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PAPERS

Feedback stabilization for one sided Lipschitz nonlinear systems in reciprocal state space: Synthesis and experimental validation

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This paper proposes a design of a stabilization control for one-sided Lipschitz (OSL) nonlinear systems in new reciprocal state space (RSS) framework. The main objective is to extend the state derivative feedback stabilization methods for a class of nonlinear systems where the nonlinearity of derivatives state satisfies the OSL properties in RSS. The presented controller is composed of a state derivative feedback approach in order to ensure asymptotic stability in the sense of Lyapunov. The first approach deals with the synthesis of a basic controller by adopting a simple transformation of Linear Matrix Inequality (LMI) to standard algebraic Ricatti equation (ARE). The second is an extension to adaptive version with adjustment parameters. High performances are shown through real-time implementation with a hardware in the loop (HIL) mode using digital signal processing (DSP) device (DSpace DS 1104).

K e y w o r d s: reciprocal state space (RSS), state derivative feedback, ons-sided Lipschitz systems, adaptive control, DSpace DS 1104

1 Introduction

During the past decades, the control of nonlinear systems has been fully exploited in many industrial control and monitoring areas. One of the key elements in the control-design is the feedback principle (using state or/and output) [1–6] in Standard State Space (SSS) form. Effectively, particular attention has been devoted by the scientific community, in recent years, to: the feedback stabilization [7–9] ; tracking and control problems [10] due to the importance of their applications.

Among the classes of the treated systems, the control of OSL nonlinear systems becomes an interesting field of research. In fact, this importance is since several physical systems satisfy the OSL properties (such as one-link flexible joint robot, the inverted pendulum system, ...). In addition, several researchers have largely addressed the control problem of this type of nonlinear systems in a SSS framework using the feedback principle; which has led to the development of many recent approaches such as: stabilization version [11]; robust version developed by [12]; learning control approach [13]; sliding window method [14] and extensions to the fault-tolerant control [15, 16].

Despite the large number of works dealing with the stabilization and control of this class of systems in SSS, several applications and systems provide measurements and information describing their operating principles according to derivatives state (Electrical system with impulse mode [17], acceleration sensors, mobile robot [18], ...). So, the application of control synthesis methods in the SSS is not feasible [17, 19] for these types of systems where only the nonlinearity of state was considered.

Hence, the need to use a new RSS framework that takes in account these classes of systems with derivative state presents a judicious solution. Furthermore, the studies of Lipschitz nonlinearity [20] of state derivative were few [21] in control problem, while, the studies of OSL do not exist in the literature. These limitations motivate the synthesis of feedback stabilization for OSL nonlinear systems in RSS form. This paper is an extension and generalization of the synthesis method proposed by [20, 22] for nonlinear systems where the nonlinearity satisfies the OSL proprieties in the RSS framework. First, a basic feedback control is designed to stabilize the OSL nonlinear system. Lyapunov's asymptotic stability is ensured through the transformation of an LMI resolution problem to standard ARE. Second, an adaptive approach is presented with adjustment parameters. The update methodology considered in this case, is extracted from the Lyapunov function to reduce the parametric errors which ensures the stability of the closed loop system.

First, the problem and the preliminary will be presented. Next, the method of control designed to stabilize an OSL nonlinear system, through the transformation of an LMI resolution problem to standard ARE, will be given in details. After, the next section presents the design of the approach of adaptive stabilization of OSL nonlinear systems in RSS form. Finally, high performance

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of the proposed methods through Real Time Implementation using the DSpace DS 1104 kit will be shown in the last section.

2 Problem statement and preliminaries

The problem of synthesis of a control law in many practical applications, is that the output of the system is formulated with state derivatives [22, 21]. That's why a new RSS has been developed and applied on Lipschitz nonlinear systems (considering the Lipschitz nonlinearity of state derivative) with real-time implementations [20]. Hence, with this same approach, the purpose of this paper is to extend the study for general class of nonlinear systems by adopting new OSL properties in the RSS framework. For that, and in a similar way to the work of [20, 22, 23], the considered nonlinear system in RSS form is

$$x = \overline{A}\dot{x} + \overline{B}u + \overline{f}(\dot{x}) \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ are respectively the state and input vectors. A and B are constant matrices of appropriate dimensions.

