Signal periodicity detection using Ramanujan subspace projection

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Signal periodic decomposition and periodic estimation are two crucial problems in the signal processing domain. Due to its significance, the applications have been extended to fields like periodic sequence analysis of biomolecules, stock market predictions, speech signal processing, and musical pitch analysis. The recently proposed Ramanujan sums (RS) based transforms are very useful in analysing the periodicity of signals. This paper proposes a method for periodicity detection of signals with multiple periods based on autocorrelation and Ramanujan subspace projection with low computational complexity. The proposed method is compared with other signal periodicity detection methods and the results show that the proposed method detects the signal period correctly in less time.

K e y w o r d s: Ramanujan sums, subspace decomposition, periodicity detection, autocorrelation

1 Introduction

Many real world signals such as music, speech, agriculture, medicine, finance, sales data, stock market, astronomy exhibit periodic and repetitive behaviour. Periodicity detection is important in understanding signal properties, finding the hidden relationship between different signal features, and assists in signal compression and classification techniques. Periodic signals repeat at regular intervals, and this regular interval is termed as period. A discrete signal $x[n]$ is periodic, with period P if

$$x[n + P] = x[n]; \ \forall n \in Z, \tag{1}$$

where P is the smallest positive integer that satisfies (1). Several methods have been proposed to identify the hidden periodic components of a signal. These methods can be mainly classified [1] as time domain methods and frequency domain methods. Time domain methods are based on the autocorrelation function (ACF). In these methods, the threshold value needs to be specified to determine the dominant period. Besides, the peak value of autocorrelation function may be obtained at period P and multiples of period P. Frequency domain methods are based on spectral decomposition of signals. In the spectral decomposition method, Fourier transform (FT) is applied to signal, and from the inverse of fourier coefficients of signal, fundamental period is calculated. But for signals with more than one period, this method is not reliable.

A method for periodicity detection combining both periodogram and autocorrelation technique is proposed in [2] and it is termed as autoperiod. In this method, first, the periodogram of the signal is calculated to get an estimate of the potential periods. If these potential periods lie on the hill of ACF, then these periods are considered

to be valid periods, otherwise; they are discarded. But multiple periodicity detection of signals is not addressed in this method. Sethares and Staley [3] proposed a periodicity detection method based on periodic subspace projection. This method is based on Periodic transforms (PT), and it finds its own set of data-dependent basis elements. But periodic subspaces constructed using this technique lacks orthogonality.

The concept of exactly periodic signal and exactly periodic decomposition (EPSD) based periodicity detection is proposed by Muresan and Parks [4]. Period estimation of any signal, using EPSD is done by computing exactly periodic subspace (EPS) projections. These subspaces are orthogonal to each other only when their periods are divisors of signal length. Recently, P. P. Vaidyanathan has introduced the concept of Ramanujan subspace (Sq) [5–7], a subspace based on Ramanujan sums (RS). Two transforms based on RS, Ramanujan FIR transform (RFT) and Ramanujan periodic transform (RPT) are also proposed. Dictionary based approaches are used for identifying hidden periodicities [8, 9] and these frequency dictionaries are designed based on DFT (discrete Fourier transform), RPT, natural basis and random basis. RPT and DFT based dictionaries yield better performance at lower SNR (signal to moise ratio) values. RPT is used to detect and analyse steady state visual evoked potential of brain signal [10, 11]. RPT is also used to reduce noise in electrocardiogram [12, 13].

Ramanujan filter bank (RFB) based on Ramanujan sums [14, 15] is used to detect periodic patterns such as detection of absence seizures in electroencephalogram [16], detection of periodic segments in DNA [17] and detection of protein repeats [18]. Non adaptive comb filters are designed using RS in order to estimate the periodicity of signal. Time varying periodicity nature of

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signal is successfully represented using RFB. RFB is implemented using a multiplier less and computationally efficient structure [19]. Ramanujan subspace pursuit (RSP) algorithm is proposed for period estimation and periodic decomposition [20]. RSP is a greedy iterative algorithm offering low computational cost.

