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A COEFFICIENT DIAGRAM METHOD CONTROLLER WITH BACKSTEPPING METHODOLOGY FOR ROBOTIC MANIPULATORS

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A new robust control procedure for robot manipulators is proposed in this paper. Coefficients diagram method controllers CDM and Backstepping methodology are combined to create the novel control law. Two steps of backstepping on the resulting system are used to design a nonlinear CDM-Backstepping controller. Simulations on a PUMA robot including external disturbances, parametric uncertainties and noises are performed to show the effectiveness and feasibility of the proposed method.

K e y w o r d s: manipulators, backstepping approach, coefficients diagram method controller, robustness

1 INTRODUCTION

At present, robot manipulators are the most important instruments used in manufacturing industry. One of the most important challenges in the field of robot manipulators is to design robusts controllers [1], in particular when manipulators are required to maneuver very quickly under various disturbances. These systems are multivariable, nonlinear, strongly coupled, and its highly nonlinear dynamics changes rapidly and some dynamic parameters are uncertainty, e.g., unknown loads and disturbance. As a consequence, it is hard to find an exact mathematical model. The traditional proportional and derivative (PD) controller is very simple and does not require any knowledge of the robot dynamics. However, it requires very large actuation to achieve precise control, which is not practical but highly demanded in many cases. This is due to the fact that robotic arms constantly move among widely separated regions of their workspace such that no linearization valid for all regions can be found. Computed torque method utilizes mathematical model and parameters of the robot manipulator to cancel the non linearities. Howevers, du to the requirement of precise knowledge of the systeme structure and parameters [1], the computational task is very extensive. Although adaptive controllers can realize fine control and compensate for partially unknown manipulator dynamics [3], they often suffer from heavy computational burden and this hinders their real time applications. Another technique is called variable structure control with this, the system state are driven to a switching surface designed to make the state converge to the origin. As the system state cross the switching surface, the state become insensitive to system parameters variations, this method does not require knowledge of exact system parameters; it only requires the possible uper bound of uncertainty. A disadvantage of this method is that due to the discontinuous control activity, it may excite the unmodeled dynamic, and has the possibility of chattering problem [7]. The sliding mode control method share the common feature of using a discontinus control law although the design appрraoch is quite different. As a result, the chattering of the control signal is a common drawback. A control system with severe chattering is impractical because it stresses actuators even to a point of distruction and it may excite unmodelled plant dynamic [8]. Furthermore, many mathematical theories are used in new control methodologies to design nonlinear robust controller for robot manipulators.

The success of the CDM control is attributed to its simplicity, stability, and robustness in presence of external disturbance, parametric uncertainties and noises. Different CDM controllers have been proposed for linear system [4–6]. But, CDM controllers' essential shortage is her limitation to linear system and the needing of exponential stability for a given nonlinear systems.

Our goal in this paper is to eliminate this insufficiency by proposing a non linear robust controller CDM-Backstepping applied to robot manipulators. The controller is synthesized by joining a backstepping procedure with a CDM composition. In particular the controllers are designed by imposing the positions tracking with exact gains that are nonlinear functions of the system state. As a result, the proposed nonlinear backstepping control design is not only to stabilize the robot system, but also to oblige the tracking errors to converge to zero exponentially, then the novelty scientific in this work is that non linear CDM has not used previously and this controller summarize the performance of CDM and Backstepping.

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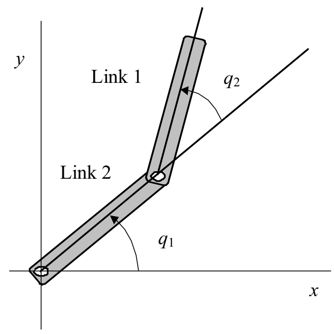

Fig. 1. Tow link rigid robot manipulator

2 ROBOT STATE SPACE MODEL

Consider the robot manipulator with rigid links and rotary joints. Furthermore, it is assumed that each degree of freedom of the manipulator is powered by an independent torque source [1]. The equations of motion for n degree-of-freedom of manipulator are formulated by using the lagrangian formulation and may be expressed by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + f(\dot{q}) = \tau(t)\,. \qquad (1)$$

Where q, $\dot{q}$ and $\ddot{q}$ are $n \times 1$ vectors of joint positions, velocities and accelerations [2], $M(q)$ is a $n \times n$ symmetric and positive definite matrix function which is also called generalized inertia matrix, $C(q,\dot{q})\dot{q}$ is $n \times 1$ vector resulting from Coriolis and centripetal accelerations, moreover $f(\dot{q})$ is $n \times +1$ vector of friction, $g(q)$ is vector of generalized gravitational forces and $\tau(t)$ is the $n \times 1$ vector of joint torque supplied by actuators.

