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ELECTROMECHANICAL ACTUATORS DYNAMICS

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The focus of the study is the numerical investigating of dynamic and static characteristics of electromechanical actuators with a linear motion, and determining the capacitance of the capacitor that can be applied to feeding the actuator. The applied method is fairly exact as to respecting (i) nonlinear properties of the magnetic circuit, (ii) incidental complexity of the shape, and (iii) complicated kinematic behaviour of the controlled mechanism. The suggested method may be used as the basis for optimizing the projects of actuators.

K e y w o r d s: actuators, dynamic system, dynamic characteristic, static characteristic, numerical analysis

1 INTRODUCTION

The actuators with the linear motion rank among those most frequently used. The linear motion is actuated via a magnetic field induced by a coil on a ferromagnetic core. The basic attributes of actuators are

- static characteristics expressing the dependence of the generated force $F_m(\zeta, I_c)$ on the position ζ of the ferromagnetic core carrying the actuator, and
- dynamic characteristics of the actuator (including characteristics of the electromechanical circuit interconnected with the actuator) expressing the dependence of velocity v, and actuator core position ζ on time t, current I_c and external forces F_{ext} acting on the core.

The aim of the paper is to show how to set up a mathematical model of the actuator with a nonlinear magnetic circuit, and to give propositions on determining its static and dynamic characteristics via professional programmes. The final aim of analysing the magnetic fields is to cope with a minimum capacity of the feeding capacitor.

2 FORMULATION OF THE PROBLEM

In what follows we consider an electromechanical actuator of a symmetrical rotational disposition as shown in Fig. 1 The magnetic circuit is constituted by casing 2 and armature 3, taken in direction ζ_0 through the agency of the magnetic field induced by coil 1. The armature is connected via regulation nut 4 and elastic clutch 6 with nonmagnetic pull rod 5, that controls an external body (*eg* lever mechanism, *etc*). Spring 7 returns the movable system (3, 4, 5) back to the starting position $\zeta = 0$, provided the coil carries no current. In the starting position the system is arrested by strap plate 9.

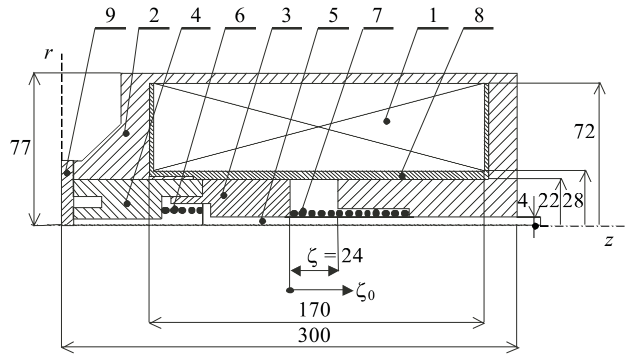

Fig. 1. Disposition of electromechanical actuator: 1 — exciting coil, 2 — ferromagnetic casting, 3 — ferromagnetic armature, 4 — regulation matrice, 5 — pull rod, 6 — elastic clutch, 7 — spring, 8 — isolation box of coil, 9 — strap plate

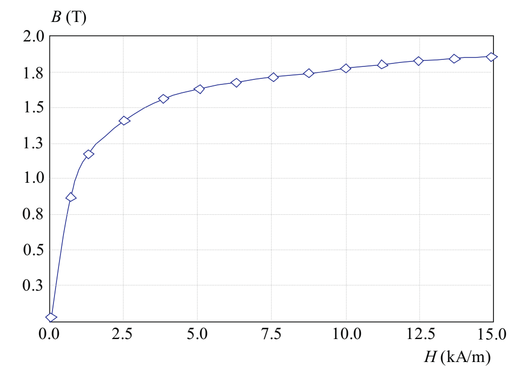

Fig. 2. Dependence of $B(H)$ steel

The following values are introduced:

- geometric dimensions of actuator, including air gap $\zeta_{\max} = 24$ mm,
- physical parameters of used materials, see Tab. 1 and Fig. 2,
- exciting current i_z, or supply voltage u_0. In case of failure the actuator is fed from capacitor with voltage u_0,
- external force mastered by the actuator.