Another efficient method for periodicity estimation based on RS and variance of data matrix is proposed in [21]. In this method, data of length N is partitioned into a matrix of size $\lceil NP/P \rceil \times P$, where P is the assumed period. The variance of all columns of this data matrix and the corresponding dips are calculated. Dip refers to a point where the magnitude of variance is less than its immediate neighbours. The highest dip is observed at composite period length. Period estimation is done by RS subspace projection to all the factors of this composite period.

Period estimation using the subspace projection method is time consuming as the signal is projected onto subspaces of dimension 1 to maximum expected period Q. In this paper, candidate periods are identified using autocorrelation function, and then subspace projection is done to these selected periods. The final period estimation is done based on subspace projection energies of these selected periods. This approach effectively reduces the number of projections and hence the computation time.

2 Ramanujan sums

Ramanujan sum, denoted by $c_q(n)$, is a function of two positive integer variables q ($q \geq 1$) and n and it is defined [22] as

$$c_q\ (n) \sum_{\substack{k=1 \\ (k,q)=1}}^{q} e^{j2\pi kn/q}, \tag{2}$$

where $(k, q) = 1$ means k only takes values coprime to q. $c_q(n)$ is periodic with period q, *ie* $c_q(n + q) = c_q(n)$.

First few RS are

$$c_1(n) = 1\,,$$
$$c_2(n) = 1, -1\,, \tag{3}$$
$$c_3(n) = 2, -1, -1\,,$$

2.1 Some properties of Ramanujan sums

Some properties of Ramanujan Sums [5] which are significant in signal processing domain are listed below.

Ramanujan sequences, $c_q(n)$ are always integer valued, hence (2) can be also written as

$$c_q(n) = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} \cos \frac{2\pi kn}{q}\,. \tag{4}$$

Multiplicative Property: Ramanujan sums, $c_q(n)$ are multiplicative in q

$$c_{q_1}\, c_{q_2}(n) = c_{q_1}(n)c_{q_2}(n)\,, \tag{5}$$

where q_1 and q_2 are coprime.

Prime: If period q is a prime number then

$$c_q(n) = \begin{cases} q-1 & \text{if } q \text{ is multiple of } n\,, \\ -1 & \text{otherwise.} \end{cases} \tag{6}$$

Power of prime: For any prime q,

$$c_{q^l}(n) = q^{l-1} c_q \Big(\frac{n}{q^{l-1}} \Big), \tag{7}$$

where $l \geq 0$. *ie* higher order RS are up sampled and scaled versions of lower order Ramanujan sums [23]. Other properties of RS and their proofs have been detailed in [5].

2.2 Efficient computation of Ramanujan sums

Computation of Ramanujan sums using direct expression is time consuming; especially for higher values of q. RS can be computed efficiently based on its properties.

The steps for this efficient calculation are given below: If q is prime number, $c_q(n)$ is found using Prime property. For example, for $q = 3$, $c_q(n) = \{2, -1, -1\}$. For q, which can be factored, $c_q(n)$ is found using both power of prime and multiplicative properties.

For *eg*, $q = 4$, $c_4(n)$ can be expressed using power of prime as

$$c_{2^2}(n) = 2c_2\big(\tfrac{n}{2}\big), \tag{8}$$
$$c_2(n) = \{1, -1\}\,, \tag{9}$$
$$c_2\big(\tfrac{n}{2}\big) = \{1, 0, -1, 0\}\,, \tag{10}$$
$$c_{2^2}(n) = 2c_2\big(\tfrac{n}{2}\big) = \{2, 0, -2, 0\}\,. \tag{11}$$

Consider $q = 18$. It can be factorised as 2×3^2. Using multiplicative property, $c_{18}(n)$ can be found by following steps

$$c_{3^2}(18) = \{6, 0, 0, -3, 0, 0, -3, 0, 0, 6, 0, \\ 0, -3, 0, 0, -3, 0, 0\}\,, \tag{12}$$

$$c_2(18) = \{1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, \\ -1, 1, -1, 1, -1, 1, -1\}\,, \tag{13}$$

$$c_{18}(18) = c_{3^2}(18)c_2(18) = \{6, 0, 0, 3, 0, 0, \\ -3, 0, 0, -6, 0, 0, -3, 0, 0, 3, 0, 0\}\,. \tag{14}$$

