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ROBUST MIMO PID CONTROLLER DESIGN USING ADDITIVE AFFINE–TYPE UNCERTAINTY

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The paper deals with an application of the recently developed robust decentralized controller design approach. The innovation consists in using a new type of plant uncertainty description — namely the additive affine-type uncertainty. Related robust stability conditions are developed and modified for the application in the decentralized controller design. A case study shows application of the proposed approach.

K e y w o r d s: decentralized control, frequency domain, robust stability, additive affine type uncertainty

1 INTRODUCTION

MIMO systems usually arise as interconnection of a finite number of subsystems. In the case of such systems practical reasons often make restrictions on controller structure necessary or reasonable. The controller split into several local feedbacks becomes a decentralized controller (DC). Compared with centralized full-controller systems the DC structure brings about certain performance deterioration; on the other side there are important benefits, *eg* hardware, operation and design simplicity, and reliability improvement. In principle, the DC design comprises two steps: 1) selection of control configuration 2) design of local controllers. In the Step 2, the independent design (*eg* [3], [5], [9]), the sequential design (*eg* [2]), or some appropriate detuning method ([1], [12]) may be applied.

In this paper, the independent design philosophy has been applied; according to it local controllers are designed without considering interactions with other subsystems. The effect of interactions is transformed into bounds for individual controller designs to guarantee robust stability and desired performance of the overall system.

The paper proposes a frequency domain approach to the robust decentralized controller design for continuous-time systems described by a set of transfer function matrices. To describe such systems a new uncertainty model has been introduced, namely the additive affine-type uncertainty. For this uncertainty type, the specific M-Δ structure based robust stability conditions have been developed and modified for the decentralized controller design. Unlike standard robust approaches to the DC, the proposed one allows to use a full nominal model including the nominal interactions thus relaxing the conservatism of the robust stability conditions.

The paper is organized as follows: preliminaries and problem formulation are given in Section 2; the additive affine type uncertainty is introduced in Section 3 along with the robust stability conditions modified for the decentralized controller design. A case study dealing with robust PID controller design for the glass tube manufacturing plant follows in Section 4. Conclusions are drawn at the end of the paper.

2 PROBLEM STATEMENT AND PRELIMINARIES

Consider a MIMO system $G(s)$ and the controller $R(s)$ in the standard feedback configuration (Fig. 1), where $G(s) \in R^{m\times m}$ and $R(s) \in R^{m\times m}$ are transfer function matrices and w, u, y, e, d are respectively vectors of reference, control, output, control error and disturbance of compatible dimensions. The return difference matrix $F(s) \in R^{m\times m}$ is

$$F(s) = [I + Q(s)] \tag{1}$$

where $Q(s) \in R^{m\times m}$, $Q(s) = G(s)R(s)$ is the open-loop transfer function matrix. The *Nyquist D-contour* comprises the imaginary axis $s = j\omega$ and an infinite semicircle into the right-half plane avoiding locations where $Q(s)$ has $j\omega$-axis poles. $N[k, g(s)]$ denotes the number of anticlockwise encirclements of the point $(k, j0)$ by the Nyquist plot of $g(s)$. The closed-loop characteristic polynomial of the system in Fig. 1 is

$$\det F(s) = \det\,[I + Q(s)] = \det[I + G(s)R(s)]\,. \tag{2}$$

If $Q(s)$ has n_q unstable poles, closed-loop stability can be verified using the Generalized Nyquist Stability Theorem.

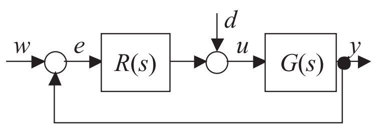

Fig. 1. Standard feedback configuration

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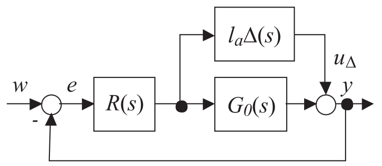

Fig. 2. Standard feedback configuration with additive unstructured uncertainty

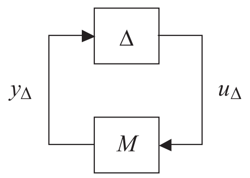

Fig. 3. $M-\Delta$ structure

Theorem 1(Generalized Nyquist Stability Theorem). *The feedback system in Fig.* 1 *is stable if and only if* $\forall s \in D$

$$\begin{aligned} &1.\ \det F(s) \neq 0\,, \\ &2.\ N\left[0, \det F(s)\right] = n_q\,, \end{aligned} \tag{3}$$