$\overline{f}(\dot{x}) \in \mathbb{R}^{n\times m}$ is the nonlinear vector field which satisfies the following proposed One Sided Lipschitz (OSL) and Quadratic Inner-Bounded (QIB) properties [14, 11] in RSS.

Properties:

$\overline{f}(\dot{x})$ satisfies the OSL propriety in RSS framework, with respect to $\dot{x}$ *ie* :

$$\langle \overline{f}(\dot{x}_1) - \overline{f}(\dot{x}_2), \dot{x}_1 - \dot{x}_2\rangle \le \bar{\rho}\|\dot{x}_1 - \dot{x}_2\|^2 \tag{2}$$

$\forall \dot{x}_1, \dot{x}_2 \in \mathbb{R}^n$; where $\overline{\rho}$ is the OSL constant.

$\overline{f}(\dot{x})$ is QIB in RSS framework with respect to $\dot{x}$, *ie*

$$\|\overline{f}(\dot{x}_1)-\overline{f}(\dot{x}_2)\|^2 \le \bar{\beta}\|\dot{x}_1-\dot{x}_2\|^2+\bar{\gamma}\langle \dot{x}_1-\dot{x}_2\,,\, \overline{f}(\dot{x}_1)-\overline{f}(\dot{x}_2)\rangle \tag{3}$$

$\forall \dot{x}_1, \dot{x}_2 \in \mathbb{R}^n$; where $\bar{\beta}$ and $\bar{\gamma}$ are real scalars.

These constants can be positive, zero or negative, contrary to the Lipschitz constant which can be only positive.

Furthermore, if $\overline{f}(\dot{x})$ is Lipschitz [20], it is also OSL $\bar{\rho} > 0$ and QIB ($\bar{\beta} > 0$ and $\bar{\gamma} = 0$. Now, according to properties (2), (3) and assuming $\overline{f}(0) = 0$ with $\dot{x}_1 = \dot{x}$ and $\dot{x}_2 = 0$, then we can obtain the following inequalities.

OSL condition of $\overline{f}(\dot{x})$:

$$\bar{\mu}_1\bar{\rho}\dot{x}^\top\dot{x} - \bar{\mu}_1\dot{x}^\top\overline{f} \ge 0\,. \tag{4}$$

QIB condition of $\overline{f}(\dot{x})$:

$$\bar{\mu}_2\bar{\beta}\dot{x}^\top\dot{x} + \bar{\mu}_2\bar{\gamma}\dot{x}^\top\overline{f} - \bar{\mu}_2\overline{f}^\top\overline{f} \ge 0\,. \tag{5}$$

OSL and QIB conditions of $\overline{f}(\dot{x})$:

$$(\bar{\mu}_1\bar{\rho} + \bar{\mu}_2\bar{\gamma})\dot{x}^T\dot{x} + (\bar{\mu}_2\bar{\gamma} - \bar{\mu}_1)\dot{x}^\top\overline{f} - \bar{\mu}_2\overline{f}^\top\overline{f} \ge 0 \tag{6}$$

where $\bar{\mu}_1 > 0$, $\bar{\mu}_2 > 0$ are chosen arbitrarily.

These properties and assumptions will then be the key elements of the synthesis of stabilizing control law in the RSS form. Indeed, through these proprieties, we present in the following two methods of stabilization. The first method, the gain is calculated through a transformation of an LMI to an ARE. The second includes in its synthesis an adaptive mechanism.

R e m a r k 1. The controllability and observability analyses for nonlinear system in RSS framework, investigated in [24, 25], remain valid in this paper.

3 Feedback stabilization with LMI transformation

Using the properties and assumptions mentioned in section above and in order to guarantee the asymptotic stability in the sense of Lyapunov, the following state derivative feedback control law is proposed.