The method described above saves significant amount of time for large values of q. The time taken to calculate RS of period 900 using direct expression and the aforementioned method are 41.7781 s and 0.035375 s respectively.
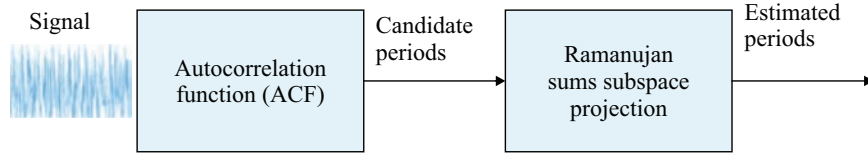
Fig. 1. Block diagram of the proposed method for period estimation

3 Ramanujan subspace (Sq)

Ramanujan subspace Sq is constructed from $c_q(n)$ and its shifted versions. For a given period q, construct a $q \times q$ integer circulant matrix B

$$B_q = \begin{bmatrix} c_q(0) & c_q(q-1) & \dots & c_q(1) \\ c_q(1) & c_q(0) & \dots & c_q(2) \\ c_q(2) & c_q(1) & \dots & c_q(3) \\ \vdots & \vdots & \ddots & \vdots \\ c_q(q-2) & c_q(q-3) & \dots & c_q(q-1) \\ c_q(q-3) & c_q(q-4) & \dots & c_q(0) \end{bmatrix}. \quad (15)$$

For example, for $q = 3$

$$B_3 = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}. \quad (16)$$

Ramanujan subspace $Sq \subset Rq$ is spanned by columns of B. The projection matrix for S is given by

$$P_q = \frac{1}{q} B_q, \quad (17)$$

which satisfies the conditions for projection matrix, $P_q^2 = P_q$ and $P_q^\top = P_q$.

3.1 Orthogonal projection of signal onto S_q

The orthogonal projection x_q of signal onto Sq is represented as

$$x_q = \text{Proj}\,(x, S_q). \quad (18)$$

Let signal x, be of any length, N (not necessarily a multiple of period q). Let $M = \left\lceil \frac{N}{q} \right\rceil$ where $\left\lceil \frac{N}{q} \right\rceil$ is the smallest integer greater than or equal to $\frac{N}{q}$.
The orthogonal projection matrix P_q given in (17) is modified to matrix $\tilde{P}_q$ as

$$\tilde{P}_q = \frac{1}{M} \begin{bmatrix} P_q & \dots & P_q \\ \vdots & \ddots & \vdots \\ P_q & \dots & P_q \end{bmatrix}, \quad (19)$$

where, $\tilde{P}_q$ has a dimension $qM \times qM$.

To construct $\tilde{P}_q$, P_q is repeated $\left\lceil \frac{N}{q} \right\rceil$ times horizontally and vertically. $\tilde{P}_q$ also satisfies $\tilde{P}_q^2 = \tilde{P}_q$, $\tilde{P}_q^\top = \tilde{P}_q$. ie $\tilde{P}_q$ represents an orthogonal projection.

The projection of x into Ramanujan subspace S_q can be written as

$$x_q = \tilde{P}_q x. \quad (20)$$

Before finding the projection, signal $x \in R^n$ is modified into $\tilde{x}$ of length qM by padding $qM - N$ zeros at the end. The signal projection x_q is calculated using (20) and only first N elements of x_q are preserved.

3.2 Exactly periodic subspaces

A signal x is of exactly period P if x is in $R(\Psi^P)$ and the projection of x onto is zero for all $R(\Psi^{\tilde{P}})$ is zero for all $\tilde{P} \leq P$ [4]. A signal with exactly P period is not exactly periodic $2P, 3P, 4P, etc$, though it continues to be of period $2P, 3P, 4P, etc$. Projection of a signal into Ramanujan subspace decomposes the signal into exactly periodic subspaces.