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Table 1. Material properties of components of actuator

	component	material	parameter		value	unit
1	exciting coil	Cu conductor	conductor diameter	D_v	1	mm
			number of windings	N_z	7500	–
			coefficient plneni	k	0.784	
			permeability	μ_r	1	–
2	ferromagnetic casting	carbon steel [4] ČSN 12 040	characteristic	$B(H)$	see Fig. 2	
3	ferromagnetic armature	carbon steel [4] ČSN 12 040	characteristic	$B(H)$	see Fig. 2	
4	regulation matrice	brass	permeability	μ_r	1	–
5	pull rod	brass	permeability	μ_r	1	–
6	elastic clutch	austenitic steel ČSN 17 021 [4]	permeability	μ_r	1	–
7	spring	austenitic steel ČSN 17 021 [4]	permeability	μ_r	1	–
8	isolation box of coil	silon [5]	permeability	μ_r	1	–

ČSN = Technical Standards of the Czech Republic

The aim rests on determining a dynamic characteristic of the actuator.

3 STATIC CHARACTERISTIC OF ACTUATOR

Static mathematical model

It is necessary to determine a steady magnetic field excited by current i_z, in the exciting coil of the actuator.

Definition region is given by the cross-section of the actuator in Fig. 1,

Differential equation for vector magnetic potential A:

$$\operatorname{rot}\frac{1}{\mu}\operatorname{rot}A(R)=\boldsymbol{J}_c \tag{1}$$

where
$\boldsymbol{J}_c=\boldsymbol{r}_0 0+\boldsymbol{z}_0 0+\boldsymbol{\varphi}_0 J_{c\varphi}(r,z)$,
$\boldsymbol{A}=\boldsymbol{r}_0 0+\boldsymbol{z}_0 0+\boldsymbol{\varphi}_0 A_{\varphi}(r,z)$,
$\boldsymbol{r}_0$, $\boldsymbol{z}_0$ and $\boldsymbol{\varphi}_0$ are base vectors in cylindrical coordinates, $\boldsymbol{r}$ [m] is radius vector defining position of a general point.

With regard to Tab. 1, (1) takes in particular subregions 1 to 9 (Fig. 1) the following forms

- in the region of exciting coil

$$\operatorname{rot}\operatorname{rot}A=\boldsymbol{J}_c \tag{2}$$

  where $\boldsymbol{J}_c=\dfrac{N_z I_z}{S_1}\boldsymbol{\varphi}_0$, and S_1 is the cross-section of the exciting coil,
- in the region of ferromagnetic components 2, 3, 4 of magnetic circuit

$$\operatorname{rot}\frac{1}{\mu}\operatorname{rot}A=0\,, \tag{3}$$

- in the region of nonmagnetic components of the actuator

$$\operatorname{rot}\operatorname{rot}A=0\,. \tag{4}$$

Boundary conditions

Supposing the casing of the actuator is sufficiently thick, its surface outline stands for the line of force, implying for (1) and (4) the boundary condition

$$rA(\boldsymbol{r})=0\,. \tag{5}$$

Vector of magnetic induction $\boldsymbol{B}(r,z)$ of the magnetic field under examination, is

$$\boldsymbol{B}=\operatorname{rot}A \tag{6}$$

where $\boldsymbol{B}=\boldsymbol{r}_0 B_r(r,z)+\boldsymbol{z}_0 B_z(r,z)+\boldsymbol{\varphi}_0 0$.

Vector of force $\boldsymbol{F}_m(\zeta,i_z)$ acting on armature 3 in position ζ, if the coil carries current i_z, is given by the relation

$$\boldsymbol{F}_m(\zeta,i_z)=\frac{1}{2}\int_{S_3}[H(\boldsymbol{n}\cdot\boldsymbol{B})+\boldsymbol{B}(\boldsymbol{n}\cdot\boldsymbol{H})-\boldsymbol{n}(\boldsymbol{B}\cdot\boldsymbol{H})]\mathrm{d}S \tag{7}$$

following from the Maxwell's stress tensor. Here
$\boldsymbol{F}_m=\boldsymbol{r}_0+\boldsymbol{z}_0 F_{mz}+\boldsymbol{\varphi}_0 0$,
S_3 is the surface of the armature, $\boldsymbol{n}$ is vector of external normal line towards surface S_3.

