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DETERMINATION OF SELF AND MUTUAL INDUCTANCES OF A DOUBLE–HELIX COIL

Jaroslav Franek — Mojmír Kollár

This paper is concerned with determination of self and mutual inductances in two cases of a double-helical coils with accordingly (spiral) or oppositely (anti-spiral) twisted wires. The former of these instances is the twisted-pair lead (TPL) pair, here described as a spiral double-helix. Parametric description of appropriate line integral is used to determine the inductance per unit length by evaluating the vector potential integral along the edge of the twisted conductors. As a variable parameter the twist-angle or the number of the turns per unit length (turning rate) is introduced. There are two limit cases of this task. The first, with zero twist-angle, will lead to a well known two-wire or twin-lead (transmission) line. The second, with relatively high "turning rate" is approaching to case of a long solenoidal coil.

K e y w o r d s: double-helix coil, twisted pair lead (TPL), twin lead, numeric solution, magnetic field, inductance per-unit-length

1 INTRODUCTION

In recent years the twisted-pair lead (TPL) has been used in many applications (*eg* in telecommunication) instead of previously wide-spread coaxial cables. TPL are nowadays used in a wide frequency range in telecommunication and IT networks – ranging from low up to GHz frequencies. They are fabricated in various modifications from several bunches up to several hundreds of single pairs bundled each to other in parallel in one single cable.

There are at leas two reasons for using this type of conductors: (i) simple fabrication (in comparison with coaxial cable) which is reflected in economic benefit, (ii) highly suppressed mutual inductance (and the so called cross-talks) between neighbouring pairs that is inevitable if one wants to have many pairs incorporated in one cable without taking additional means.

In this work we deal with numerical determination of inductance per unit length in a more general case of coil winding and their eventual cross-talk suppression. Analytic expression for this due to their anticipated complexity are hardly to be given. In various technical sheets [1] in case of a TPL the damping per unit length is given usually along with approximate values of the characteristic (or wave) impedance and the phase velocity, sometimes also the specific (per unit length) capacity is added. From four parameters characterizing the transmission line (inductance, capacity, resistance and conductivity per unit length) in practice the conductivity can be excluded since using current insulation materials this is negligible even at very low frequencies if compared with the actual capacitive admittance. The resistance can be expressed as in relevant cases (*eg* two-wire line or twin lead) as the surface resistance depending on the used material shape and the signal frequency. Usually lacking parameter is the inductance per unit length – which is also the merit of this approach.

2 PRELIMINARY CONSIDERATIONS

Let us first consider a two-wire line with axis-to axis distance b and r_0 being the wire diameter, see Fig. 1. The line is of arbitrary length Na – where N is an integer and a is the length of its part (hereafter called a-segment).

We start with this single case since that is the limit of a double-helix with negligible small twisting rate (turns per length a). In an analytical approach it is possible to determine the inductance per unit length of a sufficiently long line $l = Na$ simply by evaluating the magnetic field density that would be caused by infinitely long conductors and integrate the flux density, or the vector potential, over appropriate but finite area or length pertaining to the part of the line called here as the a-segment. However, in a numerical treatment one can not use infinitely long sources – especially if the analytical expression (*eg* in case of twisted wires) is not known. Thus the field density or the vector potential – to be further integrated – must be first evaluated as if caused by finite length conductors. The question of primary importance is the necessary length of conductors that will provide a sufficient precision of determined per-unit-length inductance if this is evaluated over the considered a-segment.

We propose here the following treatment. On a line of total length ($l = Na$) divided to equal a-segments, the inductance L can be interpreted as a sum

$$L = L_a + \sum_{\text{left}} M_{i,a} + \sum_{\text{right}} M_{i,a} \tag{1}$$

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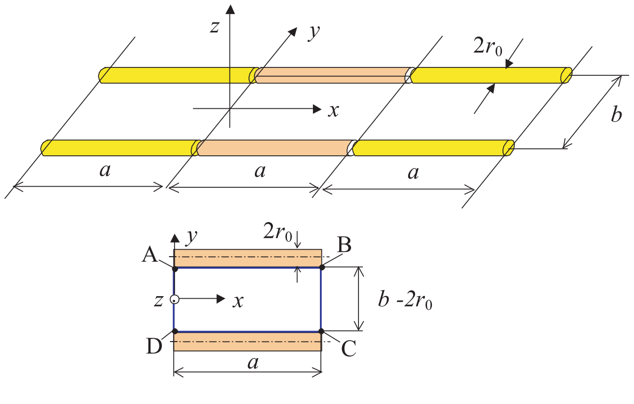