4 Proposed method

The proposed method employs a two tier approach for periodicity estimation. The first phase consists of finding candidate periods using autocorrelation and then estimating the correct period from the candidate periods by subspace projection into RS subspace.

The block diagram of the proposed method is shown in Fig. 1.

4.1 Autocorrelation

Autocorrelation function measures the similarity between the signal and its delayed version. For a discrete time signal $x(n)$, defined for all n, the autocorrelation function is

$$R_{xx}(l) = \frac{1}{N-l} \sum_{n=0}^{N-l-1} x(n)x(n+l), \quad (21)$$

where l is the lag time. Autocorrelation function based periodicity detection method relies on the property that if signal $x(n)$ is periodic with period N, then the autocorrelation of the signal, $R_{xx}(l)$ is also periodic with the same period. Autocorrelation value peaks at $l = 0$, where l is the lag time, since the signal is being correlated with itself. The peaks also occur at values of l that correspond to period N and multiples of N [1]. Let x be a periodic signal of length N with an unknown period p. Potential periods of signal is identified by applying autocorrelation to signal $x(n)$ for lag values up to $\frac{N}{2}$.
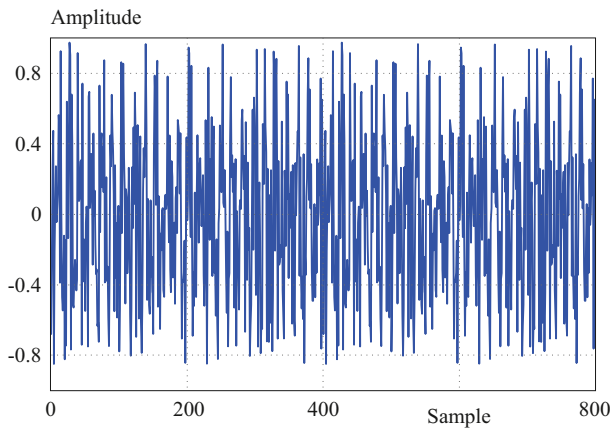
Fig. 2. Mixed periodic signal of length 800 with periods 16 and 25

Fig. 3. ACF of the signal

4.2 Candidate period selection from ACF

After the autocorrelation is calculated for input signal, peak values are selected based on the threshold value. The threshold for selecting peaks from ACF values is chosen, [24]

$$V = (V_{\mathrm{mean}} - V_{\mathrm{min}})k + V_{\mathrm{mean}}, \qquad (22)$$

where, V_{min}, V_{mean} are minimum value and mean value of signal ACF values and k, coefficient for threshold calculation. Since ACF shows peaks at signal period and its multiples, factors of these candidate periods need to be considered for periodicity estimation.

4.3 Subspace projection onto RS

Let q be the actual period of the signal whose periodicity needs to be estimated. The autocorrelation function may give peak values for multiples of q. In order to correctly identify the actual period, the factors of candidate periods obtained from ACF are projected into RS subspace. The projection of signal into Ramanujan Subspace (Sq) decomposes the signal into q exactly periodic subspaces. After the subspace projection, projection energy for that particular period is calculated. The projection energy is calculated based on the periodic norm. Periodic norm is the norm of vector calculated using inner product defined on p-periodic subspaces.

Periodic norm of a vector x is calculated as

$$\|x\| = \frac{\mathrm{norm}(x)}{\sqrt{\mathrm{length}(x)}}. \qquad (23)$$

The subspace projection is done iteratively, *ie* after projecting the period into the RS subspace, the corresponding periodic component is removed from the current signal. This residue is used for the next projection. This process is repeated until the subspace projection is done for all candidate periods. Periods corresponding to projection energies greater than 10 % of the signal energy are selected. The period with the highest projection energy is selected as the estimated period.
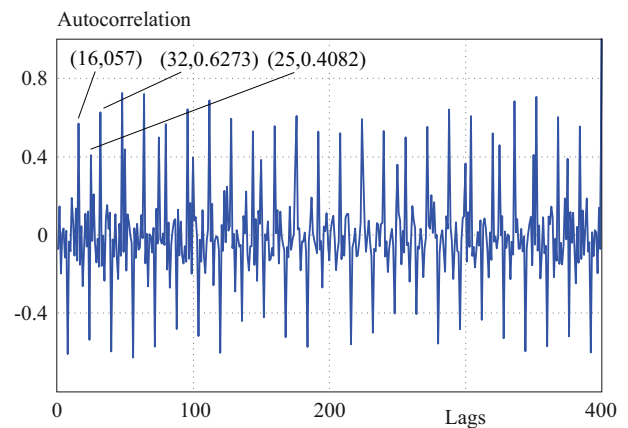
Algorithm for periodicity detection using proposed method is given below