Eigenvalues of $Q(s)$ called *characteristic functions of* $Q(s)$ are defined to be the set of m algebraic functions $q_i(s)$, $i = 1, \dots, m$ [8] given as

$$\det\left[q_i(s) I_m - Q(s)\right] = 0\,, \quad i = 1, \dots, m\,. \tag{4}$$

Considering characteristic functions of $Q(s)$, the closed-loop characteristic polynomial can be rewritten as

$$\det F(s) = \det\left[I + Q(s)\right] = \prod_{i=1}^{m}\left[1 + q_i(s)\right]\,. \tag{5}$$

Characteristic loci (CL) are the set of loci in the complex plane $q_i(s)$, $i = 1, 2, \dots, m$ traced out by the characteristic functions of $Q(s)$, $\forall s \in D$. In terms of CLs Theorem 1 reads as follows.

Theorem 2. *The feedback system in Fig.* 1 *with the open-loop transfer function matrix* $Q(s)$ *is stable if and only if* $\forall s \in D$

$$1.\ \det\left[I + Q(s)\right] \neq 0\,, \tag{6}$$

$$2.\ \sum_{i=1}^{m} N\left\{0, \left[1 + q_i(s)\right]\right\} = n_q\,, \tag{7}$$

ie the sum of anticlockwise encirclements of $(0, j0)$ *contributed by the CL's of* $[I + Q(s)]$ *is to be* n_q.

When designing a controller, a major source of difficulty is the plant model inaccuracy. Then, instead of a single model the behaviour of a class of models is considered. Let $\widetilde{G}(s) \in \Pi$ be any member of a set of possible plants Π and $G_0(s) \in \Pi$ be the nominal model. A simple uncertainty model is obtained using unstructured uncertainty, *ie* a full complex perturbation matrix that can specify six commonly used perturbation types [10]. In the next development only the additive perturbation E_a is considered defining a family of perturbed plant models Π_a. Any member of the Π_a family satisfies

$$\widetilde{G}(s) = G_0(s) + E_a(s)\,. \tag{8}$$

In case of unstructured uncertainty, the magnitude of $E_a(s)$ is measured in terms of upper bound on its maximum singular value $\sigma_M(E_a)$

$$\sigma_M(E_a) \leq \ell_a(\omega) = \max_{\widetilde{G} \in \Pi_a} \sigma_M(E_a) \tag{9}$$

and the family of plants is defined as

$$\begin{aligned} \Pi_a &= \left\{\widetilde{G}(s)\colon \left|\widetilde{G}(s) - G_0(s)\right| \leq \ell_a(\omega)\right\}, \\ \ell_a(\omega) &= \max_{\widetilde{G} \in \Pi_a} \sigma_M\left[\widetilde{G}(s) - G_0(s)\right] \end{aligned} \tag{10}$$

where $\ell_a(\omega)$ is as a scalar weight on a normalized perturbation $\Delta(s)$

$$E_a(s) = \ell_a(s)\Delta(s)\,, \tag{11}$$

$$\sigma_M[\Delta(s)] \leq 1 \quad \forall s \in D\,. \tag{12}$$

Standard feedback configuration with the uncertain plant model (8) and (11) is depicted in Fig. 2.

The block scheme in Fig. 2 can be rearranged to obtain the general $M-\Delta$ structure where $M(s)$ represents the nominal model and $\Delta(s)$ the normalized perturbation (12) (see [10]).

Theorem 3 (Robust stability for unstructured perturbations, [10]). *Assume that the nominal system M(s) is stable (nominal stability) and the perturbations* $\Delta(s)$ *are stable. Then the* $M-\Delta$ *system in Fig. 3 is stable for all perturbations* $\Delta(s)$ *that satisfy* (8) *if and only if*

$$\sigma_M\left[M(j\omega)\right] < 1 \quad \forall \omega\,. \tag{13}$$

For the additive perturbation (11), $M = \ell_a M_a$ where (disregarding the negative sign which does not affect the resulting robustness condition).

$$M_a = R(s)\left[I + R(s)G(s)\right]^{-1}. \tag{14}$$

Problem Formulation. Consider a system with m subsystems given by a set of N transfer function matrices $G^k(s)$, $k = 1, \dots, N$ A robust decentralized controller is to be designed

$$\begin{aligned} R(s) &= \operatorname{diag}\{R_i(s)\}_{i=1,\dots m}\,, \\ &\det R(s) \neq 0 \quad \forall s \end{aligned} \tag{15}$$

where $R_i(s)$ is the transfer function of the i-th local controller. The designed controller has to guarantee 1) stability of the controlled plant over the entire operating range specified by the N transfer function matrices and described by means the additive affine type uncertainty (*ie* we have robust stability, RS), and 2) specified nominal performance (NP).