The robot manipulator that we are going to use for our application is called PUMA robot as shown in Fig. 1, it is characterized by two rotary joints identified by $n = 2$ variables q_1 and q_2 where

$$q = (q_1 \quad q_2)^\top, \qquad \tau = (\tau_1 \quad \tau_2)^\top, \qquad (2,3)$$

$$M = \begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix};\ C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix},\ f = (b_1\dot{q}_1\ b_2\dot{q}_2)^\top \quad (4)$$

$$\text{with}\quad \alpha = \frac{1}{3}l^2\big(m_1 + 4m_2 + 3m_2\cos q_2\big),$$
$$\beta = m_2 l^2\Big(\frac{1}{3} + \frac{1}{2}\cos q_2\Big), \quad \gamma = 13 m_2 l^2\,,$$
$$c_{11} = -m_2 l^2 \dot{q}_2 \sin(q_2), \quad c_{12} = c_{12}/2,$$
$$c_{21} = \frac{1}{2} m_2 l^2 \dot{q}_1 \sin(q_2), \quad c_{22} = 0.$$
$$g_1 = \frac{1}{2} m_1 g l \cos(q_1) + m_2 g l\Big(\frac{1}{2}\cos(q_1 + q_3) + \cos q_1\Big),$$
$$g_2 = \frac{1}{2} m_2 g l \cos(q_1 + q_2),$$
$$\text{it comes out:}\quad \tau(t) = M(q)\ddot{q} + h(q,\dot{q}) \qquad (5)$$

Denoting

$$(x_1, x_2, x_3, x_4)^\top = (q_1, \dot{q}_1, q_2, \dot{q}_2)^\top \qquad (6)$$

we have

$$h(x) = \begin{bmatrix} c_{11}(x)x_2 + c_{12}(x)x_4 + g_1(x) + b_1 x_2 \\ c_{21}(x)x_2 + c_{22}(x)x_4 + g_2(x) + b_2 x_4 \end{bmatrix}$$

and the state space representation is

$$\begin{aligned} &\dot{x}_1 = x_2, \quad \dot{x}_3 = x_4, \\ &\dot{x}_2 = \frac{1}{(1 - \frac{\beta}{\alpha\gamma}\beta)}\Big(\frac{\tau_1 - h_1(x)}{\alpha}\Big) + (-\tau_2 + h_2(x))\frac{\beta}{\alpha\gamma} \\ &\dot{x}_4 = \frac{1}{(1 - \frac{\beta}{\alpha\gamma}\beta)}\Big(\frac{\tau_2 - h_2(x))}{\alpha} + (-\tau_1 + h_1(x))\frac{\beta}{\alpha\gamma}\Big) \end{aligned} \qquad (8)$$

$$\begin{aligned} &\dot{x}_1 = x_2\,, \quad \dot{x}_2 = F_1(x) + G_1(x)\tau\,, \\ &\dot{x}_3 = x_4 \quad \dot{x}_4 = F_2(x) + G_2(x)\tau \end{aligned} \qquad (9)$$

$$F_1(x) = \frac{\frac{-h_1(x)}{\alpha}}{(1 - \frac{\beta}{\alpha\gamma}\beta)} + \frac{h_2(x)\frac{\beta}{\alpha\gamma}}{(1 - \frac{\beta}{\alpha\gamma}\beta)}, \qquad (10)$$

$$F_2(x) = \frac{\frac{-h_2(x)}{\alpha}}{1 - \frac{\beta}{\alpha\gamma}\beta} + \frac{h_1(x)\frac{\beta}{\alpha\gamma}}{1 - \frac{\beta}{\alpha\gamma}\beta}, \qquad (11)$$

$$G_1(x) = \frac{\frac{1}{\alpha}}{(1 - \frac{\beta}{\alpha\gamma}\beta)}\ \frac{-\frac{\beta}{\alpha\gamma}}{(1 - \frac{\beta}{\alpha\gamma}\beta)}, \qquad (12)$$

$$G_2(x) = \frac{-\frac{\beta}{\alpha\gamma}}{(1 - \frac{\beta}{\alpha\gamma}\beta)}\ \frac{\frac{1}{\alpha}}{(1 - \frac{\beta}{\alpha\gamma}\beta)}. \qquad (13)$$

3 CDM CONTROL DESIGN

Coefficient diagram method is an algebraic approach with polynomial form, it alow to design easily the controller under the conditions of stability, time domain performance and robustness. The performance specification, equivalent time constant and stability index are specified in the closed loop transfer function and related to the controller parameters algebraically. Habitually, the order of the controller is less than the order of the plant.