Input: Signal $x \in R^n$ Output: Estimated periods of signal

- Compute autocorrelation function (ACF) of the input signal;
- Extract q corresponds to ACF values greater than threshold, th given by (22)
- Find factors of q from step 2. Let n be the number of factors.
- Initialize: Set $r = x$
- For $i \leftarrow 1$ to n do
  If $\|x_q\| \geq 0.1\,\|x\|$
  then Save x_q and q;
  end
- Update residue $r = r - x_q$;
  end

- Sort q in descending order of x_q

5 Evaluation

The proposed method is applied to a set of mixed periodic signals. The performance is compared with the results obtained from other periodicity detection techniques such as MBest, Small2Large, Mbestgam and BestCorrelation methods as proposed in [3].

5.1 Illustration of the algorithm

Three different cases of input signal periods are considered for illustration of the proposed method.
Case 1: Individual signal periods are co-prime to each other, *eg* 25 and 16.
Case 2: Signal periods are prime, *eg* 13 and 19.
Case 3: Signal periods are multiples of each other *eg* 32 and 64.

In the first case, a synthetic composite signal of length 800 samples with periods 16 and 25 is considered. Signal length is twice the length of the least common multiplier of the periods 16 and 25. The input signal is in Fig. 2.

Table 1. Period detection methods comparison for signal with periods 16 and 25

Methods	Periods detected	Time taken(sec)
Proposed method	25, 16, 8, 5, 2, 4	0.3012
Small2large	16,25	0.5658
mbest	1,5,25,2,10,50,4,100,200,16	2.5270
mbestgam	16,1,5,25,2,8,16,40,200	2.5240
bestcorrelation	8,16,25	0.5475
RSP	25,16,8,5,4,2	0.3440

Table 2. Period detection methods comparison for input signal with periods 19 and 13

Methods	Periods detected	Time taken(s)
Proposed method	19,13	0.1566
Small2large	13,19	0.2296
mbest	19,7,133,78,156,76,152,163,107,161	1.0695
mbestgam	1,13,19,19,13,5,10,20,41,17	1.0445
bestcorrelation	13,19	0.2286
RSP	19,13,1	0.1364

Table 3. Period detection methods comparison for input signal with periods 32 and 16

Methods	Periods detected	Time taken(sec)
Proposed method	32,16,64,4,8,5,20	0.0145
Small2large	16,22,29,30,38,49	0.0222
mbest	4,8,16,32,7,14,42,40,38,39	0.0838
mbestgam	4,32,5,11,7,23,	0.0733
bestcorrelation	8,5,32,11	0.0305
RSP	16,32,8,4,1	0.0183

The ACF of the input signal is computed for lags from 1 to $N = 2$. ACF for lag 0 is not computed since the signal is always perfectly correlated with itself. ACF plot for the signal is shown in Fig. 3.

The threshold value for extracting candidate periods from ACF is calculated using (22); constant k value is set as 1.1 for experiments. The factors of candidate periods are computed. The factors of candidate periods also need to be considered since; the autocorrelation is not a good indicator of true period. The value of ACF is high for true period of signal and its multiples. Here, ACF of signal shows a higher value for period 32, which is a multiple of original signal period 16. Comparison between the proposed method and methods based on periodicity transforms [3] and RSP [20] is given in Table 1, 2 and 3.

In the first case (Table 1) the proposed algorithm detects distinct periods faster than other methods. The methods Small2Large, Best Correlation and RSP also detected correct periods 16 and 25.