Fig. 1. Coordinates and partitioned twin-wire line

where $M_{i,a}$ – are the *mutual inductances* of a given a-segment and the adjacent a-segments on both sides – the nearest and further neihghbours. This may allow to determine also the inductance of parts that are closer to the line ends taking approximately into account the *edge effect*. Specifically for a-segments that are near middle of the line (apart from the line edges) the left and right sums will be due to symmetry almost equal, for the central $a-$segment giving exactly

$$L = L_a + 2\sum M_{i,a} \tag{2}$$

If we want to express the self-inductance (L_a) of an $a-$segment, we need to know the magnetic flux trough area $S = a(b - 2r_0)$ – given by the wire axes and two transversal join lines – or the vector potential along the circumferential path of this area ABCDA, see Fig. 1. This flux, needless too say, must be caused only by the current flowing in given $a-$segment *ie* in a part of the twin-wire length a. While determining the mutual inductances $M_{i,a}$ we need to know essentially the same, except that the current flowing consecutively in individual adjacent $a-$segment is now to be considered only.

It can be verified that the vector potential in a case of long enough line (total length Na) in a middle point of AB or CD abscissae is

$$A_x = \frac{\mu_0}{4\pi}\ln\left(\frac{1+\sqrt{1+(\frac{2r_0}{Na})^2}}{1+\sqrt{1+(\frac{2b-2r_0}{Na})^2}}\,\frac{1-\sqrt{1+(\frac{2b-2r_0}{Na})^2}}{1-\sqrt{1+(\frac{2r_0}{Na})^2}}\right) \tag{3}$$

giving for $(\frac{2r_0}{Na})^2 < 1$ and as well $(\frac{2b-2r_0}{Na})^2 < 1$

$$A_x = \frac{\mu_0}{2\pi}\left[\ln(\frac{b}{r_0}-1) - \frac{1}{2}\ln\left(1+(\frac{b}{a}-\frac{r_0}{Na})^2\right)\right] \tag{4}$$

what for $N >> r_0/b$ simplifies to

$$A_x = \frac{\mu_0}{2\pi}\left[\ln(\frac{b}{r_0}-1) - \frac{1}{2}\ln\left(1+(\frac{b}{a})^2\right)\right]. \tag{5}$$

Note that if the wires were "*in touch*" b would be equal to $2r_0$, thus in any actual case $r_0/b < 0.5$.

It is now interesting to compare the last result with that for infinitely long twin-line at which the vector potential along the same path is given simply by

$$A_x = \frac{\mu_0}{2\pi}\left[\ln(\frac{b}{r_0}-1)\right] \tag{6}$$

and one can see that the difference caused by a finite length of the considered line is given by the second term in (5) vanishing for $a > b$.

To determine the flux coupled by loop ABCDA we simply multiply (5) by the length of $a-$segment and by factor of 2 - taking thus into account parts AB and CD – where the contributions will be added while integrating the vector potential along circumferential path (ABCDA). Contrary to those (due to symmetry) the contributions along CD and DA are canceled. To obtain the inductance (L) per unit length of the considered line we divide the above flux again by its length (a) to get

$$L = \frac{\mu_0}{\pi}\left[\ln(\frac{b}{r_0}-1) - \frac{1}{2}\ln\left(1+(\frac{b}{a})^2\right)\right] \tag{7}$$

and for $a/b \to \infty$ we have

$$L_\infty = \frac{\mu_0}{\pi}\left[\ln(\frac{b}{r_0}-1)\right]. \tag{8}$$

From obvious reasons – since we intend to evaluate numerically the inductance of a twisted line where the next and next a-segments are always distorted with respect to the previous – we have tested first the proposed procedure at a collinear line. It was found that using (2) with three terms was sufficient for our purposes, thus we have taken

$$L = L_a + 2(M_{1a} + M_{2a} + M_{3a}). \tag{9}$$

The self inductance of a given $a-$segment with its increasing length tends towards the inductance of a twin-wire line and the mutual inductances are becoming negligible as expected by theory, see Fig. 2. As the result we can conclude that with $a > 2b$ and $b/r_0 > 8$ the sum (9) gives up to 99% of the theoretical value. In numerical solutions presented further, we have used $a = 5b$.