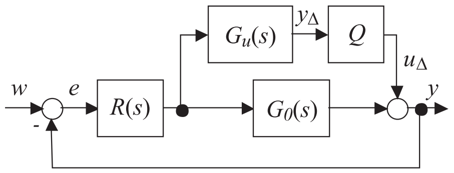

Fig. 4. Standard feedback configuration with additive affine type uncertainty

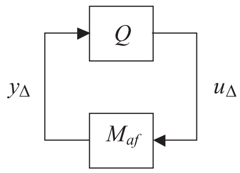

Fig. 5. $M_{af}-Q$ structure for the additive affine type uncertainty

3 MAIN RESULTS

First, it is necessary to find an appropriate frequency domain model of the uncertain plant. In previous works on that topic (*eg* [5], [6]) the nominal model used to be selected as the mean value parameter model and the uncertainty modelled as either additive or multiplicative (input and output) form with respect to the nominal model.

By analogy with the time domain, a novel type of frequency domain uncertainty has been proposed in this paper, namely the additive affine-type uncertainty. For such uncertainty type robust stability conditions are developed and modified for application in the decentralized controller design.

3.1 Robust stability condition for uncertain systems with additive affine type uncertainty

The additive affine type structure of $E_a(s)$ in (11) is as follows

$$E_a(s) = \sum_{i=1}^{p} G_i(s) q_i \tag{16}$$

where $G_i(s) \in R^{m\times m}$ for $i = 1\dots,p$; p is the number of uncertainties; $q_i \in [q_{i\min}, q_{i\max}]$ are parameters from a specified interval whereby $q_{i\min} + q_{i\max} = 0$. Transfer function matrices $G_i(s)$, $i = 1,\dots,p$ are assumed to have their poles located in the left half-plane. Substituting (16) to (8) yields

$$\widetilde{G}(s) = G_0(s) + \sum_{i=1}^{p} G_i(s) q_i \tag{17}$$

or in vector form

$$\begin{aligned}\widetilde{G}(s) = G_0(s) + [\, I_{q1} \quad \dots \quad I_{qp} \,]\,[\, G_1(s) \quad \dots \quad G_p(s) \,]^\top \\ = G_0(s) + QG_u(s) \quad (18)\end{aligned}$$

where $I_{qi} = q_i I_{m\times m}$, $Q = [\, I_{q1} \quad \dots \quad I_{qp} \,] \in R^{m\times(mp)}$, $G_u(s) = [\, G_1(s) \quad \dots \quad G_p(s) \,]^\top \in R^{(mp)\times m}$.

Standard feedback configuration from Fig. 2 modified for the additive affine type uncertainty is shown in Fig. 4.

Similarly as in case of the $M-\Delta$ structure, breaking the loop before and after Q, the above block scheme can be rearranged into the $M_{af}-Q$ structure shown in Fig. 5. where (without considering the negative sign)

$$M_{af} = G_u R(I + G_0 R)^{-1} = G_u (I + RG_0)^{-1} R\,. \tag{19}$$

By analogy with Theorem 3, the necessary and sufficient stability condition for the $M_{af}-Q$ structure is

$$\sigma_M\big(M_{af} Q\big) < 1\,. \tag{20}$$

Using norm properties, (20) can further be modified

$$\sigma_M\big(M_{af}\big)\sigma_M(Q) < 1 \tag{21}$$

where

$$\sigma_M(Q) = \lambda_M^{1/2}\big(QQ^\top\big) = \lambda_M^{1/2}\Big(\sum_{i=1}^{p} q_i^2 I\Big) = \sqrt{\sum_{i=1}^{p} q_i^2} \tag{22}$$

and $I \in R^{m\times m}$ is the identity matrix.

If absolute values of the lower and upper bounds for q_i, $i = 1,\dots,p$ equal, *ie* if

$$q_0 = \big|q_{i\min}\big| = \big|q_{i\max}\big| \tag{23}$$

then (21) can be modified to obtain robust stability condition stated in the next Corollary 1.

