

# Fast off grid compressed sensing ISAR imaging algorithm

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To solve the off grid problem in compressed sensing (CS) based inverse synthetic aperture radar (ISAR) imaging, a fast and accurate algorithm has been proposed in the paper. By jointly estimating the off grid error and the sparse solution, off grid ISAR imaging is transformed into a joint optimization problem. Interestingly, it can be solved efficiently through two least squares problems based on first order Taylor approximation. When applied to complex sinusoids and quasi real ISAR data, the proposed algorithm has got better results than the conventional algorithm. Therefore, it is a promising off grid CS based ISAR imaging algorithm.

**Key words:** inverse synthetic aperture radar (ISAR), compressed sensing (CS), off grid

## 1 Introduction

In compressed sensing (CS) based inverse synthetic aperture radar (ISAR) imaging, continuous parameters are discretized to grids and the strong scattering points are supposed in the grid centers. However, the strong scattering points may not be in the grid centers. So there is mismatch between the predefined dictionary and actual dictionary. It will degrade the imaging performance significantly, which is named the off grid problem [1, 2].

To solve the problem, a direct method is to refine the grid, but it will increase the coherence in the sparsifying dictionary, which is adverse to reliable sparse recovery. Recently, methods based on nuclear norm optimization [3] and local optimization [4] have been proposed, however there is a requirement in the minimal distance between scatterers. Most of off grid CS methods jointly estimate the off grid error and the sparse solution. Bayesian method [5], matching pursuit algorithm [6] and alternating convex search algorithm [7] can be used with this model. However, their computation burdens are big. To decrease the heavy calculations, the first order Taylor approximation is used [5, 6]. In [6], a novel compressive sensing technique is proposed to alleviate the issues related with the reconstruction of the radar targets whose positions do not coincide with the assumed delay-Doppler grid. However, the computation burden is still very big. To reduce the computation burden further, we present a fast and accurate off grid ISAR imaging method in this work.

## 2 CS based ISAR imaging model

Supposing linear frequency modulated (LFM) signal is transmitted by radar, after dechirping, removing the

residual video phase and range compression, the signal in a range cell can be expressed as [8]

$$\mathbf{z}(\tau) = \sum_{k=1}^K \xi_k \exp(-j2\pi f_k \tau) \quad (1)$$

where  $\tau$  is azimuth time,  $K$  is the number of scattering points in the range cell,  $\xi_k$  and  $f_k$  are complex coefficient and Doppler frequency of the  $k$ -th scattering point respectively.

Then, ISAR imaging in cross-range can be transformed into a CS problem. Equation (1) can be written in matrix form

$$\mathbf{z} = \Phi \mathbf{s} \quad (2)$$

where  $\mathbf{z} \in \mathbf{C}^{N \times 1}$  is signal in the range cell,  $N$  is the total number of samples in cross-range,  $\Phi \in \mathbf{C}^{N \times N}$  represents Fourier basis matrix, and  $\mathbf{s} = [s_1, s_2, \dots, s_i, \dots, s_N] \in \mathbf{C}^{N \times 1}$  is  $K$  sparse coefficient vector which indicates the scattering points distribution of a target. By employing measurement matrix  $\Psi \in \mathbf{C}^{M \times N}$  ( $M < N$ ), where  $M$  is the dimension of measurements, signal is observed compressively

$$\mathbf{y} = \Psi \mathbf{z} + \mathbf{n} = \Theta \mathbf{s} + \mathbf{n} \quad (3)$$

where  $\mathbf{y}$  is measurements vector,  $\Theta = \Psi \Phi$ , and  $\mathbf{n}$  is noise.  $\mathbf{s}$  can be recovered by a compressed sensing recovery algorithm.

## 3 Off grid CS ISAR imaging algorithm

As the frequency components of  $\mathbf{z}$  are not known in advance, a predefined Fourier dictionary is employed, which can not guarantee all the frequency components of  $\mathbf{z}$  exactly lie on the grid points. As a result, the CS based

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imaging performance degrades significantly, which is the off grid problem.

To solve the problem, the following will give a fast off grid CS ISAR imaging algorithm. ISAR imaging requires identification of frequency grid nodes at which scatterers are present, *ie* the index set of “non-zero” coefficient, which can be accomplished by a CS recovery algorithm such as OMP algorithm. Then, the selected frequency grids are perturbed to get a better fit to the measurements, *ie* the new optimization problem is [6]

$$\arg \min_{s_i, \delta f_i} \left\| \mathbf{y} - \sum_{i=1}^K s_i \Theta(f_i + \delta f_i) \right\|_2 \quad \text{s.t. } |\delta f_i| \leq \Delta/2 \quad (4)$$

where  $i$  is scatterer index,  $s_i$  is coefficient of the  $i$ -th scatterer,  $\delta f_i$  is unknown perturbation of the  $i$ -th scatterer from its closest grid node  $f_i$ ,  $\Delta$  is the size of grid, and  $\Theta(f_i + \delta f_i)$  is the column of  $\Theta$  with parameters  $f_i + \delta f_i$ . To get the joint minimization, coefficients and frequency perturbation can be updated sequentially in the following way.