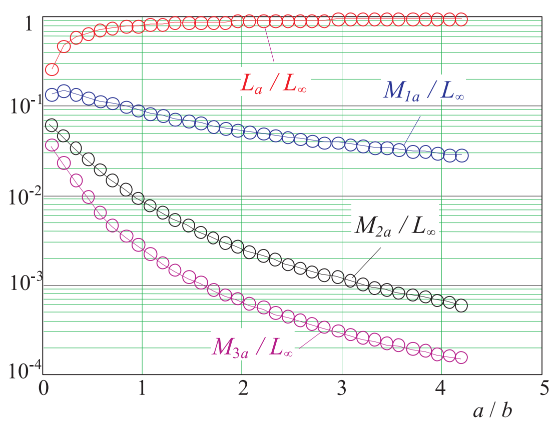

Fig. 2. Reduced inductances (*self* and *mutual*) *vs* a/b ratio for $b/r_0 = 8$

3 DOUBLE–HELIX COIL CONFIGURATIONS

First, let us consider two single helical coils, see Fig. 3 - one is wound clockwise (dextrorotatory) the other anti-clockwise (laevorotatory).

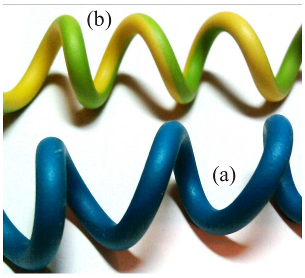

Fig. 3. Two types of winding: (a) – dextrorotatory coil (clockwise), (b) – laevorotatory coil (anticlokwise)

Two such windings can be placed "inside" one each other, see Fig. 4 up, or (again) simply separated, see Fig. 3 — unless they were wound around a rod core as shown in Fig. 4 down. They form spatially an un-mated double-helix — that will be here referred to as anti-spiral — since to get such a configuration one has to turn each of the individual wires around the central (hypothetical or real) rod in opposite direction during the winding process. Note that this case suffers from a coincidence-conflict because of some parts of the windings would have to share the same space to preserve the symmetry of the whole configuration (encircled in Fig. 4 down). This is the less important the finer wires are wound around the thicker rod.

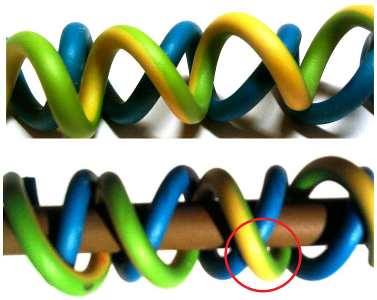

Fig. 4. Anti-spiral coil, spatially un-mated windings

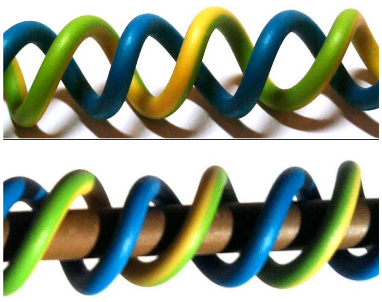

Fig. 5. Spiral coil, spatially mated windings

Contrary to this, the windings shown in Fig. 5, can not be placed one "inside" each other nor can be simply separated. They form mechanically mated double-helix (the individual windings can be joined or separated by screwing on unscrewing only, and due to no coincidence-conflict the symmetry of this configuration is always preserved). Windings of that type will be here referred to as spiral. While winding this configuration, one has to turn each of the individual wires around the (hypothetical) central rod in the same direction.

The TPL is made just in this way while the diameter of a particular central rod around which the wires are likely to be wound - tends to zero. In Fig. 6 there are shown TPLs with different turning rate or the turn angle papameter δ — that is defined below.