Corollary 1 (Robust stability for additive affine type uncertainty). *Assume that the nominal system M_{af} (19) is stable (nominal stability). Then the $M_{af}-Q$ system with the additive affine type uncertainty* (16) *in Fig. 5 is stable if and only if*

$$\sigma_M\big(M_{af}\big)\sqrt{\sum_{i=1}^{p} q_i^2} < 1\,. \tag{24}$$

If, moreover $\big|q_{i\min}\big| = \big|q_{i\max}\big|$ then (24) reads

$$\sigma_M\big(M_{af}\big) q_0 \sqrt{p} < 1\,. \tag{25}$$

Consider the nominal model $G_0(s) \in R^{m\times m}$ with m subsystems that can be split into the diagonal and the off-diagonal parts describing respectively models of decoupled nominal subsystems and nominal interactions $G_m(s)$.

$$G_0(s) = G_d(s) + G_m(s) \tag{26}$$

where $G_d(s) = \operatorname{diag}\{G_{di}(s)\}_{i=1,\dots,m}$, and $G_d(s) \neq 0$ $\forall\, s \in D$.

Factorizing (19) in terms of the split system (26) and the diagonal controller (15) yields

$$M_{af} = G_u\big(I + RG_0\big)^{-1} = G_u\big[R\big(R^{-1} + G_d + G_m\big)\big]^{-1}R = G_u\big(P + G_m\big)^{-1}. \quad (27)$$

where $R^{-1}(s)$ is supposed to exist and

$$P(s) = R^{-1}(s) + G_d(s) \quad (28)$$

is a diagonal matrix $P(s) = \operatorname{diag}\{p_i(s)\}_{i=1,\dots,m}$.

Conditions for stability of M_{af} (nominal stability) have been developed by further manipulating (27).

$$M_{af} = G_u\big(P + G_m\big)^{-1} = G_u \frac{\operatorname{adj}(P + G_M)}{\det(P + G_M)}. \quad (29)$$

If G_u is stable, nominal stability depends on the stability of the closed-loop characteristic polynomial

$$CLCP = \det\left[P(s) + G_m(s)\right]. \quad (30)$$

Factoring out $P(s)$ in (30) yields (s is omitted to simplify the notation)

$$CLCP = \det[P + G_m] = \det P \det\left[I + P^{-1}G_m\right]. \quad (31)$$

If simultaneously numerators of all entries of $P(s)$ are stable polynomials and $G_m(s)$ is stable, by applying the Small Gain Theorem [10] to (31) the sufficient stability condition is obtained in the following form [6]

$$\left\|P^{-1}G_m\right\| < 1 \text{ or equivalently } \left\|P^{-1}\right\| < \frac{1}{\|G_m\|}. \quad (32)$$

Applying the equivalence between spectral norms and singular values (32) can be rewritten as follows

$$\sigma_m(P) > \sigma_M\big(G_m(s)\big) \quad (33)$$

where $\sigma_m(\cdot)$ denotes the minimum singular value of the corresponding matrix.

Another formulation of nominal stability condition along with guaranteed nominal performance conditions under a decentralized controller are given in the next section.

3.2 Decentralized controller design for nominal stability

Nominal stability (NS) is a prerequisite for guaranteeing both the nominal performance (NP) and the robust stability (RS). Nominal stability of the uncertain feedback system in Fig. 2 means stability of M_{af} (19). If using decentralized controller, M_{af} is given by (27); its nominal stability will be examined by applying Theorem 1 to the characteristic polynomial (30). A detailed description of this approach can be found in [3], [5], [6].

A simple manipulation of (28) yields

$$I + R(s)[G_d(s) - P(s)] = 0 \quad (34)$$

where

$$G^{eq}(s) = G_d(s) - P(s) \quad (35)$$

is a diagonal matrix of collecting the so-called equivalent subsystems. The i-th equivalent subsystem is given by

$$G_i^{eq}(s) = G_{di}(s) - p_i(s), \quad i = 1, 2, \dots, m \quad (36)$$

and the i-th equivalent closed-loop characteristic polynomial is

$$CLCP_i^{eq} = 1 + R_i(s)G_i^{eq}(s), \quad i = 1, 2, \dots, m. \quad (37)$$

Conditions for nominal stability under a decentraized controller are obtained by applying Theorem 1 to (30).