To calculate the dynamic characteristics of the actuator, we need to formulate the relation for *inductance of exciting coile*. The energy density of magnetic field w_m is

$$w_m=\int_0^B\boldsymbol{H}\cdot\mathrm{d}\boldsymbol{B} \tag{8}$$

and the energy of the magnetic field in the whole actuator, whose volume is V is

$$W_m=\int_V w_m\mathrm{d}V\,. \tag{9}$$

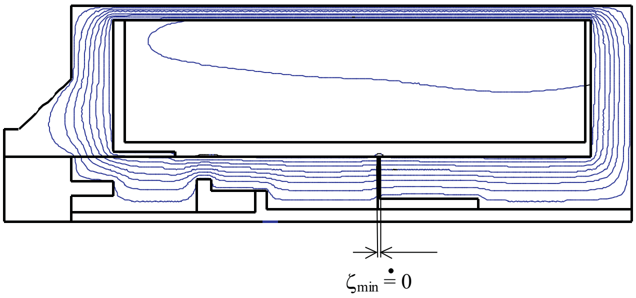

Fig. 3. Line of magnetic field intensity in actuator at starting position: $\zeta = 0$ mm, $i_z = 1$ A

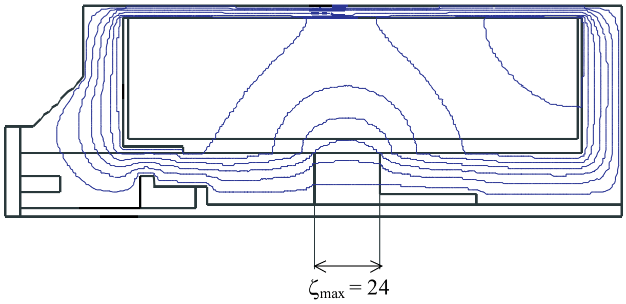

Fig. 4. Line of magnetic field intensity in actuator: $\zeta = \zeta_{max} = 24$ mm, $i_z = 1$ A

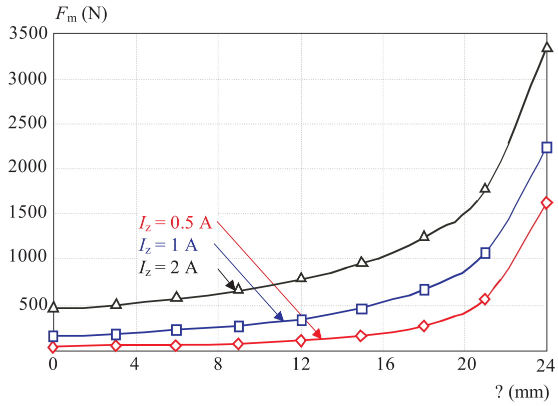

Fig. 5. Static characteristic of actuator, *ie* dependence of the force on position ζ

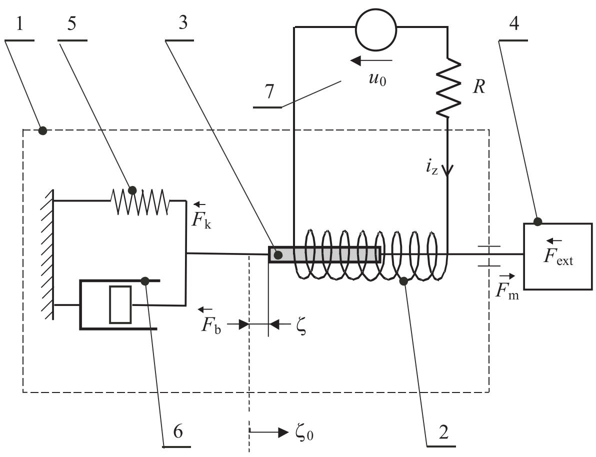

Fig. 6. Disposition of electromechanical circuit of actuator for calculation of dynamic characteristics: 1 — electromechanical actuator, 2 — coil, 3 — ferromagnetic armature, 4 — controlled mechanism, 5 — spring, 6 — damper, 7 — supply circuit of voltage source