The output of the controlled closed-loop system is

$$y = \frac{N(s)F(s)}{P(s)}r + \frac{A(s)N(s)}{P(s)}d\,, \qquad (14)$$

where y is the output, r is the reference input, u is the control and d is the external disturbance signal, $N(s)$ and $D(s)$ are the numerator and the denominator of the transfer function of the plant, respectively, $A(s)$ is the denominator polynomial of the controller transfer function, while $F(s)$ and $B(s)$ are called the reference numerator and the feedback numerator polynomials of the controller transfer function.

Also $P(s)$ is the characteristic polynomial and given by

$$P(s) = D(s)A(s) + N(s)B(s) = \sum_{i=0}^{n} \mu_i s^i. \qquad (15)$$

The nominal mathematical model is

$$R(s)=\frac{N(s)}{D(s)}=\frac{a_m s^m+a_{m-1}s^{m-1}+\cdots+a_0}{b_n s^n+b_{n-1}s^{n-1}+\cdots+b_0}\,. \tag{16}$$

The controller polynomials $A(s)$ and $B(s)$ are

$$A(s)=\sum_{i=0}^{n} l_i s^i \qquad B(s)=\sum_{i=0}^{n} k_i s^i\,. \tag{17}$$

The equivalent time constant T_0 indicate the time response speed and the stability indices γ_i give the stability and the waveform of the time response. They are defined in terms of the coefficients of the characteristic polynomial in (15) as

$$\begin{aligned} T_0&=\frac{\mu_1}{\mu_0}\,,\\ \gamma_i&=\frac{\mu_i^2}{\mu_{i-1}\mu_{i+1}}\,,\ \text{for } i=1\ldots(n-1)\,. \end{aligned} \tag{18}$$

The settling time and the equivalent time constant is defined as

$$\begin{aligned} &T_0=\frac{t_s}{2.5\sim 3}\,,\\ &\gamma_1=2.5\,,\ \gamma_i=2\,,\ ;i=2\sim(n-1)\,,\ ;\gamma_0=\gamma_n=\infty\,. \end{aligned} \tag{19}$$

The last values can be adjusted to assure the required performance, so that $\gamma_i>1.5$ for all $i=1\sim(n-1)$.

Then the characteristic polynomial to be used to design the parameters of a controller is

$$P(s)=\mu_0\Big[\Big\{\sum_{i=2}^{n}\Big(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\Big)(T_0 s)^i\Big\}+T_0 s+1\Big]. \tag{20}$$

Finally $F(s)$ which is usually defined as the pre-filter used for reducing the steady state error to zero and is selected as a constant defined by

$$F(s)=\frac{P(s)\big|_{s=0}}{N(s)}\,. \tag{21}$$

4 CONTROL OBJECTIVE

In this section, we use the CDM-Backstepping algorithm to develop the positions control law. These positions will converge exponentially to the reference value. The error positions are defined as.

$$e_1=q_1-q_1^d=x_1-q_1^d\,,\quad e_2=q_2-q_2^d=x_3-q_2^d \tag{22}$$

and their derivatives are

$$\dot e_1=x_2-\dot q_1^d\,,\quad \dot e_2=x_4-\dot q_2^d\,. \tag{23}$$

$$\text{Let}\qquad \zeta=(x_2\ \ x_4)^\top. \tag{24}$$

$$\text{then}\qquad \dot\zeta=F(x)+G(x)\tau \tag{25}$$

$$\begin{aligned} G(x)&=\begin{pmatrix} G_1(x) & 0\\ 0 & G_2(x)\end{pmatrix},\\ F(x)&=(\,F_1(x)\quad F_2(x)\,)^\top\,. \end{aligned} \tag{26}$$

We can concluded that the positions errors e_1 and e_2 can be controlled using the auxiliaries variables ζ_1 and ζ_2 respectively, which can be controlled using the real control signal τ.

