top) power flow orientation. These layers are alternating with each other being separated by points with zero Poynting real components.

4 DISCUSSION

Result shown in Fig. 5 has no analogy with "classical" case of a conductor placed in magnetic groove driven by a harmonic voltage source. The depth of the conductor or frequency does not matter. It is quite understandable since in the "classical" case of a groove — the electromagnetic wave propagates from the upper surface to the bottom of the conductor and due to the losses the its amplitude is monotonously decreasing. In here investigated case the surface inductive excitation is used. The consequence is that the energy is supplied to a wider area (at arbitrary x) and consequently there may be places where the mean value of the power flow changes its sign.

Concerning the analogy between the transmission line and propagating (plane) electromagnetic wave, it is important to notice that here investigated configuration is the case where (analogous) homogeneous line fed only by a source at one of its ends is not appropriate. Attempting to use (useful) analogy of that kind, one must rely on theory of homogeneous transmission line with continuously distributed sources of voltage or current [2-4].

There may arise a question: How it is possible that the energy propagates from the source to the conductor through its side wall, if there is clearly magnetic field component in z axis direction while electric component is in y direction giving rise to Poytnig vector with x axis component only. The answer is in that the side wall of the conductor is not in close contact with magnetic environment since there is an air gap between the conductor and magnetic media. Magnetic flux is thus partly closed trough the air gap causing magnetic field component also to the x axis and consequently non-zero z component of the Poynting vector, that is – oriented from the both side walls into the conductor. It should be noted that a discrepancy of this kind exists even in the "classical" case as far as the width of the conductor is less than that of the groove in magnetic media [5].

4.1 Impedance

The foregoing discussion aims to the difference in the space distribution of the field (current density and magnetic intensity) in described case if inductively excited conductor and in case of "classical" conductor in magnetic groove driven by external voltage source. This difference of course will have impact also on the "circuit impedance" ($\mathcal{Z}$), which will be here defined as ratio of the voltage-phasor on a unit length of conductor and the current-phasor of the total conductor current. Thus

$$\mathcal{Z} = \frac{\mathcal{E}|_{x=0} \times \text{unit length}}{\mathcal{H}|_{x=0}) \times a} \tag{13}$$

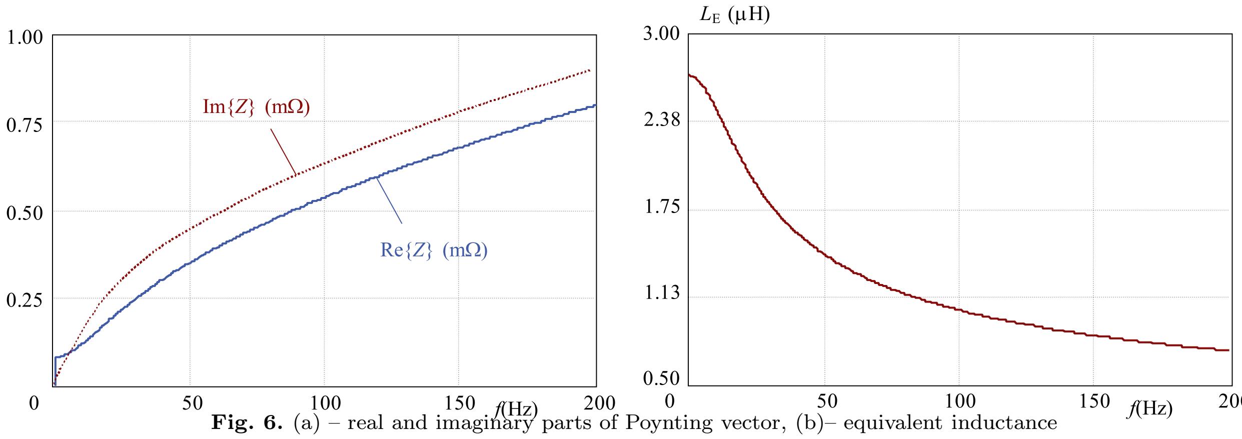


Fig. 6. (a) – real and imaginary parts of Poynting vector, (b)– equivalent inductance

with a being the conductor width. Inserting (7),(8) and (11),(12) too (13) we have

$$\mathcal{Z} = \frac{\mathcal{Z}_0\,\gamma\, b(1+\exp(2\gamma b)\times \text{unit length}/a}{(\gamma b+1)\exp(2\gamma b)-2\exp(\gamma b)-\gamma b} \tag{14}$$

where

$$\mathcal{Z}_0 = \sqrt{(\kappa + j\omega\mu)/j\omega\epsilon}$$

is the well known wave impedance of a conductor and $\omega = 2\pi f$ with f being the frequency.

After rewriting $\mathcal{Z}$ according to (14) as a function of frequency one can get the real and imaginary components dependeces resembling those of a "classical case, "however the real part of impedance (14) is in limit $f \to 0$ identical with the "classical case" impedance

$$\mathcal{Z}_C = Z_0 \coth(\gamma b) \times \text{unit length}/a$$

Frequency dependences of the real (resistivity) and imaginary (reactance) parts of the impedance, in the discussed case, are in Fig. 6 together with the equivalent inductance L_E.

5 CONCLUSION

Described model of inductively excited short-cut turn made from a strip conductor placed in a magnetic groove gives expected results as far as the conductor depth b is small compared to the sp called penetration depth δ. This is a common condition for the induction heating utilization in practice. Investigated model has shown surprising, nevertheless physically correct features if the last condition is not met, *eg* when using higher excitation frequency. Analysis has revealed that currently used analogy between the homogenous transmission line and the plane EM wave may fail even in an one dimensional approach. The solution to the given task can be attained by means of wave equations with a right side (nonhomogeneous) containing external driving magnetic field component. If we wish rather to rely on an analogy between the field equations and a transmission line then these must have homogeneously distributed driving sources (and not only passive elements).

Interesting is the resulting dependence of the real part of the Poynting vector (Fig. 5) having in some places opposite orientation than it is at the surface. This is due to penetration of the energy produced by external driving into the conductor volume through the side walls .

Acknowledgements

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References

[1] HARTSHORN, L. : Inductionn heat, Radio-frequency heating. London, G. Allen & Unwin, 1949.

[2] FRANEK, J.—KOLLÁR, M. : Active DC Line with Linear Density Distribution of Current Sources, Journal of Electrical Engineering.

[3] FRANEK, J.—BOJNA, I. : Model of Active Transmission Line with Prescribed Source Density, In: Meeting of Electromagnetic Field Theory and Measurement Departments of the Czech Republic and Slovakia, Prague, 12–13 September 2000, 40–43.

[4] FRANEK, J.—KOLLÁR, M. : Steady State Model of a DC Line with Distributed Sources, Journal of Electrical Engineering **46** No. 12 (1995), 389–393.

[5] DĚDEK, L.—DĚDKOVÁ, J. : Elektromagnetismus (2-nd edition), Vutium, Brno, 2000. (in Czech)

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