Inductance L of the coil of the actuator, carrying current i_z at position ζ can be now determined from the relation

$$W_m = \frac{1}{2} L i_z^2 . \tag{10}$$

The numerical solution of the magnetic field was carried out via programme *QuickField* [3]. To obtain the accuracy of three valid figures for the determining of both force $\boldsymbol{F}_m(\zeta, i_z)$ and inductance L, we used a triangle network with 150 thousand nodal points.

Results and their discussion

The partial results are shown in Fig. 3 and Fig. 4. illustrating the distribution of the lines of magnetic field intensity for various positions of the armature. The density of those lines that are closed outside the air gap ζ, is falling with decreasing the gap, *ie* the magnetic flow from the casing as well as the force $\boldsymbol{F}_m$ are on increase.

Final results are shown in Fig. 5, that illustrates the static characteristics of the actuator, namely the dependence of the force developed by nonmagnetic pull rod on an external body. It is evident that force $\boldsymbol{F}_m$ strongly depends on values $\zeta \in \langle 0, \zeta_{\max} \rangle$. This feature may be welcome, or unwelcome. There is also evident the dependence of $\boldsymbol{F}_m$ on current i_z.

4 DYNAMIC CHARACTERISTICS

A dynamic regime of electrical circuit with actuator is defined by a system of differential equations with variable values of voltage, current, displacement, velocity and acceleration. Dynamic characteristics express the responses of these quantities by connecting the actuator on the net.

Dynamic mathematical model

The armature of an actuator moves in the direction ζ_0 along a trajectory $\zeta \in \langle 0, \zeta_{\max} \rangle$ and together with the external mechanism it possesses mass m. At zero position it is held by spring whose elastic constant is K. The motion of the spring is damped by an absorber with the coefficient of damping B. The armature is mechanically connected with an external mechanism whose action needs force F_{ext}. At the instant $t = 0$ the coil is connected across resistance R to voltage supply. The solution of the differential equations gives the following responses: current i_z in the coil, displacement ζ and velocity v of the armature.

The electrical circuit with the actuator (Fig. 6) observes the equation

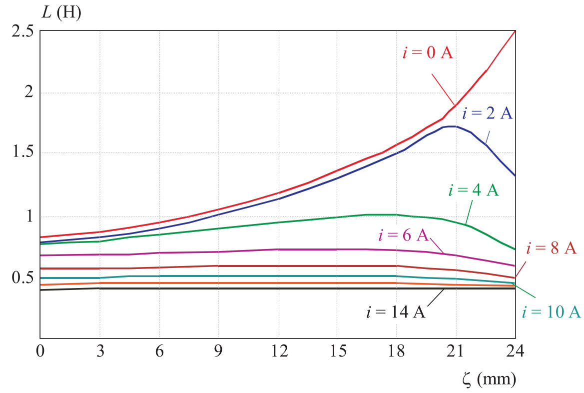

Fig. 7. Graphical matrice presentation of M1 – numerically calculated dependence $L(\zeta, i_z)$

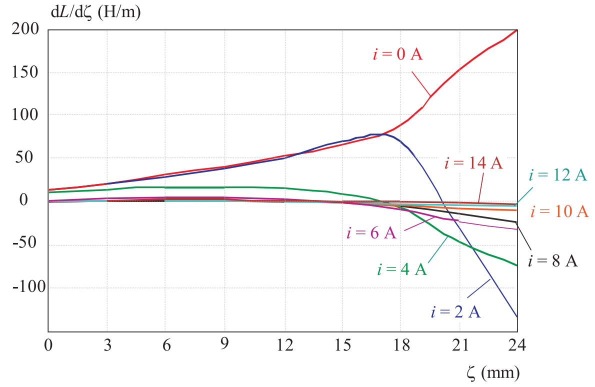

Fig. 8. Graphical matrice presentation of M2 – numerically calculated dependence $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}\zeta}$