Let ζ_1^d and ζ_2^d be the values of ζ_1 and ζ_2 respectively, which ensuring the stabilization of the positions tracking error e_1 and e_2, also these desired values are determined using Lyapunov approach by considering the dynamic equation of e_1 and e_2, consequently $e_3=\zeta_1-\zeta_1^d$ and $e_4=\zeta_2-\zeta_2^d$ with $E=(e_3\quad e_4)^\top$. Then

$$\zeta_d=(\,\zeta_1^d\quad \zeta_2^d\,)^\top\,,\qquad E=\zeta-\zeta^d\,. \tag{27}$$

The control signal is written as follows

$$A_1(x)\tau+A_2(x)\frac{\mathrm{d}\tau}{\mathrm{d}t}=E_c(t) \tag{28}$$

$$E_c(t)=C_0(x)\zeta^d-B_0(x)\zeta-B_1(x)\dot\zeta\,, \tag{29}$$

$A_1(x)$, $A_2(x)$, $C_0(x)$, $B_0(x)$ and $B_1(x)$ are nonlinear matrix gains of multivariable nonlinear CDM controller introduced in (28) and (29).

A backstepping procedure [9–20] is proposed to determinate the gains matrix assuring the exponential stability result for the links positions tracking errors.

Step 1: Firstly we design the virtual control law $\zeta_1^d(t)$ then $\zeta_2^d(t)$, theirs positions error must asymptotically converge to zero.

Step 2: secondly we choose the gains matrix $A_1(x)$, $A_2(x)$, $C_0(x)$, $B_0(x)$ and $B_1(x)$ by employing the augmented Lyapunov function that oblige the errors to track an exponential convergence.

5 CDM–BACKSTEPPING CONTROL DESIGN

Proposition 1. *The positions tracking error e_1 and e_2 are exponentially stable with the following condition*

$$\zeta_1^d=-\lambda_1 e_1+\dot q_1^d\,,\quad \zeta_2^d=-\lambda_2 e_2+\dot q_2^d\,. \tag{30}$$

Proof 1. The Lyapunov formulation can be written as

$$V_1=\frac{1}{2}e_1^2+\frac{1}{2}e_2^2, \tag{31}$$

where its time derivative can be represented as

$$\dot V_1=e_1\dot e_1+e_2\dot e_2\,. \tag{32}$$

Since the virtual control ζ_1 and ζ_2 track the desired value specified in (30), the derivative of the Lyapunov function become negative and takes the next form.

$$\dot V_1\le-\lambda_1 e_1^2-\lambda_1 e_2^2. \tag{33}$$

Then

$$\dot{V}_1 \le 0 . \tag{34}$$

As a result, the exponentially stability can be achieved for e_1 and e_2.

The control signal $\tau = (\tau_1 \quad \tau_2)^\top$ that oblige the errors e_3 and e_4 to converge to zero will be now deducted. Let

$$A_1(x) = -K\frac{\mathrm{d}G(x)}{\mathrm{d}t}, \quad A_2(x) = -KG(x) \tag{35}$$

with any positive definite matrix K.

Combining equations (27) with (29) gives

$$E = \big(B_0^{-1}C_0 - I\big)\zeta^d - B_0^{-1}E_c \tag{36}$$

And tacking

$$C_0(x) = B_0(x) = C_0 = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}. \tag{37}$$

Then

$$E_c = -C_0 E . \tag{38}$$

Its second derivative is

$$\ddot{E}_c(t) = C_0\ddot{\zeta}^d(t) - C_0\ddot{\zeta}(t) . \tag{39}$$

Combining equations (28), (29) and (35) gives

$$\ddot{\zeta}(t) = \dot{F}(x) + K_1 E_c \tag{40}$$

$$K_1 = K^{-1}. \tag{41}$$

Substituting equation (40) into (39), we obtain

$$\ddot{E}_c(t) = C_0\ddot{\zeta}^d(t) - C_0(\dot{F}(x) + K_1E_c) . \tag{42}$$

Then

$$\dot{E}_c(t) = C_0\dot{\zeta}^d(t) - C_0\big(F(x) + K_1\int_0^t E_c(\rho)\mathrm{d}\rho\big) \tag{43}$$

and using equation (38)

$$\dot{E}(t) = H(x) - K_2\int_0^t E(\rho)\mathrm{d}\rho \tag{44}$$

with

$$H(x) = F(x) - \dot{\zeta}^d(t) , \quad K_2 = C_0K_1 . \tag{45}$$

And

$$H(x) = \begin{bmatrix} H_1(x) \\ H_2(x) \end{bmatrix} = \begin{bmatrix} F_1(x) - \dot{\zeta}_1^d \\ F_2(x) - \dot{\zeta}_2^d \end{bmatrix} \tag{46}$$

tacking

$$K_1 = \begin{bmatrix} \delta_1 \operatorname{sign}(z_1) & 0 \\ 0 & \delta_2 \operatorname{sign}(z_2) \end{bmatrix} \qquad z = e_{i+1}\int_0^t e_{i+1}(\rho)\mathrm{d}\rho , \quad i = 1, 2 . \tag{47}$$