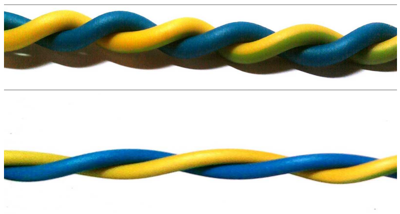

Fig. 6. TPL windings with different turning rate δ

Hereafter we anticipate that both wires are closely "in contact" or that the insulation is of negligible dimension (see Fig. 7) then $r_0 \to b/2$.

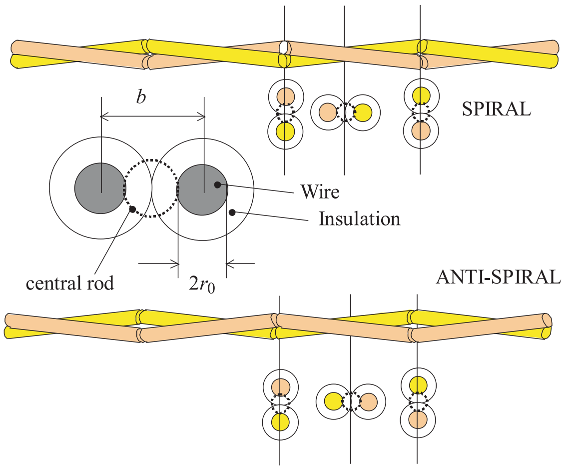

Fig. 7. Spiral and anti-spiral coil, spatially mated and un-mated windings

The position vectors of surface points on respective arms (wires) of the double-helix are expressed in parametric form where the origin in each separately, is somewhere on the wire axis

$$\vec{r}_A(\xi) = \frac{b}{2}\begin{vmatrix} \xi \\ \cos(\frac{\delta\xi}{\lambda}) \\ \sin(\frac{\delta\xi}{\lambda}) \end{vmatrix} \tag{10}$$

$$\vec{r}_B(\xi) = \frac{b}{2}\begin{vmatrix} \xi \\ -\cos(\frac{\delta\xi}{\lambda}) \\ \pm\sin(\frac{\delta\xi}{\lambda}) \end{vmatrix} \tag{11}$$

and where $\lambda = 2a/b$ and the parameter $\xi \in (0, \lambda)$. Here δ is the spiral turn-angle along the length of $a-$segment. Respective signs in (11) belong to spiral or anti-spiral wound in the plain perpendicular to $x-$ axis.

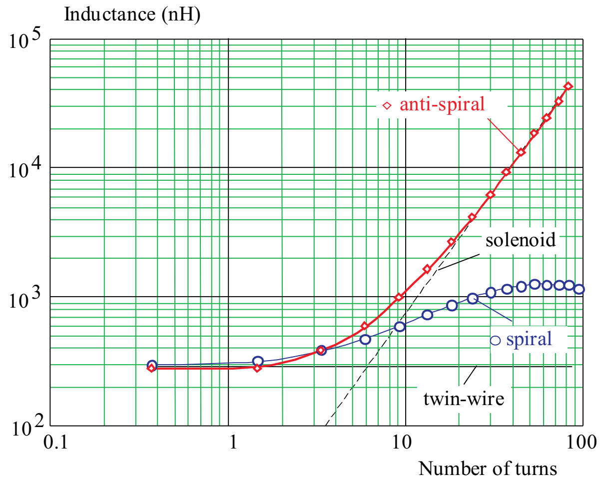

Fig. 8. Inductance *vs* number of turns for different winding configurations

If both arms are twisted accordingly (in the same direction) we get the *spiral* and plus sign in (11), otherwise – if twisted in opposite directions each to other – we talk about the *anti-spiral* and the minus sign in (11) is appropriate.