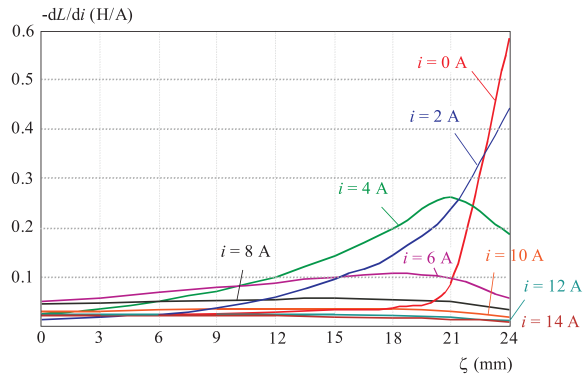

Fig. 9. Graphical matrice presentation of M3 – numerically calculated dependence $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}i_z}$

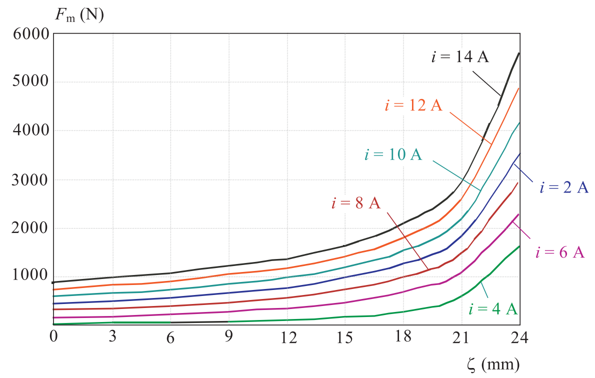

Fig. 10. Graphical matrice presentation of M4 – numerically calculated dependence $F_m(\zeta, i_z)$

$$u_0 = Ri + \frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{11}$$

where R is resistance of the supply circuit, $\Phi = L(\zeta, i_z)$ is magnetic flux of the coil, $L(\zeta, i_z)$ is inductance of the coil.

Rearranging equation (11) we get

$$\frac{\mathrm{d}i_z}{\mathrm{d}t} = -\frac{R}{\overline{L}} i_y - \frac{1}{\overline{L}} \frac{\mathrm{d}\overline{L}}{\mathrm{d}\zeta} v i_z + \frac{1}{\overline{L}} u_0 \tag{12}$$

where $\overline{L} = L + i_z \frac{\mathrm{d}L}{\mathrm{d}i_z}$.

If the actuator is feeded by capacitor with capacity C, equation (12) becomes

$$\frac{\mathrm{d}i_z}{\mathrm{d}t} = -\frac{R}{\overline{L}} i_z - \frac{1}{\overline{L}} \frac{\mathrm{d}\overline{L}}{\mathrm{d}\zeta} v i_z + \frac{1}{\overline{L}} \Big(u_0 - \frac{1}{C} \int_0^t i_z \mathrm{d}t \Big). \tag{13}$$

The mechanical quantities of the circuit with the actuator observe the equation

$$m \frac{\mathrm{d}^2\zeta}{\mathrm{d}t^2} = F_m(\zeta, i_z) - F_B - F_K - F_{\mathrm{ext}}\,, \ \zeta \in \langle 0, \zeta_{\max} \rangle, \tag{14}$$

where $F_B = Bv$ is friction force, $F_K = K\zeta$ is elastic force and F_{ext} is the force controlling external mechanism.