In the second case (Table 2) all algorithms except Mbest detected the exact periods correctly, while Mbest detected period 19 but not 13.

In the third case (Table 3)), where one individual signal period is multiple of the second signal period both the proposed algorithm and RSP detect all the periods correctly. In all the three cases of input signal combinations, the proposed method detects the actual periods with less computational time.

The proposed method is compared with other period detection methods such as EPSD [4], RPT [5,6] and RSP [20]. The EPSD period detection is done by projecting signal onto orthogonal periodic subspaces. But this method has a limitation that, the exactly periodic subspaces are orthogonal to each other only when their periods are divisors of the signal length. Period detection using RPT is done by first finding orthogonal projections $x_{qi}(n)$ that has period exactly equal to q_i and then period is determined from q_i corresponding to non-zero projections $x_{qik}(n)$. The time taken by RPT is very less compared to other methods, since the number of projections needed is less. For example in case 1, where the length of the signal is 800, the number of projections carried out in RPT is 18, since the number of factors of 800 is 18. But RPT fails to find the period correctly, if the periodic signal has a period, which is not a divisor of the signal duration.

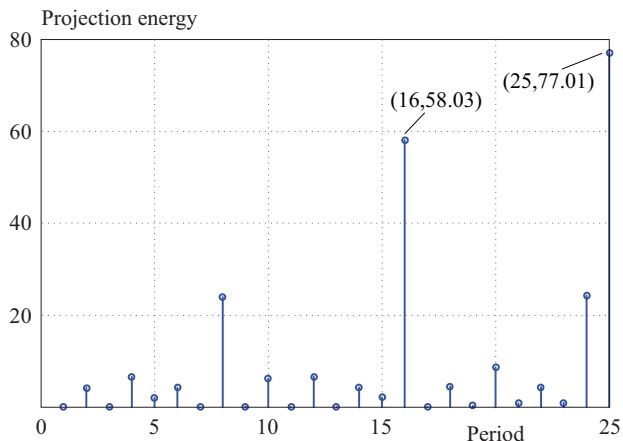The RSP finds any periodic components corresponding to periods from 1 to a maximum of Q using maximum
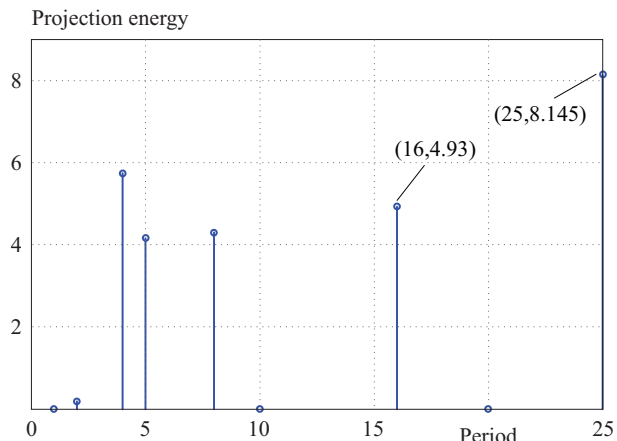
Fig. 4. Projection energy *vs* period using EPSD

Fig. 5. Projection energy *vs* period using RPT
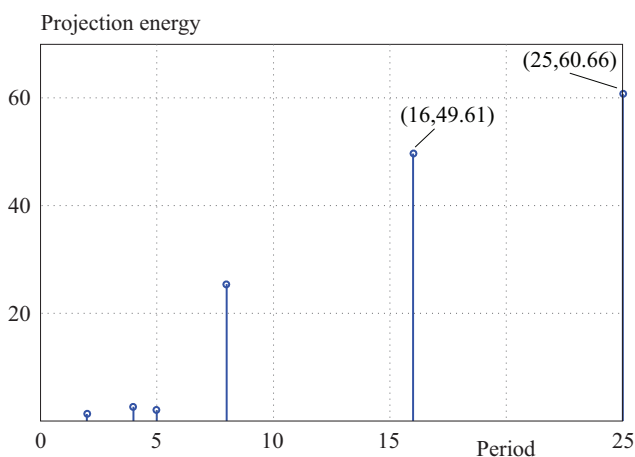

