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Decentralized control of nonlinear complex systems

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In the paper, a novel approach to decentralized controller design for nonlinear systems is introduced. The proposed method is based on the relationship between the stability of complex nonlinear systems and the stability of their subsystems. The design procedure of the decentralized controller consists of three steps. In the first step, the stability of the complex nonlinear system is calculated. In the second step, stability conditions at the subsystem level are obtained such that guarantee the stability of the complex nonlinear system. Finally, in the third step, a controller design method is used to ensure that the subsystem stability conditions obtained in the second step are met. As an example, to better understanding the proposed method two simple nonlinear models are used to demonstrate the effectiveness of the proposed method.

Keywords: output feedback, decentralized control, nonlinear control systems, Lyapunov method

1 Introduction

Real systems are becoming large and more complex because their must meet the increasing quantity and quality demands. Todays are experiencing significant transformations due to the integration of different energy sources and advancements in different digital technologies. These changes introduce new challenges to global system, especially in maintaining quality, stability and reliability in such environments. Centralized control frame works often cannot adapt efficiently to guarantee in real situation all above demands. Therefore, decentralized control is considered as an effective method to control of large-scale real processes [1]. In recent years fruitful results in the field of decentralized control of nonlinear systems has been obtained in [2-4] and [7]. In the paper [6] each subsystem is described by a nonlinear state model, such that interaction between different subsystems is linear. Decentralized feedback control is constructed such that stabilize the complex system for all initial conditions. In the paper [5] robust control law is design for systems processing similar subsystems. It is shown that design process for decentralized controller may be simplified using similar subsystems structure. Above works are based on the assumption that full state information is available.

In the time domain, three groups of decentralized control methods have been developed: stability analysis and decentralized control design using the aggregation matrix approach [10], the Vector Lyapunov function approach, [9], and significant progress has been made in the control of LSS through the use of LMI-BMI, as seen in the review article [11]. Unfortunately, when the above approaches are used for stability analysis and decentralized controller design, a complete complex model of LSS needs to be applied.

More simple decentralized controller design procedures for linear systems with small conservatism are obtained in [8, 12, 13].

In this article, we pursue the idea given in above papers for linear systems and we have proposed an original procedure for designing decentralized controllers for nonlinear complex systems. The idea of the decentralized controller design methodology is based on the relationship between the stability of subsystems and the stability of the complex system. The method proposed in this paper for designing a decentralized controller consists of three steps, as follows.

In the first step, the stability of the nonlinear and non-controlled complex plant model (when u_1, u_2 are constants) is calculated. In the second step, subsystems without interaction and their such stability boundary are calculated, that ensures the stability of the complex system, is obtained. Based on the results obtained in the second step, an appropriate method for decentralized controller design will be selected to design a decentralized controller that adheres to the calculated subsystems' stability boundaries.

In the proposed design methods, the notion of strong or weak interaction between subsystems not play any role. The proposed method based on the relations between subsystem stability and stability of complex system. The proposed decreases the conservatives of the decentralized controller design procedure. In this paper, the decentralized procedure is applied only on the subsystem level.

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2 Preliminaries, system model and stability

We are given the nonlinear second-order system in the following form:

$$\dot{\mathbf{x}} = [\mathbf{A} + \mathbf{D}(\mathbf{x})]\mathbf{x} + \mathbf{B}\mathbf{u}, \quad \mathbf{y} = \mathbf{C}\mathbf{x}, \tag{1}$$

where $\mathbf{D}(\mathbf{x})$ is nonlinear function matrix, should be choose by controller designer on the base of the real plant. Suppose that $\|\mathbf{D}(\mathbf{x})\|$ approaches to zero as x goes to zero. Here, $\mathbf{x} \in \mathrm{R}^{\mathrm{n}}, \mathbf{u} \in \mathrm{R}^{\mathrm{m}}, \mathbf{y} \in \mathrm{R}^{\mathrm{l}}$ are the states, inputs, and outputs of the system, respectively. Let us consider a candidate Lyapunov function for system (1) as follows:

$$\mathbf{V}(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}, \quad \mathbf{P} > 0. \tag{2}$$

Assume, that system matrix $\mathbf{A}$ is asymptotically stable. The first time derivative of (2) with respect to linear part of (1) gives the Lyapunov equation as follows:

$$\dot{\mathbf{V}}(\mathbf{x}) = \mathbf{x}^T(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A})\mathbf{x} = -\mathbf{x}^T\mathbf{Q}\mathbf{x}, \quad \mathbf{Q} > 0. \tag{3}$$