COROLLARY 2 (Nominal stability under a DC). *The nominal closed-loop system M_{af} (19) under a decentralized controller (15) is stable if and only if $\forall\, s \in D$*

- *there exists a diagonal matrix $P(s) = \operatorname{diag}\{p_i(s)\}_{i=1,\dots,m}$ such that all equivalent subsystems $G_i^{eq}(s)$ (36) can be stabilized by their related local controllers $R_i(s)$, ie all equivalent closed-loop characteristic polynomials (37) have roots located in the open left half-plane;*
- *the following two conditions are met $\forall s \in D$:*

$$1.\ \det[P(s) + G_m(s)] \neq 0, \quad (38)$$

$$2.\ N[0, \det[P(s) + G_m(s)] = n_m \quad (39)$$

where n_m is the number of unstable poles of the matrix $M(s) = P(s) + G_m(s)$.

According to the Corollary 2, the problem to be solved in the robust decentralized controller design has reduced to finding a diagonal matrix $P(s) = \operatorname{diag}\{p_i(s)\}_{i=1,\dots,m}$ that simultaneously guarantees stability and specified nominal performance of the controlled plant over its entire operating range given by the additive affine-type uncertainty (16) (robust stability). This approach allows considering the full mean parameter value model as the nominal system.

A general method for choosing $P(s)$ is not available yet; this issue is subject of an ongoing research. Thus far, two basic methods for selecting $P(s)$ have been developed based upon the conditions stated in Corollary 1.

A. Choosing diagonal $P(s)$ with different entries

In [6], [11] two possible heuristic approaches have been proposed to find coefficients of $p_i(s)$. According to the independent design philosophy, $p_i(s)$ actually represent bounds for local controller design that are to be met to guarantee the full system performance (including stability).

General suggestions for choosing $P(s)$ are as follows: for stable $P^{-1}(s)$ and $G_m(s)$ the necessary and sufficient closed-loop stability condition is (33). A detuning method for choosing $P(s)$ with different entries is proposed in [7]).

In all cases, numerators of the chosen $p_i(s)$, $i = 1,\dots,m$ are to be stable polynomials. To guarantee robust stability, parameters of $p_i(s)$, $i = 1,\dots,m$ are to be further adapted so as to meet conditions of Corollary 2.

B. Choosing diagonal $P(s) = p(s)I$ with identical entries

This approach allows to achieve a specified performance in terms of the degree of stability for the full nominal system [3]. Entries of the diagonal matrix $P(s)$ are chosen such as to appropriately account for interactions $G_m(s)$.

Substituting $P(s) = \operatorname{diag}\{p_i(s)\}_{i=1,\dots,m}$ to (30) and equating to zero yields

$$\det[p_i(s)I + G_m(s)] = 0\,, \quad i = 1,\dots,m \tag{41}$$

which actually defines the m characteristic functions $g_i(s)$, $i = 1,\dots,m$ of the matrix $[-G_m(s)]$. If, moreover, $p(s)$ is taken to be any of the characteristic functions of $[-G_m(s)]$ then for fixed $\ell \in \{1,\dots,m\}$ and $p(s) = -g_\ell(s)$ we obtain

$$\det[P(s)+G(s)] = \prod_{i=1}^{m}[p(s)+g_i(s)] = \prod_{i=1}^{m}[-g_\ell(s)+g_i(s)] = 0\,. \tag{42}$$

For $P(s) = \operatorname{diag}\{g_\ell(s)\}_{m\times m}$ the closed-loop system is at the limit of instability (*ie* it has some poles on the imaginary axis and no poles in the RHP). Similarly, for $P(s) = \operatorname{diag}\{-g_\ell(s-\alpha)\}_{m\times m}$ the closed-loop will have poles with $\operatorname{Re} s \le -\alpha$ and its degree of stability will be α. Transfer functions of equivalent subsystems are

$$G_i^{eq}(s-\alpha) = G_{di}(s-\alpha) + g_\ell(s-\alpha)\,,\ i = 1,2,\dots,m\,. \tag{43}$$

Thus, for a fixed $\ell \in \{1,\dots,m\}$ and $\alpha \ge 0$ the diagonal controller $R(s)$ that stabilizes the equivalent subsystems (36) with the degree of stability α simultaneously guarantees the same degree of stability for the full system. The above results are summarized in the following Lemma.

Lemma 1 (Guaranteed nominal performance). *The nominal closed-loop system M_{af} (19) under a decentralized controller (15) is stable with the degree of stability α if and only if there exist such $\alpha \ge 0$ and $g_\ell(s)$, fixed $\ell \in \{1,\dots,m\}$, that $\forall s \in D$ and $i = 1,2,\dots,m$ the following two conditions are met $\forall i$, $\forall s \in D$*
1. $m_{i\ell}^{eq}(s-\alpha) = [-g_\ell(s-\alpha) + g_i(s)] \ne 0$
2. all equivalent closed-loop characteristic polynomials (37) have roots with $\operatorname{Re} s \le -\alpha$.