THEOREM 1. *The nonlinear RSS system* (1) *which satisfies the conditions* (2), (3), (6), *will be asymptotically stable in the sense of Lyapunov by using the derivative state feedback control* $u = \bar{K}\dot{x}$, *if there exists* $\bar{m}_1$, $\bar{m}_2$, $\overline{\eta} > 0$ *and matrix* $P = P^T > 0$, *where*

$$u = \bar{K}\dot{x} = -\frac{K}{\|\bar{B}\|^2}\dot{x} \tag{7}$$

with $K = \frac{1}{2}\bar{B}^\top P$, P *is the solution of the following ARE:*

$$P\bar{A} + \bar{A}^T P + P\Big[I - \frac{\bar{B}\bar{B}^T}{\|\bar{B}\|^2}\Big]P + (\bar{m}_1 + \bar{m}_2 + \overline{\eta})I = 0 \tag{8}$$

with $\bar{m}_1 = \frac{\bar{\mu}_1\bar{\rho}+\bar{\mu}_2\bar{\gamma}}{\bar{\mu}_2}$; $\bar{m}_2 = \frac{\bar{\mu}_2\bar{\gamma}-\bar{\mu}_1}{\bar{\mu}_2}\overline{\rho}$.

P r o o f . Using the state derivative feedback control law $u = \bar{K}\dot{x}$, the closed loop of system (1) becomes

$$x = \underbrace{(\overline{A} + \overline{B}\bar{K})}_{\bar{A}_c}\dot{x} + \overline{f}(\dot{x}) \tag{9}$$

In order to guarantee the closed-loop stability of the system (9) with the control law $\bar{K}$, the following Lyapunov function candidate is selected

$$V(x) = x^\top Px \tag{10}$$

where P is a Symmetric Positive Definite (SDP) matrix. Then, $\dot{V}(x)$ becomes:

$$\begin{aligned}\dot{V}(x) &= \dot{x}^\top Px + x^\top P\dot{x} = \\ &\dot{x}^\top P\big(\bar{A}_c\dot{x} + \overline{f}(\dot{x})\big)+\big(\bar{A}_c\dot{x} + \overline{f}(\dot{x})\big)^\top P\dot{x} = \\ &\dot{x}^\top\big(P\bar{A}_c + \bar{A}_c^\top P\big)\dot{x} + \underbrace{\dot{x}^\top P\overline{f}(\dot{x}) + \overline{f}^\top(\dot{x})P\dot{x}}_{\bar{D}}\,.\end{aligned} \tag{11}$$

Now, applying the lemma $X^TY + Y^TX \leq X^TX + Y^TY$ on the term $\bar{D}$, we obtain

$$\dot{x}^\top P\overline{f}(\dot{x}) + \overline{f}^\top(\dot{x})P\dot{x} = \dot{x}^\top PP\dot{x} + \overline{f}^\top(\dot{x})\overline{f}(\dot{x}) \tag{12}$$

hence, $\dot{V}(x)$ becomes

$$\dot{V}(x) = \dot{x}^\top \big(P\bar{A}_c + \bar{A}_c^\top P + PP\big)\dot{x} + \overline{f}^\top(\dot{x})\overline{f}(\dot{x})\,. \tag{13}$$

Moreover, by using the (4) and (5), (13) can be written as

$$\begin{aligned}\dot{V}(x) &= \dot{x}^\top(P\bar{A}_c + \bar{A}_c^T P + PP)\dot{x} + \\ &\quad \underbrace{\frac{\bar{\mu}_1\bar{\rho} + \bar{\mu}_2\bar{\gamma}}{\bar{\mu}_2}}_{\bar{m}_1}\,\dot{x}^\top\dot{x} + \underbrace{\frac{\bar{\mu}_2\bar{\gamma} - \bar{\mu}_1}{\bar{\mu}_2}\bar{\rho}}_{\bar{m}_2}\,\dot{x}^\top\dot{x} \\ &= \dot{x}^\top(P\bar{A}_c + \bar{A}_c^\top P + PP + (\bar{m}_1 + \bar{m}_2)I)\dot{x}\,. \end{aligned} \tag{14}$$