First initialize  $f_i^{(1)}$  to grid centers  $f_i$  and get an initial coefficient vector estimation

$$\mathbf{s}^{(1)} = \arg \min_{\mathbf{s}} \left\| \mathbf{y} - \sum_{i=1}^K s_i \Theta(f_i^{(1)}) \right\|_2.$$

Update from  $n = 1$  until convergence

$$[\delta f_1^{(n)} \dots \delta f_K^{(n)}] = \arg \min_{|\delta f_i| \leq \Delta/2} \left\| \mathbf{y} - \sum_{i=1}^K s_i^{(n)} \Theta(f_i^{(n)} + \delta f_i) \right\|_2, \quad (5)$$

$$\mathbf{s}^{(n+1)} = \arg \min_{\mathbf{s}} \left\| \mathbf{y} - \sum_{i=1}^K s_i \Theta(f_i^{(n+1)}) \right\|_2 \quad (6)$$

where  $n$  and  $n + 1$  denote iteration index, and  $f_i^{(n+1)} = f_i^{(n)} + \delta f_i^{(n)}$ . Equation (6) is a standard least squares problem, while obtaining solution of the constrained non-linear optimization problem (5) is not easy. But linearization of the cost function in (5) around  $f_i^{(n)}$  will reduce the optimization complexity greatly.  $\Theta(f_i^{(n)} + \delta f_i)$  is approximated by its first order Taylor series as  $\Theta(f_i^{(n)} + \delta f_i) \approx \Theta(f_i^{(n)}) + \Theta'(f_i^{(n)}) \delta f_i$ , then (5) can be re-written as

$$[\delta f_1^{(n)} \dots \delta f_K^{(n)}] = \arg \min_{\boldsymbol{\gamma}} \left\| \mathbf{y}^{(n)} - \mathbf{A}^{(n)} \boldsymbol{\gamma} \right\|_2 \quad (7)$$

where  $\boldsymbol{\gamma} = [\delta f_1 \dots \delta f_K]$ ,  $\mathbf{y}^{(n)} = \mathbf{y} - \sum_{i=1}^K s_i^{(n)} \Theta(f_i^{(n)})$ ,

$\mathbf{A}^{(n)} \in \mathbb{C}^{M \times K}$  is defined as  $\mathbf{A}^{(n)} = \left[ s_1^{(n)} \frac{\partial \Theta}{\partial f_1^{(n)}}, \dots, s_K^{(n)} \frac{\partial \Theta}{\partial f_K^{(n)}} \right]$  and can be easily calculated. Now (7) is a least squares problem, so its closed-form solution is

$$[\delta f_1^{(n)} \dots \delta f_K^{(n)}] = ((\mathbf{A}^{(n)})^H \mathbf{A}^{(n)})^{-1} (\mathbf{A}^{(n)})^H \mathbf{y}^{(n)}. \quad (8)$$

As perturbations should be real, (8) is modified to  $[\delta f_1^{(n)} \dots \delta f_K^{(n)}] = \text{Re}\{((\mathbf{A}^{(n)})^H \mathbf{A}^{(n)})^{-1} (\mathbf{A}^{(n)})^H \mathbf{y}^{(n)}\}$ . So the solution to the main problem (4) can be written as

$$\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} + \text{Re}\{((\mathbf{A}^{(n)})^H \mathbf{A}^{(n)})^{-1} (\mathbf{A}^{(n)})^H \mathbf{y}^{(n)}\}, \quad (9)$$

$$\mathbf{s}^{(n+1)} = (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \mathbf{y} \quad (10)$$

where  $\mathbf{B} = [\Theta(f_1^{(n+1)}), \dots, \Theta(f_K^{(n+1)})]$ . In a word, the optimal estimation of  $\mathbf{s}$  and  $\mathbf{f}$  is calculated through (9) and (10) iteratively until convergence.

As the proposed algorithm can directly get the solutions of (5) and (6) during iterations, it is much faster than the algorithm proposed in [6]. Experiments show that the proposed algorithm only needs about 1% time of the algorithm in [6] for typical step value. Moreover, the algorithm in [6] need choose step size, which is difficult in real application.

## 4 Experiment results and discussion

The effectiveness of the proposed algorithm is demonstrated by the following experiments.