After some manipulation, we have

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = v\,, \tag{15}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{B}{m} v - \frac{K}{m} \zeta - \frac{F_{\mathrm{ext}}}{m} + \frac{F_m(\zeta, i_z)}{m}\,. \tag{16}$$

Dynamic representation of the circuit with the actuator is given by equations (12), resp. (13), further by (15) and (16) with zero initial conditions

$$i(t=0) = 0\,, \quad \zeta(t=0) = 0\,, \quad v(t=0) = 0\,. \tag{17}$$

Calculation of inductance L and force F_m

By calculating inductance L of the coil, and force F_m, acting on the mechanism, it is necessary to respect the nonlinearity of the magnetic circuit, as well as the variability of the air gap. The inductance L and force F_m are functions of current i_z and armature position ζ, $L(\zeta, i_z)$ and $F_m(\zeta, i_z)$. The calculation of these quantities is carried out in two steps:

1) Determination of functions $L(\zeta, i_z)$ and $F_m(\zeta, i_z)$. Using a suitable professional program (*eg* Quick Field [3]), we determine the distribution of magnetic potential vector $\boldsymbol{A}(r, y)$ in the whole actuator, and then the distribution of magnetic induction $\boldsymbol{B}(r, z)$ for chosen discrete

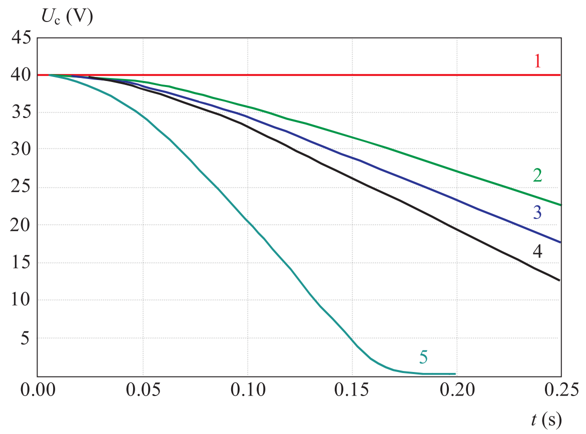

Fig. 11. Time course of voltage $u_c(t)$ on capacitor: 1 – $u_0(t) =$ 40 V, 2 – $C = 0.05$ F, 3 – $C = 0.035$ F, 4 – $C = 0.03$ F, 5 – $C = 0.01$ F

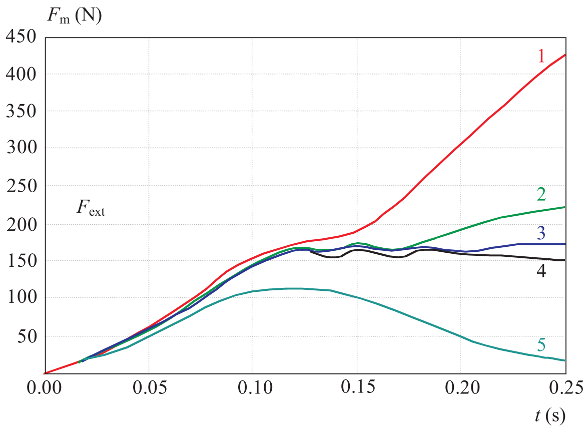

Fig. 13. Time dependence of magnetic force $F_m(t)$ acting on armature: 1 – $u_0(t) = 40$ V, 2 – $C = 0.05$ F, 3 – $C = 0.035$ F, 4 – $C = 0.03$ F, 5 – $C = 0.01$ F

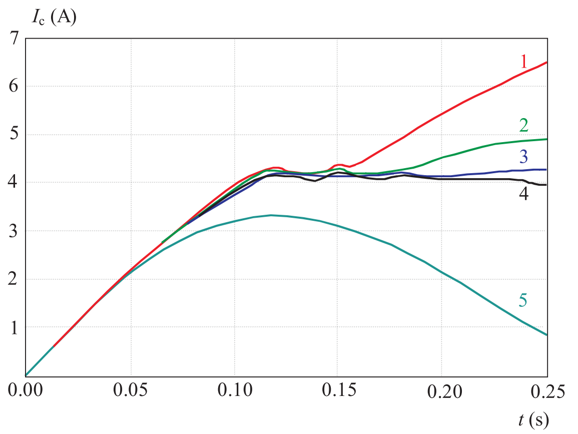

Fig. 12. Time dependence of current $i_c(t)$ in coil: 1 – $u_0(t) = 40$ V, 2 – $C = 0.05$ F, 3 – $C = 0.035$ F, 4 – $C = 0.03$ F, 5 – $C = 0.01$ F