PROPOSITION 2. *Consider the robot manipulator dynamic (8), in closed-loop with the multivariable CDM control (28), suppose that the gains δ_1, δ_2, c_1 and c_2 are such that*

$$\Big|c_1\delta_1 \operatorname{sign}(z_1)\int_0^t e_3(\rho)d\rho\Big| \succ \Delta_1 \textit{with } \Delta_1 \ge |e_1| + |H_1(x)|,$$
$$\Big|c_2\delta_2 \operatorname{sign}(z_2)\int_0^t e_4(\rho)d\rho\Big| \succ \Delta_2 \textit{with } \Delta_2 \ge |e_2| + |H_2(x)|. \tag{48}$$

According to Lyapunov stability, it implies that the tracking errors $e_1(t)$, $e_2(t)$, $e_3(t)$ and $e_4(t)$ are exponentially stable and the closed-loop system is internally stable.

P r o o f . Consider the augmented Lyapunov function [12].

$$V_2 = V_1 + \frac{1}{2}E^\top E . \tag{49}$$

Its derivative along the plant trajectories is given by

$$\dot{V}_2 = \dot{V}_1 + E^{|top}\dot{E} . \tag{50}$$

Using the expressions of $e_3(t)$, $e_4(t)$ and equation (32), we get

$$\dot{V}_2 = -\lambda_1e_1^2 - \lambda_2e_2^2 + e_1e_3 + e_2e_4 . \tag{51}$$

This gives

$$\dot{V}_2 \le -\lambda_1e_1^2 - \lambda_2e_2^2 + E^\top\left[\dot{E} + \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}\right]. \tag{52}$$

Changing the dynamics of E by (44) and the control signal by (28), then one has

$$\dot{V}_2 \le -\lambda_1e_1^2 - \lambda_2e_2^2 + v(t) \tag{53}$$

where

$$v(t) = E^\top\left[\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} + \begin{pmatrix} H_1(x) \\ H_2(x) \end{pmatrix} - K_2\int_0^t E(\rho)\mathrm{d}\rho\right]. \tag{54}$$

If $v(t) < 0$ then

$$\dot{V}_2 \le -\lambda_1e_1^2 - \lambda_2e_2^2 . \tag{55}$$

Notice that

$$v(t) = E^\top\left[\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} + \begin{pmatrix} H_1(x) \\ H_2(x) \end{pmatrix} - \begin{pmatrix} c_1\delta_1z_1 \operatorname{sign}(z_1) \\ c_2\delta_2z_2 \operatorname{sign}(z_2) \end{pmatrix}\right]. \tag{56}$$

Furthermore, to guarantee the negativity of $\dot{V}_2$, the gains δ_1, δ_2, c_1 and c_2 must be chosen from inequality (48) for the reason that $z_iS(z_i) > 0$ and $v(t) \le 0$. Therefore, it can be concluded that

