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Optimal design of digital low-pass filters using multiverse optimization

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The designs of first- and second-order digital low-pass filters with infinite impulse response (IIR) are presented in this letter, utilizing a meta-heuristic optimization technique. Firstly, the analog transfer functions of the first and second- order filters are considered, followed by the application of an L_1-norm-based multi-verse optimization algorithm to directly emulate their magnitude-frequency response in the digital domain. The obtained magnitude-frequency response shows superior matching with the analog counterpart for different cut-off frequencies of the first- and second-order filters, as well as varying quality factors for the second-order filter. In comparison to the filter's magnitude-frequency response obtained through traditional bilinear transform and advanced operators, the proposed technique accurately manifests the analog magnitude-frequency response in the digital domain.

Keywords: analog-to-digital transform, digital low-pass filter, L_1 error fitness function, optimization

1 Introduction

In signal processing, filtering is a widely used technique that modifies the input signal's spectrum to produce an output signal with desired spectral qualities. Filters are widely accepted due to their adaptability using a programmable processor, precision in performance, multiplexing, data logging, thermal stability, and perfect reproducibility. Digital filters can be designed as either finite impulse response (FIR) or infinite impulse response (IIR). While FIR systems are stable and have phase linearity, IIR systems require less memory and have a reduced level of computational complexity while providing a sharp cut-off. As a result, IIR-based designs of digital filters are typically preferred [1].

In recent years, there has been extensive research on designing digital filters, which can be done either by designing directly in the digital domain or by replicating the analog domain response in the digital domain using analog-to-digital transforms. The latter approach is most widely used, with the bilinear transform being the most popular example, as it preserves stability and corresponding filter order [2]. However, the linearity of the bilinear transform is limited to the Laplace operator only up to 0.3π of the entire digital range, requiring oversampling [3-5]. Several other transforms have been developed using different interpolation, optimization, and fractional delays, such as Al-Alaoui's interpolation between rectangular and trapezoidal rules for better magnitude approximation [6-8]. The use of fractional delay in place of unit delay, and the designs based on fractional interpolation have also been suggested as alternatives to the bilinear and Al-Alaoui transforms [9, 10]. Additionally, various designs have been developed using different optimization algorithms to extend the linearity region to accommodate a wide frequency band during transformation [11-14]. However, the use of bilinear and other transforms to convert the analog transfer function in the digital domain often leads to large-magnitude errors, particularly when the sampling frequency is relatively low. Therefore, this work proposes an alternative method to design digital filters directly in the digital domain using multi-verse optimization in L_1-sense. The examined examples [1, 6, 9, 10] demonstrate that the proposed technique out-performs traditional transformations in matching the analog magnitude-frequency response in digital filter designing.

The rest of the letter is organized as follows: Section 2 deals with the problem formulation of first- and second-order digital filter design. Section 3 briefly describes the multi-verse optimization algorithm utilized for optimizing the transfer function's coefficients. Section 4 discusses the obtained simulation results and comparisons. Section 5 provides the conclusion of the paper.

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2 Problem formulation and design

2.1 First-order filter design

The generalized first-order low-pass filter transfer function in analog domain can be defined as

$$H_{first}(s) = \frac{\omega_c}{s + \omega_c}, \tag{1}$$

where $s = j\omega$, and angular cut-off frequency $\omega_c = 2\pi fc$. The magnitude-frequency response of (1) for matching in the digital domain is [2]

$$\left|H_{first}(j\omega)\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \tag{2}$$

which has a unity gain at DC, $1/\sqrt{2}$ at the corresponding cut-off frequencies, and zero at infinity. For any cut-off frequency in (1), the operator *'S'* can directly be approximated by the first-order digital transfer function as [15]

$$s \rightarrow \frac{2}{T}\frac{a_0 + a_1 z^{-1}}{b_0 + b_1 z^{-1}}$$

Therefore,

$$H_1(z) = H_{first}(s)_{s \rightarrow \frac{2 a_0 + a_1 z^{-1}}{T b_0 + b_1 z^{-1}}} \tag{3}$$

Now, for any specific cut-off frequency, the coefficients a_0, a_1, b_0, and b_1 can be optimized. Correspondingly, the error objective function is defined as:

$$\|E_1\| = \sum_{\omega} \left|H_{first}(s) - H_1(z)_{z=e^{j\omega}}\right|, \tag{4}$$

where ||·|| provides the norm of function.