To determine the complex stability conditions of the matrix $\mathbf{A}$, let's modify the matrix $\mathbf{A}$ (3) as follows:

$$\dot{\mathbf{V}} = \mathbf{x}^T\big((\mathbf{A} + \alpha\mathbf{I})^T\mathbf{P} + \mathbf{P}(\mathbf{A} + \alpha\mathbf{I})\big)\mathbf{x} - \mathbf{x}^T\mathbf{Q}\mathbf{x} = 0 \tag{4}$$

Solve the Lyapunov matrix (4) with respect to matrix P and α. Note that if the obtained result for $\alpha < 0$, the system (1) is unstable. Denote $\mathbf{A_c} = \mathbf{A} + \alpha\mathbf{I}$. The first derivative of the candidate Lyapunov function with respect to (1) and $\mathbf{A_c}$ gives:

$$\dot{\mathbf{V}}_\mathbf{c} = \mathbf{x}^T(\mathbf{A}_\mathbf{c}^T\mathbf{P} + \mathbf{P}\mathbf{A_c})\mathbf{x} + \mathbf{x}^T\big(\mathbf{D}^T(\mathbf{x})\mathbf{P} + \mathbf{P}\mathbf{D}(\mathbf{x})\big)\mathbf{x} \tag{5}$$

For (5) we have: for any $\gamma > 0$ there exists $\mathrm{r} > 0$ such that $\|\mathbf{D}(\mathbf{x})\| < \gamma$ for all $\|\mathbf{x}\| < r$:

$$\dot{\mathbf{V}}_\mathbf{c} \leq [-\lambda_{min}(\mathbf{Q}) + 2\gamma\|\mathbf{P}\|]\|\mathbf{x}\|^2 \quad \|\mathbf{x}\| < r \tag{6}$$

Choose $\gamma < \frac{\lambda_{\min}(\mathbf{Q})}{2\|\mathbf{P}\|}$. From equation (6) it is clear that there exists such a value of $\mathbf{x} \in [0, \gamma]$ that $\dot{\mathbf{V}}_\mathbf{c} < 0$, meaning that nonlinear system (1) is asymptotically stable. Let matrix $\mathbf{A_c}$ be divided into the following matrices:

$$\mathbf{A_C}\mathbf{x} = \begin{bmatrix} A_{11}(x_1) & A_{12}(\mathbf{x}) \\ A_{21}(\mathbf{x}) & A_{22}(x_2) \end{bmatrix} \tag{7}$$

From the above Eqn. (7), one can see that we have obtained two linear subsystems with interactions $\mathrm{A}_{12}(\mathbf{x})$ and $\mathrm{A}_{21}(\mathbf{x})$. Summarizing the above results, we obtain the following Lemma:

Lemma 1: Assume that matrix $\mathbf{A_c}$ is asymptotically stable. If the subsystems parameters without decentralized controllers guarantee the stability of complex system. For this case the maximal subsystem eigenvalues are given by α for which the stability of the complex system is guaranteed. To guarantee the stability of a complex system, the subsystem decentralized controllers need to be designed such that for the maximal value of subsystem closed-loop eigenvalues β, the following inequality holds:

$$\beta \leq \alpha$$

3 Examples

In this study, we present the time control of two MIMO systems. Since these systems are abstract and not tied to real-world physical quantities, the model parameters and time axis values are dimensionless and have no associated SI units.

3.1 First example

The problem is to design two PI decentralized controllers that ensure the stability of two subsystems and the stability of complex system. Let us assume the structure and parameters of the complex linear part of system are given as follows:

System and input matrix

$$\mathbf{A} = \begin{bmatrix} -0.55 & 0.1 \\ 0.12 & -0.3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0.7 \end{bmatrix}$$

Output matrix

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix

$$\mathbf{Q} = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}$$

Note, that for simulation of the closed-loop system the nonlinear part of system $\mathbf{D}(\mathbf{x})$ should be taken from the real plant.

For the next, two scenarios will be shown. In the first scenario we assume that for matrix $\mathbf{A}$ the designed PI controller will move the maximal eigenvalue of closed-loop system to zero, that is to the boundary of stability. In this case, we need to choose the corresponding value of α such that the linear part of the system with α will be on the stability boundary. In the second scenario, we assume that the designed PI controller will move the maximal eigenvalue of matrix $\mathbf{A}$ to the left by a value of $\alpha = -0.15$.