Thereafter, assuming that it exists a scalar $\bar{\eta} > 0$ such as

$$\dot{V}(x) = \dot{x}^\top(P\bar{A}_c + \bar{A}_c^\top P + PP + (\bar{m}_1 + \bar{m}_2)I)\dot{x} = -\bar{\eta}\dot{x}^\top\dot{x} \tag{15}$$

this leads to

$$\dot{V}(x) \leq -\bar{\eta}\dot{x}^\top\dot{x} < 0\,. \tag{16}$$

By proposing the following state derivative feedback control law: $u = \bar{K}\dot{x} = -\frac{K}{\|\bar{B}\|^2}\dot{x}$, (15) becomes

$$\begin{aligned}P\bar{A}_c + \bar{A}_c^\top P + PP + (\bar{m}_1 + \bar{m}_2)I &= P\bar{A} + \bar{A}^\top P + \\ (\bar{m}_1 + \bar{m}_2 + \bar{\eta})I - P\frac{\bar{B}K}{\|\bar{B}\|^2} &- \frac{K^\top\bar{B}^\top}{\|\bar{B}\|^2}\,. \end{aligned} \tag{17}$$

by choosing $K = \frac{1}{2}\bar{B}^\top P$, (eq51a) will be in the standard form of an ARE:

$$P\bar{A} + \bar{A}^\top P + P[I - \frac{\bar{B}\bar{B}^\top}{\|\bar{B}\|^2}]P + (\bar{m}_1 + \bar{m}_2 + \bar{\eta})I = 0 \tag{18}$$

proves that the solution of (8) ensures Lyapunov's asymptotic stability for the original system. □

R e m a r k 2 [26, 23]. $\Big[I - \frac{\bar{B}\bar{B}^T}{\|\bar{B}\|^2}\Big]$ and $(\bar{m}_1 + \bar{m}_2 + \bar{\eta})I$ are SDP, therefore when the matrix $\bar{A}$ is Hurwitz, there exists a SDP matrix P (solution of (18) if the associated Hamiltonian matrix (H) is hyperbolic [26]:

$$H = \begin{bmatrix} \bar{A} & I - \frac{\bar{B}\bar{B}^\top}{\|\bar{B}\|^2} \\ -(\bar{m}_1 + \bar{m}_2 + \bar{\eta})I & -\bar{A}^\top \end{bmatrix}$$

4 Extension to adaptive design

In this section, the objective is to synthesize a stabilizing adaptive control for OSL nonlinear systems verifying the properties (2)–(6) with the same analogy of [20].

First, the same system (1) is considered in this section where the following assumption is satisfied:

Assumption 1:

Assuming there exists a matrices: $\bar{P}$ where $\bar{P} = \bar{P}^T \geq 0$; $\Theta^* \in \mathbb{R}^{m\times n}$, verifying the equation

$$\bar{P}(\bar{A} + \bar{\rho}I + \bar{B}\Theta^*) + (\bar{A} + \bar{\rho}I + \bar{B}\Theta^*)^\top\bar{P} = -\bar{Q} \tag{19}$$

where Θ^* is unknown matrix and $\bar{Q}$ is SDP.

Now, by choosing an adaptive derivative state feedback control in the following form: $u = \Theta\dot{x}$ with an adaptation technique for the row vector Θ, the closed loop system

$$x = (\overline{A} + \overline{B}\Theta)\dot{x} + \overline{f}(\dot{x}) \tag{20}$$

is asymptotically stable where $\Theta \mapsto \Theta^*$.

In order to guarantee the closed loop stability, the following state derivative feedback control law is proposed.