2.2 Second-order filter design

The generalized second-order low-pass filter transfer function in analog domain with quality factor (Q) factor can be defined as [15]

$$H_{second}(s) = \frac{{\omega_c}^2}{s^2 + \frac{s\omega_c}{Q} + {\omega_c}^2} \tag{5}$$

while the magnitude-frequency response can be expressed as [15]

$$\left|H_{second}(j\omega)\right| = \frac{{\omega_c}^2}{\sqrt{\left(\frac{\omega\omega_c}{Q}\right)^2 + ({\omega_c}^2 - \omega^2)^2}} \tag{6}$$

In the case of a first-order low-pass filter, the gain is one at DC and decreases to zero at infinity. However, for a second-order filter, the gain reaches a maximum value of Q at the cut-off frequency. For Q factors greater than, $1/\sqrt{2}$, a resonant peak forms at the cut-off, and the magnitude-frequency response is no longer monotonically decreasing. Due to the nature of the second-order low-pass filter, intermediary calculations may result in discontinuities, making more sophisticated measures necessary compared to the first-order case [15, 16]. Therefore, for a given cut-off frequency and Q factor, the coefficients a_0, a_1, b_0, and b_1 can be optimized.

The error objective function can be expressed as

$$\|E_2\| = \sum_{\omega} \left|H_{sec\,ond}(s) - H_2(z)_{z=e^{j\omega}}\right| \tag{7}$$

where

$$H_2(z) = H_{sec\,ond}(s)_{s \rightarrow \frac{2 a_0 + a_1 z^{-1}}{T b_0 + b_1 z^{-1}}} \tag{8}$$

The reason of using error function in L_1-sense is that it offers a smooth response with less overshoot and ripples at discontinuity points [17].

3 Description of L1-MVO optimization

The multi-verse optimizing (MVO) algorithm is a nature-inspired stochastic population-based algorithm that utilizes the concept of many parallel universes. This algorithm relies on three main components, namely, wormholes, black holes, and white holes. White holes, produced by the big bang or the collision of universes, play a role in the expansion of the universes. In contrast, a black hole represents the antithesis of a white hole, conferring a strong gravitational pull that attracts everything, including light. Wormholes function as time-space tunnels allowing travel within and between different universes. The inflation rates present in each universe also contribute to the expansion. A mathematical model is developed using these three ideas to evaluate local research, exploitation, and exploration. MVO utilizes wormholes to explore the search regions and employs black and white hole concepts to exploit the identified search region. Every solution and variable is interpreted as a universe and object in the search region, respectively. As a result, the fitness function directly correlates with the inflation rate [18-20].

The algorithm operates successfully by following the guidelines outlined below.

- With rising inflation rates, white holes are more likely and while black holes are less likely to exist.
- To get the possible best universe, the objects may move randomly regardless of the inflation rate.
- Through white holes and black holes, the object often moves from a higher inflation rate to a lower inflation rate.

The optimization process is initiated by randomly generating a set of universes. In each iteration, objects are moved from higher inflation rates to lower ones through white holes and black holes. In addition, wormholes are used to randomly teleport objects to obtain the best universe. The updating process is based on the following equation [18]:

$$x_i^j = \begin{cases} \begin{cases} x_j + TDR + P, & if\ r_3 < 0.5, \\ x_j - TDR + P, & if\ r_3 \geq 0.5, \end{cases} & if\ r_2 < WEP \\ x_{RW}^i, & if\ r_2 \geq WEP \end{cases} \tag{9}$$

where $P = \left(r_4\left(ub_j - lb_j\right) + lb_j\right)$ and x_j represents the j^{th} parameter of the best individual universe and x_{RW}^i is the j^{th} element of a solution picked by the roulette wheel selection mechanism. lb_j and ub_j indicate the lower and upper bounds of j^{th} variable, x^j represents the j^{th} parameter of i^{th} universe with r_2, r_3, r_4 being random numbers ranging from 0 to 1. TDR and WEP are traveling distance rate and wormhole existence probability and are defined as follows [18, 19]

$$WEP = a + t\left(\frac{b-a}{T}\right), \tag{10}$$

where a, b, t are the minimum, maximum, and current iterations. T represents the maximum number of allowed iterations.

$$TDR = 1 - \frac{t^{\frac{1}{p}}}{T^{\frac{1}{T}}} \tag{11}$$

where p indicates the exploitation accuracy [18]. The pseudocode is given in the Algorithm 1.