4 NUMERICAL EVALUATION OF DOUBLE–HELIX COIL INDUCTANCE

From(10) and (11) we can derive the vectorial length differential $d\vec{r}_A(\xi)$ and $d\vec{r}_B(\xi)$ while the conductor itself is replaced by an infinitely thin filament in its axis. Here we neglect the contribution known as the internal inductance in case of a finite-dimension wire. The former is followed by expressing the position vector of an arbitrary space point

$$\vec{r}_b(x,y,z) = \begin{vmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{vmatrix} \tag{12}$$

and the position vectors of points laying on the transversal joint lines (like BC and AD in Fig. 1)

$$\vec{r}_{bX}(x,y,z,\xi) = \vec{r}_b(x,y,z) - \vec{r}_X(\xi). \tag{13}$$

for $X = A, B$. Using the above relations we can express the vector potential by the well know formula for each of arms separately

$$\vec{A}_X(x,y,z) = \frac{\mu_0 I}{4\pi} \int_0^{\lambda} \frac{d\vec{r}_X(\xi)}{|\vec{r}_{bX}(x,y,z,\xi)|} \tag{14}$$

for $X = A, B$ to give the result

$$\vec{A}(x,y,z) = \sum_{X=A,B} \vec{A}_X(x,y,z) \tag{15}$$

Finally from (15) we determine the *self* and *mutual* inductances per unit length as a line integral along the appropriate helix-curve on the surface of respective conductors divided by current I and length a of the actual a-segment

$$M_{i,a} = \frac{1}{aI} \sum_{X=A,B} \int_{i\lambda_n}^{(i+1)\lambda_m} \vec{A}(\vec{r}_X(\xi))\, d\vec{r}_X(\xi) \tag{16}$$

using this notation $M_{0,a} \equiv L_a$.

To evaluate all this numerically, a pseudo-code was written in MathCad_12 environment [4] details of which will not be given here but are to appear at [4].

5 RESULTS AND DISCUSSION

Two curves in Fig. 8 are showing the dependences of the inductance of a line length 20 cm as a function of number of turns m wound along its length. The wire

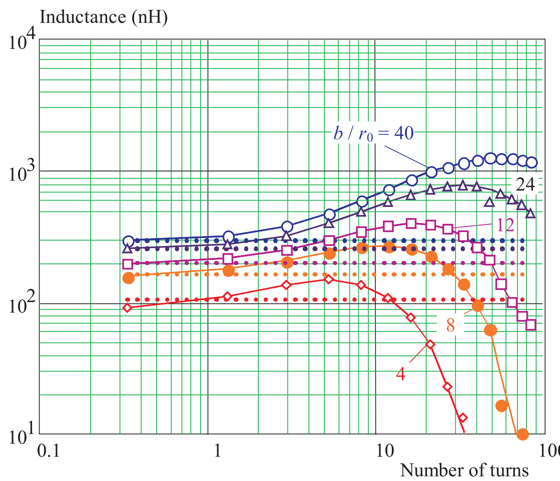

Fig. 9. Inductance *vs* number of turns for *spiral* coil (solid curves) and twin-wire (dotted lines) at different b/r_0 ratios

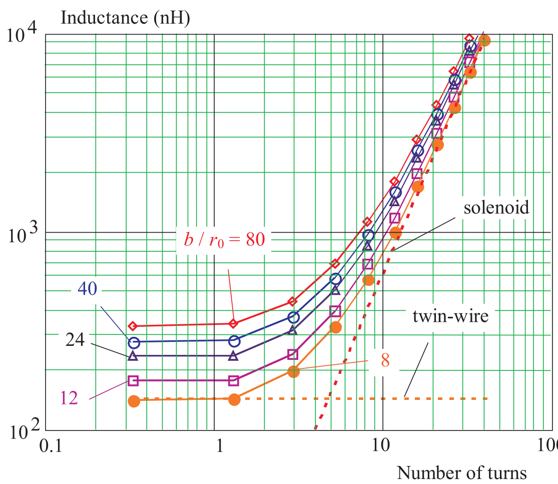

Fig. 10. Inductance *vs* number of turns for *anti-spiral*at different b/r_0 ratios (solid curves); solenoidal coil and twin-wire — dotted lines