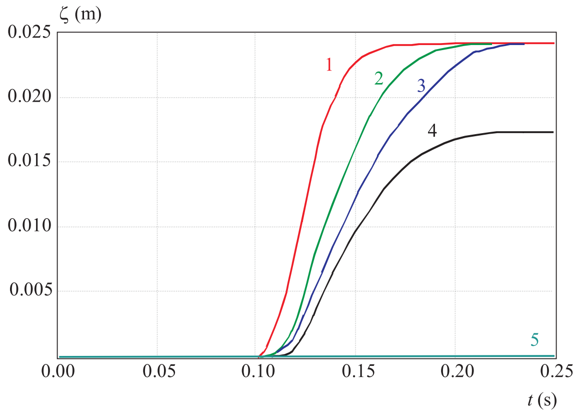

Fig. 14. Time dependence of shift $\zeta(t)$ of armature: 1 – $u_0(t) =$ 40 V, 2 – $C = 0.05$ F, 3 – $C = 0.035$ F, 4 – $C = 0.03$ F, 5 – $C = 0.01$ F

values of current $i_z \in \langle 0, i_{\max} = \frac{u_0}{R}\rangle$ and armature position $\zeta \in \langle 0, \zeta_{\max}\rangle$. From equation (8) can be determined energy of magnetic field W_m and from (10) inductance L.

$$L(\zeta, i_z) = \frac{2W_m(\zeta, i_z)}{i_z^2}\,. \tag{18}$$

Equation (7) defines $F_m(\zeta, i_z)$.

2) Determination of functions $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}\zeta}$ and $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}i}$ can be carried out by numerical derivation

$$\frac{\mathrm{d}L(\zeta, i_z)}{\mathrm{d}\zeta} \approx \frac{L(\zeta+\Delta\zeta, i_z - L(\zeta-\Delta\zeta, i_z)}{2\Delta\zeta} \tag{19}$$

resp.

$$\frac{\mathrm{d}L(\zeta, i_z)}{\mathrm{d}i_z} \approx \frac{L(\zeta, i_z+\Delta i_z) - L(\zeta, i_z-\Delta i_z)}{2\Delta i_z} \tag{20}$$

Accuracy of these approximations is of the order $(\Delta\zeta)^2$ resp. $(\Delta i_z)^2$, where $\Delta\zeta$ resp. Δi_z, are the steps changing the values $\zeta \in \langle 0, \zeta_{\max}\rangle$ and $i_z \in \langle 0, i_{\max} = \frac{u_0}{R}\rangle$ by examining magnetic fields for determination of functions $L(\zeta, i_z)$.

In this way we can evaluate the sets of the values $L(\zeta, i_z)$, $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}\zeta}$, $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}i_z}$ and $F_m(\zeta, i_z)$ for ζ, i_z from intervals $\langle 0, \zeta_{\max}\rangle$, $\langle 0, i_{\max} = \frac{u_0}{R}\rangle$ with equidistant steps $\Delta\zeta$, Δi_z. With regard on the next numerical solution of differential equations (12), (15), (16) resp. (13), (15), (16), we can put these four sets into rectangular matrices M1 to M4. These matrices are shown in Figs. 7 to 10.

Computer model

System of differential equations (12), (15), (16) resp. (13), (15), (16) was solved numerically using Runge-Kutta method of the fourth order. The program processed by language *MatLab*, had to be extended by:

- the possibility of computing either via equations (12) or (13),
- algorithm that allows, to use matrices M1 to M4 for determining the values $L(\zeta, i_z)$, $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}\zeta}$, $\frac{\mathrm{d}L(\zeta,i_z)}{\mathrm{d}i_z}$ and $F_m(\zeta, i_z)$ at arbitrary time level for numerical values ζ, i_z from intervals $\langle 0, \zeta_{\max}\rangle$, $\langle 0, i_{\max} = \frac{u_0}{R}\rangle$. What is meant is two-dimensional linear interpolation.