Fig. 6. Projection energy *vs* period using RSP
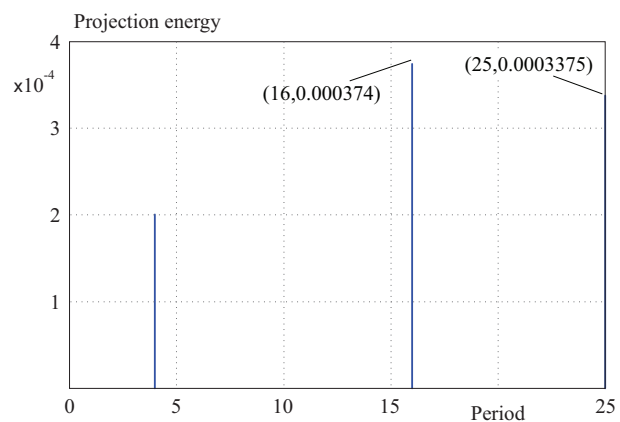

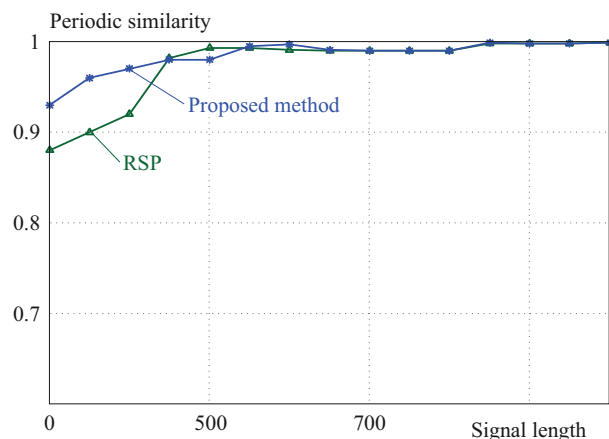**Fig. 7.** Projection energy *vs* period using proposed method

Fig. 8. Periodic similarity *vs* signal length

likelihood estimation (MLE) and RS subspace projection. In this case, the value of Q is taken as half of the signal length. RSP also finds period correctly and the time taken for period detection improves significantly if value of Q is reduced.

The plot of projection energy and the detected periods for the signal shown in Fig. 2 using EPSD, RPT, RSP and the proposed method is depicted in Fig. 4, Fig. 5, Fig. 6 and Fig. 7 respectively. In this example all four methods detect periods correctly.

The performance of proposed algorithm is also evaluated using periodic similarity metric. The similarity of signals based on their periodicity can be compared using this metric. The periodic similarity, S between two signals x and y is, [20]

$$S(x, y) = 1 - D_{\cos}(x, y), \qquad (24)$$

where, $D_{\cos}(x, y)$ is cosine Hellinger distance

$$D_{\cos}(x, y) = \frac{1}{2} \frac{\|h_x - h_y\|^2}{\|h_x\| \, \|h_y\|}. \qquad (25)$$

The proposed algorithm is applied to 15 test signals with signal length varying from 300 to 1000. These test signals are generated with two random periods selected from $[1, 100]$. It shows that the proposed algorithm achieves periodic similarity measure of 0.9 or above for test signals and as signal length increases this measure improves. Figure 8 shows the plot of periodic similarity measure of proposed algorithm and RSP.

6 Conclusion

A two-tier approach for periodicity estimation using Ramanujan sums based subspace projection is proposed. In the first phase, ACF of the signal is calculated, and projected onto the Ramanujan subspace. The autocorrelation function is used to find the potential periods, thereby reducing the number of subspace projections needed for periodicity estimation. Subspace projection is done to factors of the candidate periods and based on the projection energies, the final period estimation is done. The performance of the proposed method is compared with other period detection methods Small2Large, Mbest, Mbestgam, Bestcorrelation considering three different input combinations. The proposed method is also compared with three other subspace projection methods namely, EPSD, RPT and RSP. In terms of accurate period estimation and computational time our method outperforms other periodicity estimation techniques.

Acknowledgments

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