$$\dot{V}_2 \le 0 . \tag{57}$$

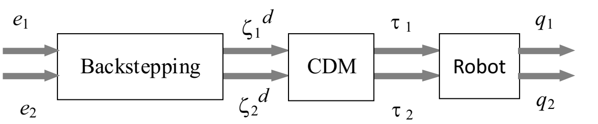

Fig. 2. CDM-Backstepping controller scheme

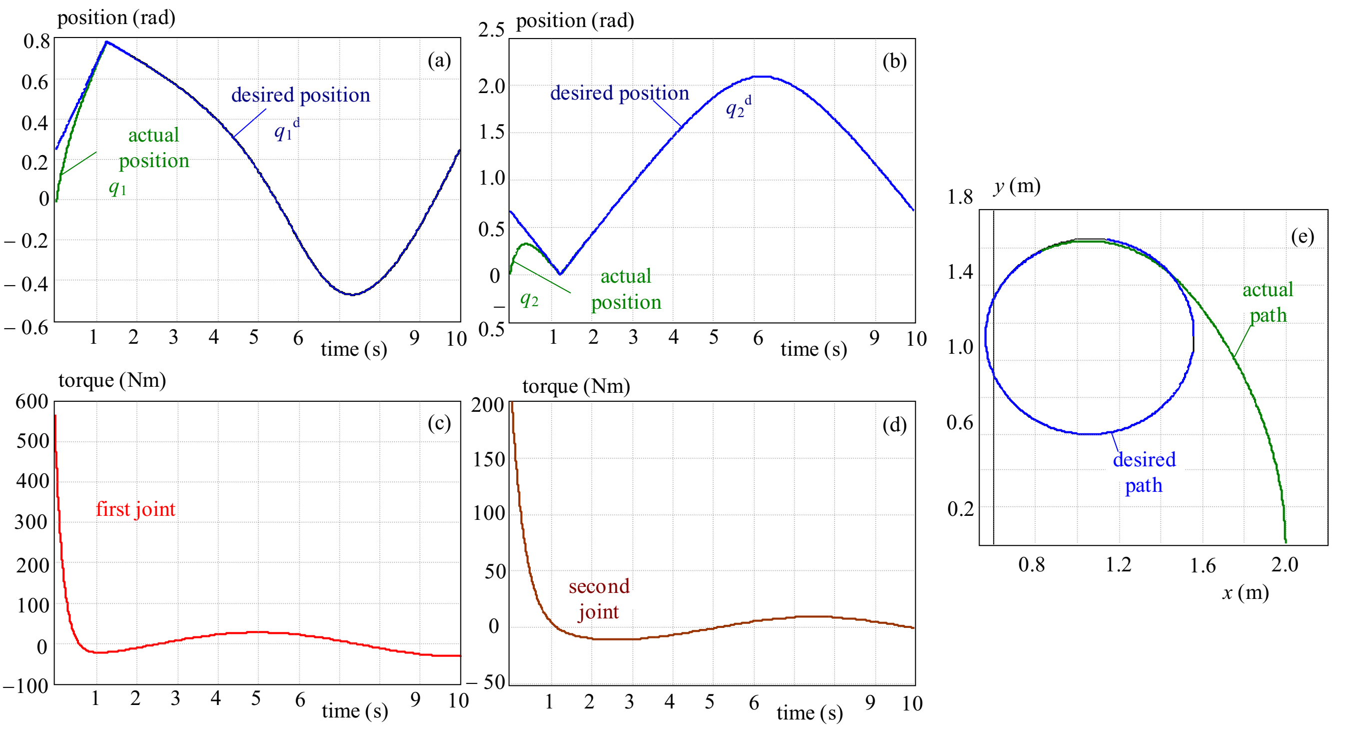

Fig. 3. CDM-Backstepping control; test one (a) — Desired and actual positions of the first joint, (b) — Desired and actual positions of the second joint, (c) — Actual torque of the first joint, (d) Actual torque of the second joint, (e) — Actual and desired path