THEOREM 2. *The nonlinear RSS system* (1) *will be asymptotically stable by using the adaptive derivative state feedback control* $u = \Theta\dot{x}$ *with* Θ *is an adjustable vector*

$$\dot{\Theta}^\top = -\lambda\dot{x}\dot{x}^\top\bar{P}\bar{B} \tag{21}$$

such that λ *is the adaptation rate.*

P r o o f . With the same approach of [20], and by adding and subtracting $\Theta^*\bar{B}\dot{x}$ in (20), we have

$$x = \big(\bar{A} + \bar{B}\Theta^*\big)\dot{x} + \bar{B}\big(\Theta - \Theta^*\big)\dot{x} + \bar{f}(\dot{x})\,. \tag{22}$$

Initially, the candidate Lyapunov function is defined by

$$V(x) = \frac{1}{2}x^\top\bar{P}x + \frac{1}{2\lambda}(\Theta - \Theta^*)(\Theta - \Theta^*)^\top. \tag{23}$$

Then, $\dot{V}(x)$ becomes:

$$\begin{aligned}\dot{V}(x) &= \frac{1}{2}[\dot{x}^\top\bar{P}x + x^\top\bar{P}\dot{x}] + \frac{1}{\lambda}(\Theta - \Theta^*)\dot{\Theta}^\top = \\ &\frac{1}{2}[\dot{x}^\top\bar{P}\{(\bar{A} + \bar{B}\Theta^*)\dot{x} + \bar{B}(\Theta - \Theta^*)\dot{x} + \bar{f}(\dot{x})\} + \\ &\{(\bar{A} + \bar{B}\Theta^*)\dot{x} + \bar{B}(\Theta - \Theta^*)\dot{x} + \bar{f}(\dot{x})\}^\top\bar{P}\dot{x}] + \frac{1}{\lambda}(\Theta - \Theta^*)\dot{\Theta}^\top. \end{aligned} \tag{24}$$

Second, by using (2) where $\bar{\rho}\dot{x}^\top\dot{x} \geq \dot{x}^\top\bar{f}(\dot{x})$ and knowing that the matrix P is SDP, we obtain $\dot{x}^\top P\bar{f}(\dot{x}) \leq \bar{\rho}\dot{x}^\top P\dot{x}$, this leads to

$$\begin{aligned}\dot{V} &\leq \dot{x}^\top\bar{P}\{(\bar{A} + \bar{B}\Theta^*)\dot{x} + \dot{x}^\top\bar{B}(\Theta - \Theta^*)\dot{x} + \frac{1}{\lambda}(\Theta - \Theta^*)\dot{\Theta}^\top \\ &+ \dot{x}^\top\bar{P}\bar{\rho}\dot{x} \leq \frac{1}{2}[\dot{x}^\top\bar{P}(\bar{A} + \bar{B}\Theta^* + \bar{\rho})\dot{x} + \dot{x}^\top(\bar{A} + \bar{B}\Theta^* + \bar{\rho})^\top\bar{P}\dot{x}] \\ &+ (\Theta - \Theta^*)\Big[\dot{x}\dot{x}^\top\bar{P}\bar{B} + \frac{1}{\lambda}\dot{\Theta}^\top\Big]. \end{aligned} \tag{25}$$