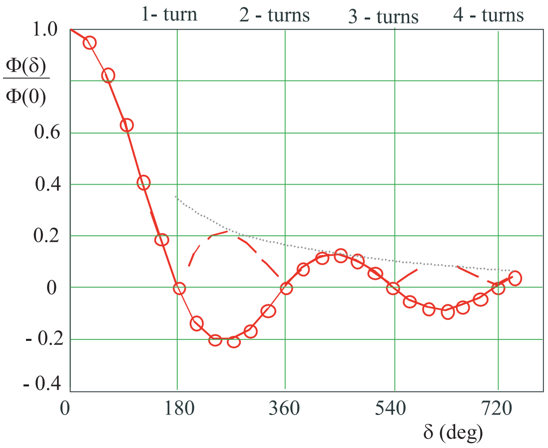

Fig. 11. Decrease of flux coupled due to external magnetic field perpendicular to the helix axis depending on number of turns N or the rotation angle δ

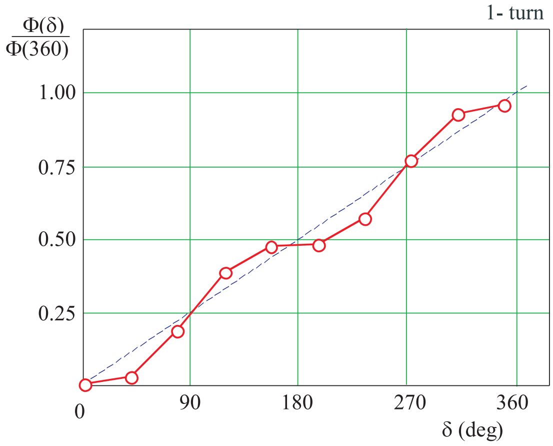

Fig. 12. Increase of flux coupled due to external magnetic field parallel to the helix axis depending on number of turns N or the rotation angle δ

axes are separated by $b = 4$ cm and the wire radius is $r_0 = 1$ mm. $(b/r_0 = 4)$.

The first case is a kind of *spiral*. When m is negligibly small this approaches to a single twin-wire line and a good agreement of the numeric and analytic solutions is obvious. Because the currents flowing in two wires are of opposite direction - the *spiral* represents the so called *bifilar* coil. The second case is kind of *anti-spiral*. If the currents flowing in paired-wires are again in opposite directions - in the *anti-spiral* both magnetic fluxes are summed and, as the number of turns per unit length grows – this is asymptotically becoming simply a long solenoid.

In this study we were partially concerned also with the influence of the b/r_0 ratio on the inductance. This is shown in Fig. 9 and 10 for the spiral and the *anti-spiral* respectively. It is interesting to see that in case of *spiral* there are local maxima of the self-inductance depending on the number of turns. With increasing number of turns the inductances in Fig. 9 tend to zero as should be expected for the bifilar windings. On the other hand, the inductances in Fig. 10 are approaching those of solenoidal coils as growing with the increased number of turns.

An attempt to evaluate the mutual inductance has in fact failed probably due to computational artefact — too complicated multiple numerical integration — obtained results had shown rather irregular character. However, the external field caused by any respective sources was considered, and the flux coupled by a straight pair $\delta = 0$ of twin-lead compared to that of a TPL with different twisting rate $\delta \neq 0$ the cross-talk suppression was estimated. These results are shown in Fig. 11 and Fig. 12. For an homogenous external field perpendicularly oriented with respect to the helix-coil axis at the rotation angle $\delta = N\pi$, where N is an integer, the coupled flux is with the computation precision equal to zero. For other rotation angles δ the absolute value of this dependence, Fig. 11, exhibits local maxima (for $\delta = 270, 440, \dots$) decreasing with increased number of turns. Thus more turns along a given length will provide less cross-talk in case of perpendicular external field. On the other hand, as shown in Fig. 12, the same will linearly increase the flux coupled due to presence of any longitudinal field. It is amazing to realize that in spite of this the cross-talk from axial field will be canceled (in case of a perfect symmetry - completely) due to opposite orientation of the voltage induced in respective wires-coils if considering appropriate loop (voltage Kirchhoff law). One can so state for a TPL line that the perpendicular field is eliminated due to (partial) magnetic flux cancelation, while the axial field is eliminated due to induced voltage cancelation.

Acknowledgements

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