To stabilize individual equivalent subsystems using local controllers, any graphical SISO frequency domain design technique can be applied (*eg* Bode plots, Neymark D-partition method).

3.3 Robust decentralized controller design

Combining results of Corollary 1 (robust stability for additive affine type uncertainty) and Corollary 2 (nominal stability under a decentralized controller) or Lemma 1 (guaranteed nominal performance under a decentralized controller), main steps of the robust decentralized controller design procedure can be deduced.

Corollary 3 (NS and RS under a DC). *The uncertain feedback system in Fig. 2 is stable under a decentralized controller (15) if conditions of Corollary 1 and Corollary 2 are satisfied simultaneously.*

Corollary 4 (NP and RS under a DC). *The uncertain feedback system in Fig. 2 described by additive affine type uncertainty (16) is stable under a decentralized controller (15) and has a guaranteed nominal performance in terms of the degree of stability α if there exist both such $\alpha \ge 0$ and $g_\ell(s)$, fixed $\ell \in \{1,\dots,m\}$, that the conditions of Lemma 1 and Corollary 1 are satisfied simultaneously.*

A case study in the next section illustrates the robust controller design.

4 CASE STUDY

Consider the glass tube manufacturing plant described in [4]. Let us recall a brief description of its operation.

The glass metal flowing out from feeder is wrapping around a rotating cylindrical blowpipe; at its lower end, a tube is continuously being drawn. Forming air is blown into the tube under a certain pressure. The produced glass tube has to have specified outer diameter and wall thickness; these quantities are manipulated through the pressure of the forming air and the drawing speed of the drawing machine.

The objective is to design local PI(D) controllers guaranteeing that the pre-set output parameters (wall thickness, outside tube diameter) are maintained over the whole operation range of the plant specified by the three operating points.

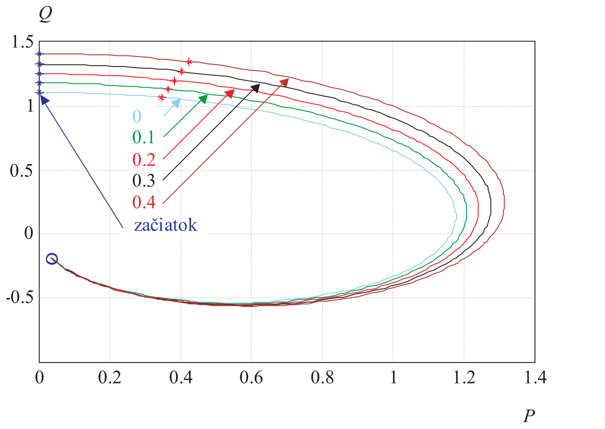

Fig. 6. Characteristic loci $g_2(s-\alpha$ of $G_m(s-\alpha)$ for $\alpha=[0,0.4]$

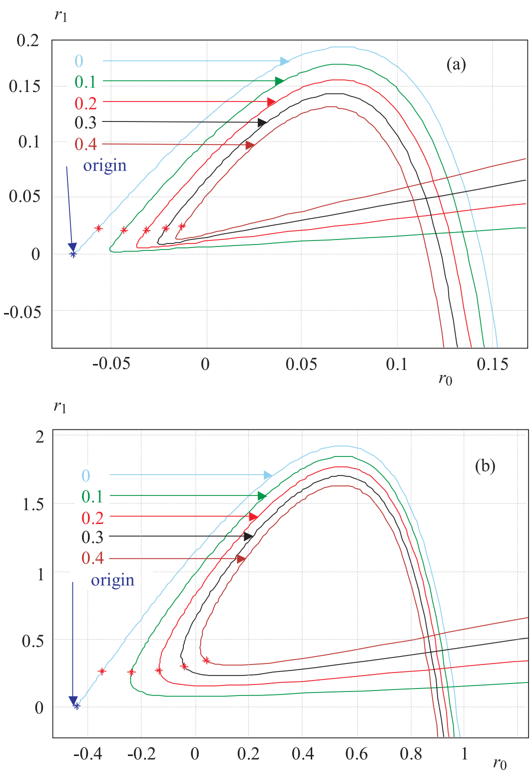

Fig. 7. D-partition of the (r_p, r_i) planes for the 1st subsystem (a) and the 2nd subsystem (b)

