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SHORT COMMUNICATION

Measuring impulse response and nonlinear distortions using exponential frequency-modulated signals

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Exponential Frequency-Modulated (EFM) signals, characterized by their exponentially changing instantaneous frequency, are valuable in radar, sonar, and communication systems. This paper explores the application of EFM signals for measuring impulse response and nonlinear distortions in electronic devices. The EFM signal testing method, which involves recording and analyzing the device's output in response to EFM signals, provides insights into amplitude-frequency, phase-frequency responses, and impulse response. The spectral density analysis reveals a 3 dB/octave decrease in high-frequency regions. An innovative measurement method is proposed, involving convolution with a time-reversed and amplitude-modulated EFM signal, simplifying traditional approaches. MATLAB simulations validate the method, highlighting its efficacy in comprehensive device performance assessment.

Keywords: exponential frequency modulation, impulse response, spectral density, convolution

1 Introduction

Exponential frequency-modulated (EFM) signals are a type of signal in which the instantaneous frequency changes exponentially with time. These signals have applications in various fields including radar, sonar, and communication systems.

Farina proposed an idea for simultaneous measurement of impulse response and nonlinear distortions using an EFM signal [1].

$$s(t) = S_m \cdot \sin\left\{2\pi \cdot \frac{f_{\text{initial}} \cdot T_{EFM}}{\ln\frac{f_{\text{final}}}{f_{\text{initial}}}} \cdot \left[\exp\left(\frac{t}{T_{EFM}} \cdot \ln\frac{f_{\text{final}}}{f_{\text{initial}}}\right) - 1\right]\right\}. \quad (1)$$

Here, f_{initial} and f_{final} are the initial and final frequencies of the test signal (selected according to the measured frequency range), T_{EFM} is duration of the test EFM signal in seconds.

Equation (1) can be rewritten as follows:

$$s(t) = S_m \cdot \sin\left[2\pi \cdot \frac{f_{\text{initial}} \cdot T_{\text{EFM}}}{ln\frac{f_{\text{final}}}{f_{\text{initial}}}} \cdot \left(\left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)^{t/T_{EFM}} - 1\right)\right]. \quad (2)$$

From this, the total phase of the oscillation is obtained

$$\theta(t) = 2\pi \cdot \frac{f_{\text{initial}} \cdot T_{EFM}}{\ln\frac{f_{\text{final}}}{f_{\text{initial}}}} \cdot \left[\left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)^{t/T_{EFM}} - 1\right] \quad (3)$$

and the instantaneous frequency

$$f(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_{\text{initial}} \cdot \left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)^{t/T_{EFM}}. \quad (4)$$

It follows from Eqn. (4) that $f=f_{\text{initial}}$ for $t=0$, and $f=f_{\text{final}}$ for $t=T_{EFM}$.

The EFM signal testing method is a technique used for measuring and analyzing the characteristics of electronic devices, systems, or circuits, involving the use of signals with exponentially changing frequency, known as EFM signals. This method is commonly employed to assess the device's response, frequency characteristics, nonlinear distortions, and more. In the EFM signal testing method, the device's output response is recorded by applying a signal with exponentially changing frequency, and further analysis is conducted on parameters such as amplitude-frequency response, phase-frequency response, and impulse response. This method can provide valuable information about the performance and characteristics of the device.

2 Method

2.1 Spectral density of the EFM signal

Let us express the specific power of the EFM signal, see Eqn. (1), assuming that its instantaneous frequency changes slowly compared to its current value at any given moment. Then, the instantaneous power of the EFM signal can be associated with the average power of a sinusoidal signal over one period, which is independent of its frequency:

$$p(t) = \frac{S_m^2}{2} \quad (5)$$

From the mathematical relationship between power and energy of a finite signal, the energy of the EFM signal

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for a differentially small-time interval *dt* can be expressed as

$$dE(t) = p(t)dt = \frac{S_m^2}{2}dt. \qquad (6)$$

Now let us take the derivative of the instantaneous frequency of the EFM signal and find the rate of change of its instantaneous frequency:

$$v_f(t) = \frac{df(t)}{dt} = \frac{f_{\text{initial}} \cdot \ln\frac{f_{\text{final}}}{f_{\text{initial}}}}{T_{EFM}} \cdot \left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)^{t/T_{EFM}}. \qquad (7)$$