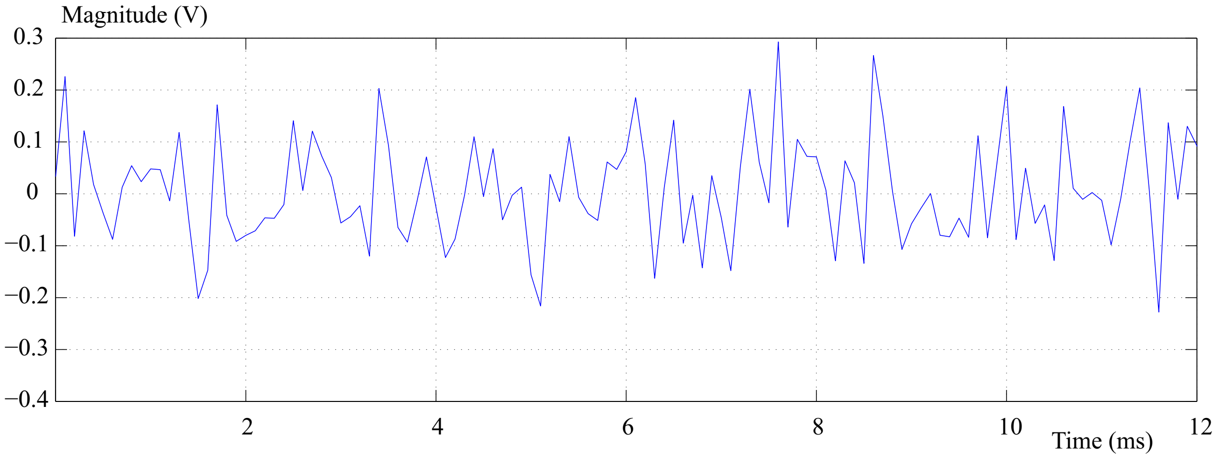

Fig. 1. Evolution of control $u_1(t)$

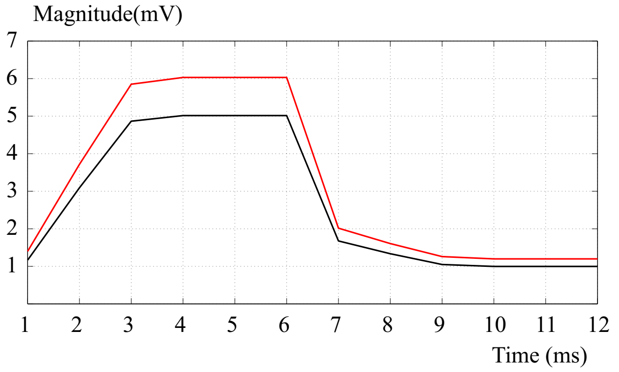

Fig. 2. Convergence of adaptive gains, Matrix Θ

Finally, by using the property (19) and the control law (21), it is easy to prove that $\dot{V} \le -\frac{1}{2}\dot{x}^\top \bar{Q}\dot{x}$. This shows that the proposed adaptive control guarantees the closed-loop stability of this class of OSL systems. □

Remark 3. *Remarks 2;3;4*, in [20] remain valid for this case of OSL nonlinear systems, then the proposed methods can be generalized to the case of (by adapting the matrices dimensions):

1) Distributed,decentralized and nonlinear singular systems [21]
2) The class of nonlinear systems in the form : $x = \overline{A}\dot{x} + \overline{f}(\dot{x}) + \overline{g}(\dot{x})u(t)$.
3) The values of the adaptation ratios which are chosen, after practical tests, are almost equal to the value of the OSL constant $\bar{\rho}$.

5 Experimental results

In this section, the example of electrical system with Impulse Mode will be treated to validate the proposed methods with a real-time implementation using a Digital Signal Processing device *DSpace*® DS 1104, similar to [28]. The dynamic equation for the system is given by

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 0 \\ 1 & -2 \end{bmatrix}}_{\overline{A}} \dot{x} + \underbrace{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}_{\overline{B}} u + \underbrace{\begin{bmatrix} 0 \\ 2sin\dot{x}_1 \end{bmatrix}}_{\overline{f}(\dot{x})}. \tag{26}$$

Using the same method of [27], $\overline{f}(\dot{x})$ satisfies OSL proprieties with $\bar{\rho} = 2$. Also, $\overline{f}(\dot{x})$ is a Lipschitz function and verify the QIB propriety where $\bar{\beta} = 2$ and $\bar{\gamma} = 0$. For the scalar variables, an arbitrary choice gives: $\bar{\mu}_1 = 1$, $\bar{\mu}_2 = 1$.

The initial condition is $x(0) = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$. Subsequently

- Using Theorem 1 with the resolution of (8) gives

$$P = \begin{bmatrix} 0.142 & 0.0334 \\ 0.0334 & 0.1292 \end{bmatrix} \text{ and } K = [\,-0.07 \quad -0.0167\,].$$

- Using Theorem 2 with the resolution of (20) gives ($\bar{Q} = 0.05\, I_2$; $\lambda = 1.87$):

$$\bar{P} = \begin{bmatrix} 2.5 & 0.025 \\ 0.025 & 0.025 \end{bmatrix}.$$

The matrices $\bar{P}$ and P are SDP which proves the feasibility of proposed approaches and conditions. Now, in this phase of implementation, a perturbation has been added to the system in the form of sinusoidal signal with variable amplitude (± 0.4 V) and frequency (between 5 Hz and 70 Hz), Figs 2-3 present respectively: the control $u_1(t)$ (using Theorem 1) and the adaptive gains of matrix Θ (using Theorem 2).