The following linearized model of the plant has been obtained via identification in several operating points:

$$G(s)=\begin{bmatrix}\frac{K_1e^{-0.5s}}{(s+8.6)(s+2)} & \frac{K_3(s-h)}{(s+8.6)(s+3.25)}\\ \frac{K_2}{s+d} & \frac{K_4}{(s+d)(s+5)}\end{bmatrix}$$

The unspecified parameters range over following intervals

$$K_1\in\langle 243\text{–}665\rangle\,;\ K_2\in\langle 25.2\text{– }40.25\rangle\,;$$
$$K_3\in\langle 1.25\text{–}5.45\rangle\,;\ K_4\in\langle 20.15\text{– }40.30\rangle\,;$$
$$D\in\langle 4.412\text{–}5.2\rangle\,;\ h\in\langle 1.2\text{–}4.5\rangle$$

From the specified intervals three transfer function matrices have been generated representing different operation points $G^k(s)$, $k=1,2,3$.

$$G^1(s):\ \begin{array}{l}K_1=243\,;\ K_2=45\,;\ K_3=1.3\,;\ K_4=21\,;\\ h=2\,;\ d=4.45\,;\end{array}$$
$$g^2(S):\ \begin{array}{l}K_1=245\,;\ K_2=40\,;\ K_3=2\,;\ K_4=34\,;\\ h=2.4\,;\ d=4.7\,;\end{array}$$
$$g^3(S):\ \begin{array}{l}K_1=249\,;\ K_2=35\,;\ K_3=3.3\,;\ K_4=41\,;\\ h=1.3\,;\ d=5.2\,.\end{array}$$

For $p=2$ and $q_I\in[-1,1]$ the nominal model has been calculated according to (17). Time delays have been approximated using the 1st order Padé approximant. All possible distributions of the three transfer matrices into the $2^2=4$ polytope vertices were examined (24 combinations) yielding 24 nominal model candidates and related transfer matrices needed to complete the description of the uncertainty region. Those possibilities that yielded unstable $G^4(s)$ were excluded and for all remaining possibilities, additive uncertainty was calculated according to (10) with $\widetilde{G}=G^k$, $k=1,2,3,4$. Finally, the nominal model $G_0(s)$ was selected guaranteeing the smallest additive uncertainty over the specified frequency range

$$G_0(s)=\begin{bmatrix}G_{110}(s) & G_{120}(s)\\ G_{210}(s) & G_{220}(s)\end{bmatrix}$$

where

$$G_{110}(s)=\frac{-246s+984}{s^3+14.6s^2+59.6s+68.8}\,,$$
$$G_{120}(s)=\frac{2.3s-3.94}{s^2+11.85s+27.95}\,,$$
$$G_{210}(s)=\frac{40s+183.3}{s^2+9.2s+21.14}\,,$$
$$G_{220}(s)=\frac{31s+144.1}{s^3+14.2s^2+67.14s+105.7}\,.$$

For the selected nominal model the design proceeds according to the procedure described *eg* in [3], [4] using the second characteristic locus $g_2(s)$ of $G_m(s)$. For $\alpha=[0,0.04]$ characteristic loci $g_2(s-\alpha)$ are plotted in Fig. 6.

Next, the D-partition method has been applied to both equivalent subsystems $i=1,2$

$$G_i^{eq}(s-\alpha)=G_i((s-\alpha)+g_1(s-\alpha)\,,\ \alpha=[0,0.4]\,.$$

Corresponding D-plots in the (r_p, r_i) plane of local controller parameters are in Fig. 7. a)

Parameters of local PI controllers have been chosen from the stable parameter region with the degree of stability $\alpha=0.4$.

$$R_1=\frac{0.106s+0.0595}{s}\qquad R_2=\frac{0.847s+0.510}{s}$$

According to Corollary 4, NS and NP are guaranteed, the achieved degree of stability of the full system is $\alpha=0.4$.