Its increment for a differentially small time interval *dt* is

$$df(t) = v_f(t)dt = \frac{f_{\text{initial}} \cdot \ln\frac{f_{\text{final}}}{f_{\text{initial}}}}{T_{EFM}} \cdot \left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)^{t/T_{EFM}} dt. \qquad (8)$$

Expressing (7) through (4), we obtain a relationship for the rate of change of the instantaneous frequency not in terms of time, but in terms of the current value of the instantaneous frequency itself:

$$v_f(f) = \frac{\ln\left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)}{T_{EFM}} \cdot f. \qquad (9)$$

Let us find an expression that approximately describes the spectral energy density of the EFM signal, using the rule of differentiating two parametric functions:

$$G(f) = \frac{dE}{df} = \frac{\frac{dE}{dt}}{\frac{df}{dt}} = \frac{p(t)}{v_f(t)} = \frac{\frac{S_m^2}{2}}{\frac{f_{\text{initial}} \cdot \ln\frac{f_{\text{final}}}{f_{\text{initial}}}}{T_{EFM}} \cdot \left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)^{t/T_{ESS}}} \qquad (10)$$

Further, we will substitute according to (9) into equation (10):

$$G(f) = \frac{dE}{df} = \frac{S_m^2 \cdot T_{EFM}}{2 \cdot \ln\left(\frac{f_{\text{final}}}{f_{\text{initial}}}\right)} \cdot \frac{1}{f} \qquad (11)$$

The frequency *f*, which serves as an argument for the energy spectral density of the EFM signal, is in the denominator of expression (11). It follows that when the frequency of the EFM signal doubles, the value of the function G(*f*) will decrease by a factor of 2. This corresponds to a 3 dB/octave (10 dB/decade) decrease in high-frequency regions. Note that the computation of the EFM signal's energy in the spectral domain gives

$$E_f = \int_{f_{\text{initial}}}^{f_{\text{final}}} \frac{S_m^2 \cdot T_{EFM}}{2 \cdot \ln\frac{f_{\text{final}}}{f_{\text{initial}}}} \cdot \frac{1}{f}\, df = S_m^2 \cdot \frac{T_{EFM}}{2}. \qquad (12)$$

This yields the same result as integrating expression (5) over the duration of the EFM signal (from 0 to T_{EFM}).

The amplitude spectral density of the EFM signal can be calculated by taking the square root of expression (11). An example of the amplitude spectral density of the EFM signal with parameters f_{initial}=11.7 Hz, f_{final}=24 kHz, T_{EFM}=43 s is shown in Fig. 1.

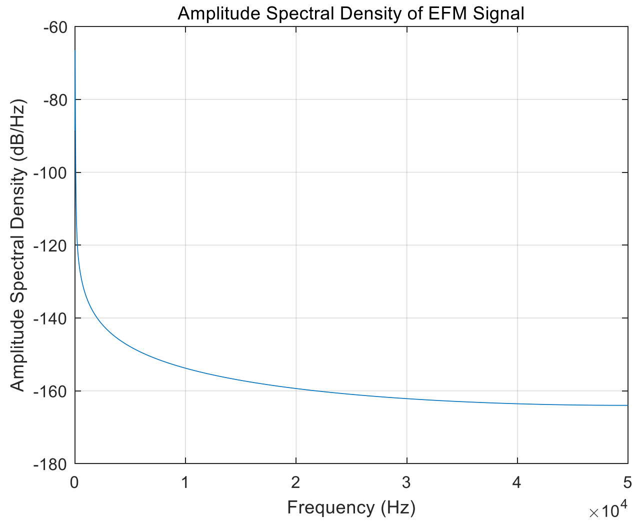

Fig. 1. Theoretical amplitude spectral density of the EFM signal

Thus, the main portion of the EFM signal's energy is concentrated in the frequency range from f_{initial} to f_{final}. In contrast to a measurement signal with linear frequency modulation (LFM), whose spectral density is uniform in a similar frequency range, the EFM signal exhibits a 3 dB/octave decrease in high-frequency regions [2].