Algorithm 1 Pseudocode

```
for each universe indexed by i do
  for each object indexed by j do
    r2 = random([0, 1]);
    if r2 < WEP then
      r3 = random([0, 1]);
      r4 = random([0, 1]);
      if r3 < 0.5 then
        U(i, j) = Best_universe(j) + TDR * ((ub(j) −
        lb(j)) * r4 + lb(j));
      else
        U(i, j) = Best_universe(j) − TDR * ((ub(j) −
        lb(j)) * r4 + lb(j));
      end if
    end if
  end for
end for
```

Firstly, a collection of universes is generated randomly as part of the optimization process. With each iteration, objects pass through white and black holes as they go from higher to lower inflation rates. Meanwhile, wormholes randomly transport stuff to obtain the best universe. For optimization control parameters, the number of universes is 50 for 200 iterations with upper and lower bounds restricted to 3 and –3, respectively.

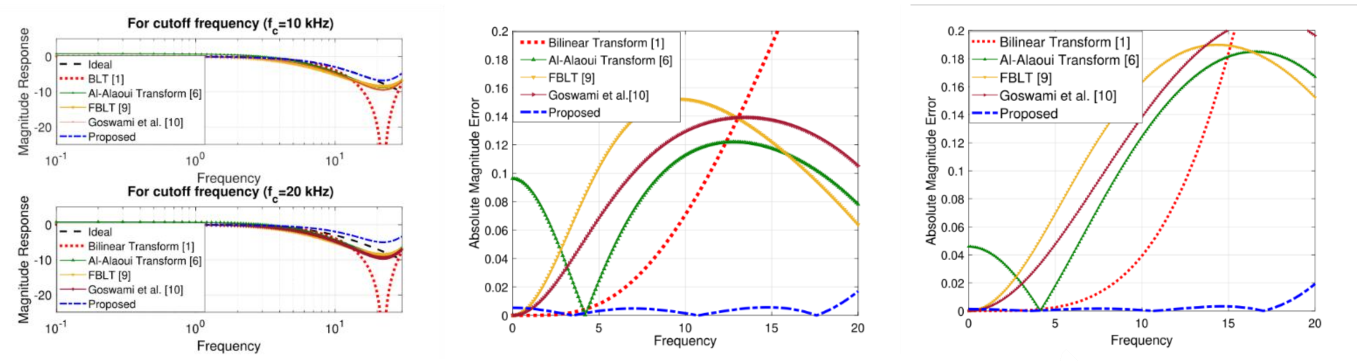
4 Simulation analysis and comparison with existing techniques

The simulations are conducted using an Intel(R) Core(TM) i5-7200U CPU processor (2.50 GHz) with 8 GB RAM, Windows 10 operating system, and MATLAB (2017a) software.

4.1 First-order filter design

The simulations were conducted by optimizing the error function defined in (4) for various cut-off frequencies with a sampling frequency of f_s=44.1 kHz. For a cut-off frequency of f_c=10 kHz, the coefficients a_0, a_1, b_0, and b_1 were optimized and yielded the values of 2.6907, –2.67748, 0.54717, and 2.9074, respectively. Similarly, for another cut-off frequency of f_c=20 kHz, the coefficients a_0, a_1, b_0, and b_1 were optimized and resulted in values –0.63958, 0.63352, –2.7921, and –0.5068, respectively. The magnitude-frequency response of the obtained transfer function is shown in Fig. 1(a). Additionally, Fig. 1(a) illustrates the magnitude-frequency responses of the same filter obtained by different transforms. The bilinear transformation, which is widely used in filter design to transform the analog domain into the digital domain, has been used along with other transforms such as the Al-Alaoui transform [6], fractional bilinear transform (FBLT) [9], and the transform proposed by Goswami et al. [10] to approximate the filter's analog magnitude-frequency response. As shown in Fig. 1(a), the proposed technique exhibits better matching of the magnitude-frequency response than the other existing methods.

The absolute magnitude error (AME) plot in Fig. 1(b) indicates that the matching magnitude error of the proposed design remains less than 0.015 for the cut-off frequency f_c=10 kHz and outperforms others. For cut-off frequency, f_c=20 kHz, the AME plot in Figure 1(c) also supports the fact that the proposed design performs better than the other techniques used for transformation. The enlisted sum of absolute magnitude error (SAME) comparison done in Tab. 1 further confirms the noteworthy improvement obtained by the proposed technique in approximating the analog magnitude-frequency response compared to others for both considered cut-off frequencies.