First, the proposed algorithm is applied to simulated complex sinusoids. Signal  $\mathbf{z}$  with length  $N = 320$  contains  $K = 5$  complex sinusoids. Their normalized frequencies are distributed randomly over  $[0, 1)$  and amplitudes are complex unit circle. Measurement matrix  $\Phi$  is random Gaussian matrix with  $M = 64$ . Gaussian white noise is added to the signal, which makes SNR to vary from 0 to 40 dB. 500 trials are run for a fixed SNR. Average frequency estimation errors are shown in Fig. 1. It can be seen that the proposed algorithm can supply more accurate frequency estimation than OMP algorithm.

Then, the proposed algorithm is applied quasi real Mig-25 aircraft ISAR data. The data is provided by the US Naval Research Laboratory. The stepped frequency radar operates at 9 GHz and has a bandwidth of 512 MHz. For each pulse, 64 complex range samples are saved. Sparse basis is  $128 \times 128$  discrete Fourier matrix and measurement matrix is  $64 \times 128$  Gaussian random matrix. Figures 2 and 3 are the imaging results. It can be seen that the proposed algorithm has got a better image than OMP algorithm, as it can solve off grid problem in ISAR imaging.

## 5 Conclusion

A novel fast and accurate off grid CS ISAR imaging algorithm has been proposed, which transforms the joint optimization problem into two least squares problems. Its effectiveness has been verified by experiments of simulated complex sinusoids frequency estimation and quasi real ISAR data imaging.

Our scientific contribution is that we propose a simple and fast off grid CS algorithm. The off grid problem in CS has attracted much attention recently. However, the existing off grid algorithms are very complex. Interestingly,

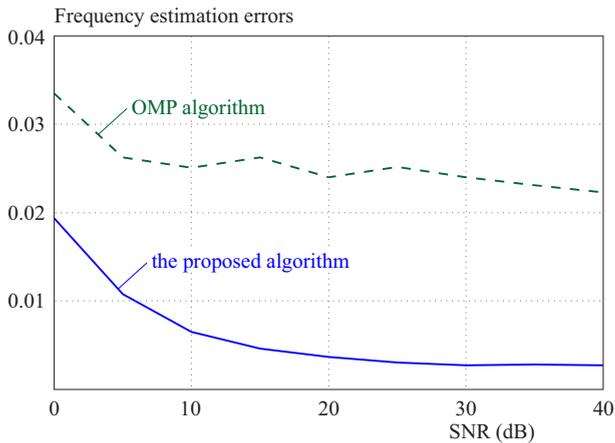


Fig. 1. Frequency estimation errors in different SNRs

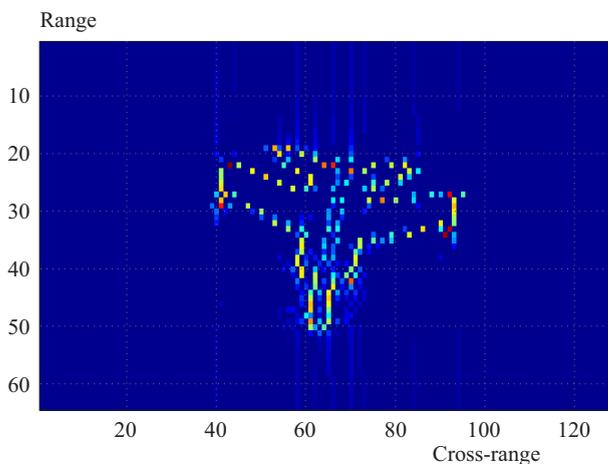


Fig. 2. Imaging result by OMP algorithm

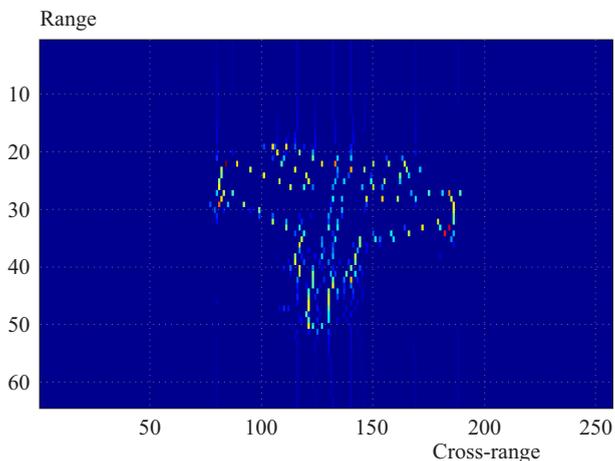


Fig. 3. Imaging result by the proposed algorithm

the proposed off grid algorithm is much simpler and faster than the existing algorithms, as it transforms the joint optimization problem into two least squares problems.

The off grid problem becomes one of the major constraints in promoting the application of CS. The proposed algorithm can solve off grid problem very well and is very

simple and fast. Moreover, the novel algorithm can be used not only for ISAR imaging but also for other CS based applications.

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