2.2 Proposed method

The measurement of the impulse response and nonlinear distortions of the device under test using the proposed method is carried out as follows:

1. Initially, it is necessary to apply the EFM signal to the input of the device under test and record its output signal.

2. Next, the convolution operation must be performed on the output signal with a specially prepared EFM signal, which is derived from the original EFM signal by time reversal and amplitude modulation. The resulting signal after convolution contains the impulse responses of the device under test.

The measurement model is schematically illustrated in Fig. 2.

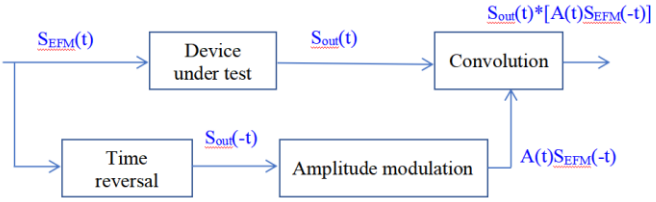

Fig. 2. Measurement model according to the proposed method (symbol * indicates the convolution procedure)

It is worth noting that traditional indirect methods of measuring the impulse response (IR) of electronic and acoustic devices involve operations such as

• transitioning from the time domain to the frequency domain

$$s_{in}(t) \rightarrow S_{in}(j\omega), \quad s_{out}(t) \rightarrow S_{out}(j\omega),$$

• determining the complex-valued frequency-dependent transfer coefficient of the device by dividing the spectral densities of the output and input signals:

$$K(j\omega) = \frac{S_{out}(j\omega)}{S_{in}(j\omega)}.$$

Finally, calculating the amplitude-frequency response (AFR), phase-frequency response (PFR), and impulse response (IR) of the device based on the found transfer coefficient. In subsequent publications, analysis will be conducted on the additional processing procedures of the output signal described in the proposed method, which represents the response of the tested device to the EFM signal: its convolution with the time-reversed and amplitude-modulated original EFM signal.

To verify the effectiveness of the above method, there is a MATLAB code that demonstrates how to evaluate the impulse response, frequency characteristics, and nonlinear distortion of testing and measuring electronic equipment using an EFM signal [3, 4].

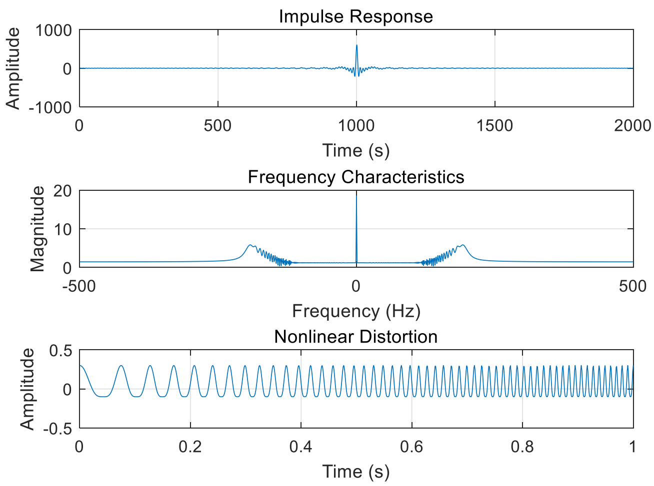

Fig. 3. Testing and measuring electronic equipment using an EFM signal

In this experiment, the code simulates an EFM signal and a measured signal with nonlinear distortion. It then calculates and plots the impulse response, frequency characteristics, and nonlinear distortion of the signals using the EFM approach. This code is a simplified illustration, and real-world scenarios may involve more advanced techniques and actual measurements.

3 Conclusion

EFM signal is commonly used to test and measure the characteristics of electronic devices, systems, or circuits, especially for evaluating their pulse response, frequency characteristics, and nonlinear distortion. By utilizing EFM signals, it is possible to acquire information about multiple parameters in a single test, thereby providing a more comprehensive assessment of device performance. This signal exhibits linear growth or decay within a certain range, which offers certain advantages in testing, such as the ability to rapidly gather frequency response data.

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