(a) (b) (c)

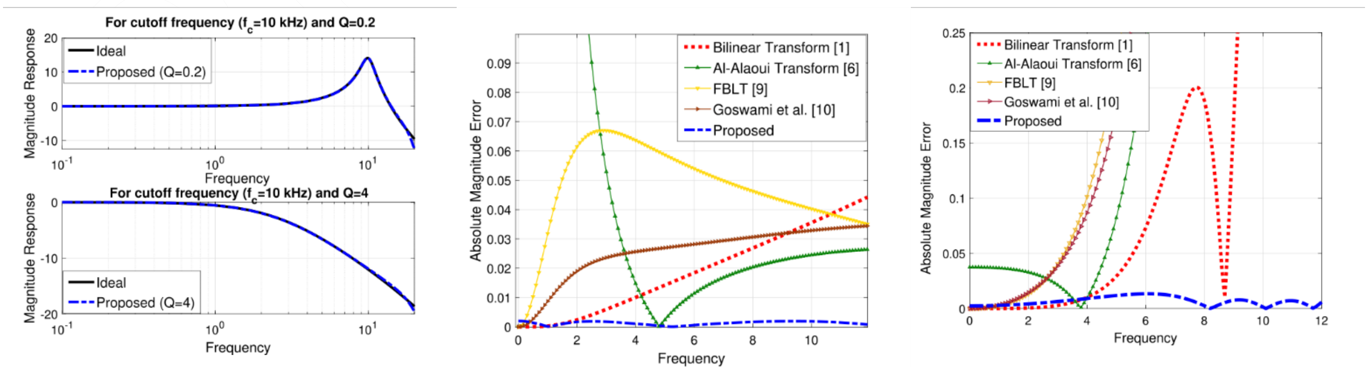
Fig. 1. (a) Magnitude-frequency response of the first-order filter for different cut-off frequencies f_c, (b) absolute magnitude error comparison of first-order filter for f_c=10 kHz, and (c) absolute magnitude error comparison of the first-order filter for f_c=20 kHz.

Table 1. Statistical comparisons for first-order low-pass filter

Method	SAME (f_c = 10 kHz)	SAME (f_c = 20 kHz)
BLT [1]	21.5167	49.0989
Al-Alaoui Transform [6]	17.6100	28.3587
FBLT [9]	20.8256	30.8143
Goswami et al. [10]	19.6677	32.3680
Proposed	0.7859	3.43

4.2 Second-order filter design

The coefficients of (7) are optimized for two different values of quality factor Q, with the cut-off frequency set to 10 kHz for the second-order filter. The corresponding coefficients a_0, a_1, a_2, b_0, b_1, and b_2 are obtained as 0.8689, –0.42339, –1.8690, 0.51631, 2.2003 and 1.83575, respectively, for Q=0.2. For Q=2, the optimum values of coefficients a_0, a_1, a_2, b_0, b_1, and b_2 are obtained as –0.8637, –0.13968, 0.99031, –0.6344, –2.36533 and –0.80692, respectively. The magnitude-frequency response obtained by the proposed technique with the ideal analog response for both values of Q is shown in Fig. 2(a), which closely matches the analog counterpart in the region of interest. After applying the other transforms to the filter for Q=0.2 and Q=2, the absolute magnitude error plots are shown in Figs. 2(b) and (c), respectively. The magnitude error plots obtained from the proposed technique are restricted up to 0.0025 for Q=0.2, and 0.015 for Q=2, which are the least among others for the respective value of Q. The statistical comparison in Tab. 2 also supports the fact of having better magnitude-frequency response matching provided by the proposed design.



(a) (b) (c)

Fig. 2. (a) Magnitude-frequency response of the second-order filter for different values of Q and f_c=10 kHz, (b) absolute magnitude error comparison of second-order filter for Q=0.2 and f_c=10 kHz, (c) absolute magnitude error comparison of second-order filter for Q=2 and f_c=10 kHz.

Table 2. Statistical comparisons for second-order low-pass filter for f_c=10 kHz

Method	SAME ($Q = 0.2$)	SAME ($Q = 2$)
BLT [1]	2.3165	26.043
Al-Alaoui Transform [6]	11.9904	58.45
FBLT [9]	5.7294	71.258
Goswami et al. [10]	3.0956	64.36
Proposed	0.1524	0.8099

5 Conclusion

In this letter, a technique has been presented for designing first- and second-order digital filters using the multi-verse optimization algorithm. The optimization process involved optimizing the coefficients of the generalized analog transfer functions for different cut-off frequencies directly in the digital domain, based on the L_1-norm error objective function. The resulting magnitude-frequency response of the transfer function accurately matched the analog response, with the least magnitude error. Hence, optimizing the filter coefficients directly in the digital domain can be considered an alternative approach to traditional methods that utilize analog to digital transforms in filter designing.

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