From Figs. 1 and 2 it is clear that the two proposed approaches ensure the stability to the original system without large perturbations and with reduced amplitudes. Effectively, the amplitudes of adaptive control gains (elements of matrix Θ) are reduced by a ratio of 0.3 compared to those of [20, 21] where the nonlinear system is treated considering only the nonlinearity which verifies the lipschitz propriety. Moreover, and since the system

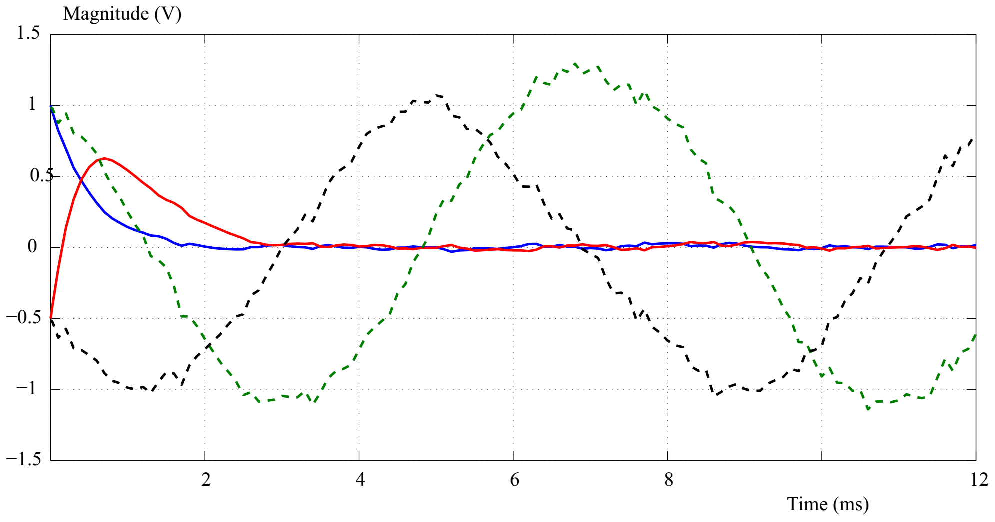

Fig. 3. Evolution of state $x_1(t)$ and $x_2(t)$

verifies the properties of OSL and QIB in RSS framework, this gives more relaxation parameters and offers more degrees of freedom in the synthesis of the control law. Hence the major interest in studying this class of systems.

Now, we are interested in the proposed adaptive approach. In this phase of implementation, two sinusoidal signals are applied on the state dynamics with an amplitude equal to ± 1.1 V with the same range of frequency variation (similar to the extreme effects of perturbation and uncertainties in industrial cases). Figure 3 shows the evolution of the states using the adaptive proposed method.

Figure 3 shows that the control law proposed in Theorem 2 ensures the well stabilization of the system with the presence of disturbing signals with high amplitudes. This also shows the robustness of the proposed adaptive approach.

6 Conclusion

In this paper, a feedback stabilization design for a class of OSL nonlinear systems in RSS form has been presented. The proposed controller is based on the choice of proper Lyapunov functions to ensure the asymptotic stability of the closed-loop system

First, a basic control is designed to stabilize the nonlinear system by transforming the problem of resolution of an LMI to ARE. After that, an extension to adaptive scheme has been proposed. Real time implementation with DSpace DS 1104 board used as an emulator has confirmed the high quality of stabilization offered by the proposed method with the presence of extreme perturbations.

The remaining opened question is: The generalization of the proposed approach for the tracking problem with an extension to the Sliding Mode Control in RSS framework? These extensions will be investigated (as part of a project) in the near future.

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