The proper numerical processing of differential equations aimed at obtaining numerical convergence of the solution. To reach the accuracy of three valid figures, the time interval $\Delta t = 10^{-6}$ sec was used.

Results and their discussion

Dynamic characteristics of the actuator by Fig. 1. and Fig. 6 are drawn up in the graphs shown in Figs. 11 and Fig. 14.

The time courses of voltage $u(t)$, current $i_z(t)$, magnetic force $F_m(\zeta, i_z)$ and displacement $\zeta(t) \approx \zeta(i_z)$ of the armature depend on the magnitude of capacity C of the capacitor. The courses of voltage $u(t)$ and current $i_z(t)$ are falling with time more rapidly than the course of capacity C. This fact is pronounced in the time course of magnetic force $F_m(\zeta, i_z)$ (its velocity is falling with decreasing capacity C) and in decelerating the motion of armature evident from Fig. 14. At a certain value of the capacity (for instance curve $\underline{4}$, $C = 0.03\,\mathrm{F}$), force $F_m(\zeta, i_z)$ is so attenuated that is capable of moving the armature only to position $\zeta \approx 0.017\,\mathrm{m} < \zeta_{\max} = 0.024\,\mathrm{m}$, as again illustrated in Fig. 14. In this position is $F_m(\zeta, i_z) < (F_{\mathrm{ext}} + F_K = K\zeta)$ so that the motion of the armature is stopped. In case of smaller capacity (for instance curve $\underline{5}$, $C = 0.01\,\mathrm{F}$), all the quantities become so small, that the condition $F_m(\zeta, i_z) < F_{\mathrm{ext}}$ remains valid, so that the motion of the armarure is stopped.

In the case that the current $i_z(t)$ is in rotating armature within the interval $\zeta \in \langle 0, \zeta_{\max} \rangle$ on increase (curves $\underline{1}$, $\underline{2}$,$\underline{3}$), the force grows too, Fig. 10. However, the magnetic circuit of the actuator may at the same instant be oversaturated which implies decreasing both the magnetic flux and the inductance. Consequently, current $i_z(t)$ and also force $F_m(\zeta, i_z)$ are decreased. As a result of these antagonistic trends there appears a local ripple effect on the course of both current $i_z(t)$ and force $F_m(\zeta, i_z)$, as shown in Figs. 11 and 12.

If current $i_z(t)$ is sufficiently great to keep the armature running throughout interval $\zeta \in \langle 0, \zeta_{\max} \rangle$, the above responses display three characteristic time intervals (Fig. 13 and 14):

a) $t \in \langle 0, t_1 \rangle$, when $F_m < F_{\mathrm{ext}} \implies$ current $i_z(t)$ and force $F_m(\zeta, i_z)$ are on increase but remain at rest,
b) $t \in \langle t_1, t_2 \rangle$, when $F_m \geq F_{\mathrm{ext}}$ and at the same time $0 \leq \zeta(t) \leq \zeta_{\max} \implies$ armature in motion,
c) $t \geq t_2$ and at the same time $\zeta = \zeta_{\max} \implies$ the motion of armature is finished, but current $i_z(t)$ and force $F_m(\zeta, i_z)$ grow.

The calculations show that the required capacity of capacitors is very great, which can be realized using the technology of supercapacitors.

5 CONCLUSION

Calculations of dynamic behaviour of actuators are usually carried out by using simplifying suppositions, for instance the linearization of the magnetic circuit or neglecting the change of the inductance of the coil, consequent upon the motion of the armature. It is true that the calculations are thus simplified, however the determination of the static and especially dynamic characteristics is not exact and remains unreliable. To optimize the calculations, the use of the suggested method is necessary.

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Bohuš Ulrych (1937), associate professor, has been working for a long time in the Department of Theory of Electrical Engineering at the Faculty of Electrical Engineering of UWB in Plzeň. His professional interests are aimed at modern numerical methods of solution of electromagnetic and coupled problems. Author and co-author of about 160 papers and several textbooks. Author of a lot of user's SW for calculation of electromagnetic fields and coupled problems.