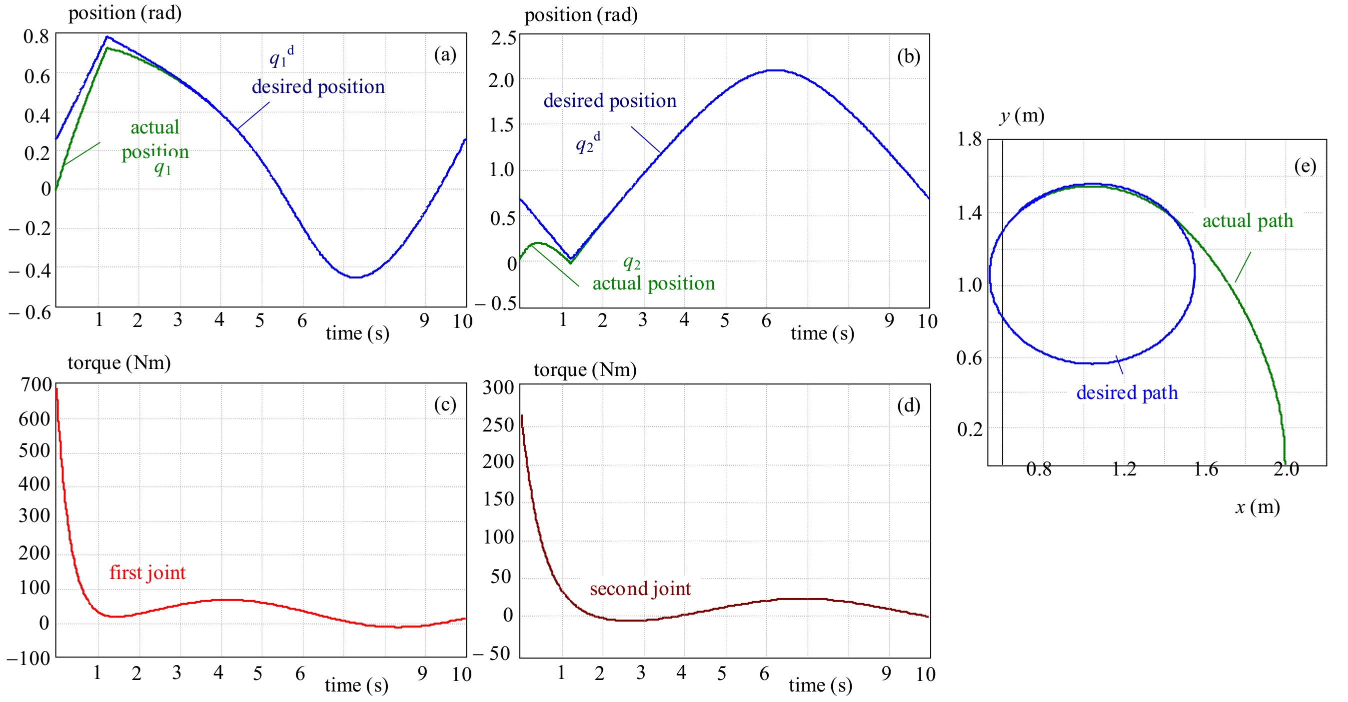

Fig. 4. CDM-Backstepping control; test two (a) — Desired and actual positions of the first joint, (b) — Desired and actual positions of the second joint, (c) — Actual torque of the first joint, (d) Actual torque of the second joint, (e) — Actual and desired path

It implies that the dynamic system is exponentially stable according to Lyapunov stability theorem.

The boundedness of state vector $X = (x_1, x_2, x_3, x_4)$ is not guarantying by the asymptotically convergence of tracking errors. q_1^d and q_2^d are bounded and the errors e_1 and e_2 are exponentially stable so that the state $\sigma = (x_2, x_4)$ is bounded, also the state $\eta = (x_1, x_3)$ is bounded; this proves that the origin of the subsystem $\sigma = \dot{\eta}$ is stable.

Remark 2. Notice that strict knowledge of the limits Δ_i and functions F_i is not required. Bounds can be employed on these variables to ensure a nonlinear robust