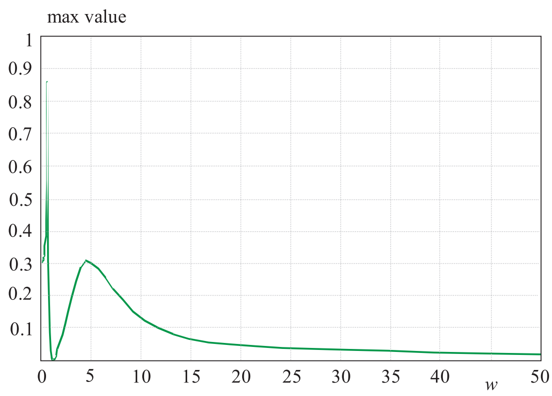

Fig. 8. Verification of the 'affine' additive robust stability condition $\sigma_M(M_{af}Q) < 1$

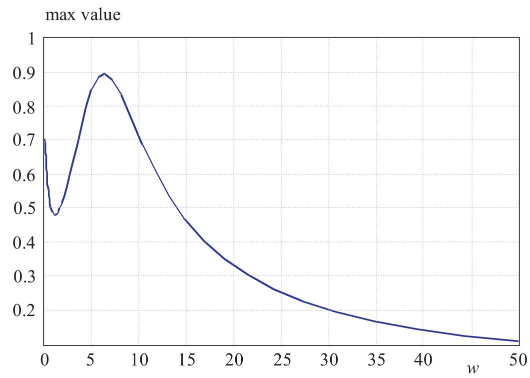

Fig. 9. Verification of the 'standard' additive robust stability condition for the mean parameter value nominal model

The 'affine additive' M_{af}–Q robust stability condition (20) is verified in Fig. 8. A comparison with the standard additive M–Δ robust stability condition in Fig. 9 obtained with the mean parameter value nominal model reveals that the novel approach to uncertainty evaluation is less conservative.

5 CONCLUSIONS

The paper presents a novel type of unstructured uncertainty description in MIMO systems and its application in the decentralized fixed structure controller design. The introduced additive affine-type uncertainty brings about a new way of nominal system computation and a new type of robust stability conditions that can be modified for the application in the decentralized controller design. In the case study a robust decentralized PI controller has been designed for the glass tube manufacturing plant. The 'affine additive' M_{af}–Q robust stability condition was evaluated and compared with the fulfilment of the 'standard additive' M–Δ robust stability condition obtained for the mean parameter value nominal model; a reduction in conservatism has been proved in the former case.

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References

[1] CHEN, D.—SEBORG, D. E.: Design of Decentralized PI Control Systems Based on Nyquist Stability Analysis, Journal of Process Control **13** (2003), 27–39.

[2] HOVD, M.—SKOGESTAD, S.: Sequential Design of Decentralized Controllers, Automatica **30** No. 10 (1994), 1601–1607.

[3] KOZÁKOVÁ, A.—VESELÝ, V.: Independent Design of Decentralized Controllers for Specified Closed-Loop Performance, In: European Control Conference ECC03, Cambridge, UK, 2003.

[4] KOZÁKOVÁ, A.: Design of a Decentralized PID Controller for the Glass Tube Drawing Process, Int. conference Cybernetics and Informatics, Dolný Kubín, Slovakia (2005).

[5] KOZÁKOVÁ, A.—VESELÝ, V.: A frequency Domain Design Technique for Robust Decentralized Controllers, In: 16th IFAC World Congress, Prague, Czech Republic (2005).

[6] KOZÁKOVÁ, A.—VESELÝ, V.: Robust Decentralized Controller Design: Independent Design, In: IFAC Workshop ALSIS06, Helsinki, Finland, 2006.

[7] KOZÁKOVÁ, A.—VESELÝ, V.: Improved Tuning Technique for Robust Decentralized PID Controllers, Accepted to 11th IFAC/IFORS/IMACS/IFIP Symposium on Large Scale Systems: Theory and Applications. 23–25 July 2007 Gdansk, Poland.

[8] MacFARLANE, A. G. J.—POSTLETHWAITE, I.: The Generalized Nyquist Stability Criterion and Multivariable Root Loci, Int. J. Control **25** (1977), 81–127.

[9] SKOGESTAD, S.—MORARI, M.: Robust Performance of Decentralized Control Systems by Independent Designs, Automatica **25** (1989), 119–125.

[10] SKOGESTAD, S.—POSTLETHWAITE, I.: Multivariable Fedback Control: Analysis and Design, 3rd edn., John Wiley & Sons Ltd., Chichester, New York, Brisbane, Toronto, Singapore, 1996.

[11] VESELÝ, V.—KOZÁKOVÁ, A.: Independent Design of Robust Decentralized Controllers, In: 1st Int. Workshop Advanced Control Circuits and Systems (ACCS05), Cairo, Egypt, 2005.

[12] XIONG, Q.—CAI, W.-J.—HE, M.: A Practical Decentralized PID Auto-Tuning Method for TITO Systems under Closed-Loop Control, Int. J. Innovative Computing, Information and Control **2** No. 2 (2006), 305–322.

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