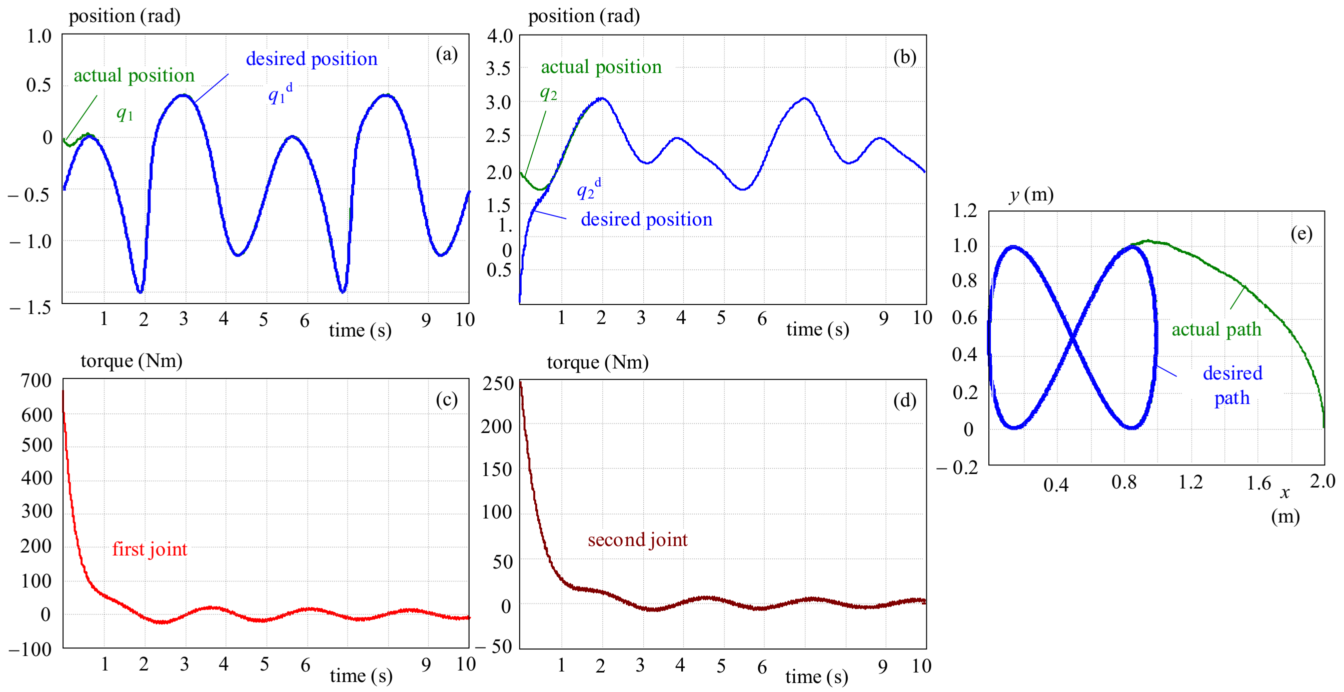

Fig. 5. CDM-Backstepping control; test three (a) — Desired and actual positions of the first joint, (b) — Desired and actual positions of the second joint, (c) — Actual torque of the first joint, (d) Actual torque of the second joint, (e) — Actual and desired path

controller under condition of disturbance, parametric uncertainties and noises. The Integral gains must be satisfactorily large to realize (48). The CDM gains are selected as designated in Proposition 2. The following CDM gains have been used for all the simulated situations.

$$(\delta_1,\ \delta_2) = (103,\ 110)\,, \quad (c_1,\ c_2) = (0.7,\ 0.6)\,. \quad (58)$$

6 SIMULATION RESULT

In order to evaluate the quality of the derived algorithms of nonlinear control, simulation tests were performed using Matlab for circular path and butterfly shape trajectory.

Test one: External disturbances

The externals disturbances that can be applied are disturbance in torques of $\tau_{d1}(t) = 10$ Nm and $\tau_{d1}(t) = 13$ Nm are applied for each joints of robotic system.

The simulation plots shown in Fig. 3 indeed verify that our CDM-Backstepping design scheme can guarantee the best performance for each joint of the robotic manipulator to track its desired trajectory exponentially and eliminate the disturbance with no overshoot and with a negligible steady state error.

Test two: Parametric uncertainties

In the second test for the robustness evaluation of the controllers, we introduced the following parametric uncertainties in the robot models.

- Tool attached to end effector, then parametric uncertainties at second link in mass $m = 4$ kg and in length $l = 0.1$ m.
- Coulomb friction and viscous friction are added to each joint of robot manipulator and given by $f_{cv1} = 2.5x_2(t) + 1.8\,\mathrm{sign}\big(x_2(t)\big)$ and $f_{cv2} = 2x_4(t) + 1.2\,\mathrm{sign}\big(x_4(t)\big)$.

The simulation results in Fig. 4 show the strong robustness of the proposed CDM-Backstepping control towards uncertainties affecting the robot mechanical parameters.

Test three: Change in the desired path

To test the controller's robustness the simulations have been executed using the same last externals disturbances and parametric uncertainties with noises applied for a butterfly shape trajectory, whish are used as a desired end-effector's path. The simulations results are depicted in Fig. 5, this ?gure show that CDM-Backstepping control present a robust path following in 2D displacement.

7 CONCLUSION

This paper reveals a new approach of robust control systems CDM for robot manipulators using backstepping design. The major distinctive of the proposed approach is the application of the novel Lyapunov functions to construct the CDM-Backstepping controller. Global stability results are obtained and the tracking errors converge to zeros with exponential forms. Simulations results have been given to demonstrate the theoretical analysis used in the controller. Further investigation can be directed to the robustness of CDM-Backstepping controller.

Appendix

Rated data of the simulated robot manipulator $b_1 = 75\,\mathrm{N/r/s}$, $b_2 = 10\,\mathrm{N/r/s}$, $l_1 = 1\,\mathrm{m}$, $l_2 = 1\,\mathrm{m}$, $m_1 = 10\,\mathrm{kg}$, $m_2 = 10\,\mathrm{kg}$, $k_1 = 40\,\mathrm{Nm/v}$, $k_2 = 20\,\mathrm{Nm/v}$, $g = 9.81\,\mathrm{ms^{-2}}$.

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