Case a of the first scenario

Calculation gives: Eigenvalues of matrix $\mathbf{A} = \{-0.5912, -0.2588\}$. Maximal eigenvalue we need to move to the stability boundary. Results for $\alpha = 0.257$ and matrix $\mathbf{P} = \begin{bmatrix} 44.845 & 105.322 \\ 105.322 & 256.6692 \end{bmatrix}$. For the above case, when the system is near the stability boundary, the obtained coefficient $\gamma_\mathrm{a} = 0.0024$. The value of γ characterizes the region, where the Lyapunov function exists or global system is asymptotically stable. The real value of state boundary depends on the structure and parameters of the nonlinear function $\mathbf{D}(\mathbf{x}) < \gamma$.

Note that obtained α is equal to the maximal eigenvalue of the stable matrix **A**. The obtained $\alpha = 0.257$ indicates that the complex system in the region defined by γ is stable. For the stable complex system, one could choose the subsystems without interaction for which the decentralized PI controller need to be designed such that the decentralized controller would move closed-loop eigenvalues to the left by α.

Case b of the first scenario

Let us assume that the designed PI controller will move the eigenvalues of the system's linear part to the left by $\alpha = -0.15$. The results of calculation system parameters are:

Lyapunov matrix for the closed-loop system is

$$\mathbf{P} = \begin{bmatrix} 0.7652 & 0.2764 \\ 0.2764 & 1.1781 \end{bmatrix}$$

The stability boundary with respect to the complex system state variables with two PI decentralized controllers is given by coefficient $\gamma_b = 0.537$. Subsystem structure and parameters for decentralized PI controller design are:

$$\dot{x}_1 = -0.55x_1 + b_1u_1, \quad y_1 = x_1$$

$$\dot{x}_2 = -0.30x_2 + b_2u_2, \quad y_2 = x_2$$

For PI decentralized controller denote $[x_i \quad z_i] = \mathbf{x}_{\text{in}}^{\text{T}}$, $y_i = x_i,\ i = 1, 2$.

$$\dot{\mathbf{x}}_{\mathbf{in}} = \mathbf{A_i x_{in}} + \mathbf{B_i u_i}, \quad \mathbf{y_{in}} = \mathbf{C_i x_{in}}$$

where $\dot{z}_1 = y_i$

$$\mathbf{A_1} = \begin{bmatrix} -0.55 & 0 \\ 1 & 0 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{C_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A_2} = \begin{bmatrix} -0.3 & 0 \\ 1 & 0 \end{bmatrix}, \mathbf{B_2} = \begin{bmatrix} 0.7 \\ 0 \end{bmatrix}, \mathbf{C_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Due to the simple matrices, the characteristic equations for the linear part of the two subsystems with $\alpha = -0.15$ and PI controllers are:

For the first and second subsystems:

$$s^2 + s(0.7 + k_{p1}) + k_{i1} = 0,$$

$$s^2 + s(0.45 + k_{p2}) + k_{i2} = 0$$

From above two characteristic equations one could obtain the set of PI controller gains. We have taken the following gains and roots of characteristic equations:

1st subsystem

$$k_{p1} = 4.3, k_{i1} = 4, \text{roots1} = \{-1, -4\}$$

2nd subsystem

$$k_{p2} = 5.0, k_{i2} = 4, \text{roots2} = \{-0.82, -4.88\}$$

If the above two subsystems' characteristic equations are asymptotically stable, then the closed-loop complex system is asymptotically stable in the region $\|\mathbf{x}\| \in \langle 0, \gamma_c \rangle$. Summarizing above results in the next Table:

Table 1. Results for all cases

Cases	α	γ
Case a	0.257	0.0024
Case b	–0.15	0.537
Case c	–1	1.045

Note that case c is when the complex system is controlled by two PI controllers.

The above results clearly indicate that increasing the dynamic quality of subsystems with decentralized controllers also increases the stability region of the complex nonlinear system $\gamma_a < \gamma_b < \gamma_c$.

3.2 Second example

The goal of this example is to show the step-by-step design of two decentralized P-controllers for a second-order system and for the case of constraints on the two output variables.

The plant model is given as follows:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}, \quad y_1 = x_1, \quad y_2 = x_2$$

Control algorithm which ensures the output constraints is given as follows

$$u_1 = D_1K_1x_1, \quad u_2 = D_2K_2x_2, \quad D_i = d_i - \bar{x}_i, i = 1,2$$

where d_i define the maximal value of soft output constraints and $\bar{x}_i$ is auxiliary variable which depends on the i-th output, see Fig. 1, and K_i is the controller gain. The control algorithm, after small manipulation is in the form

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \left\{ \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} - \begin{bmatrix} \bar{x}_1 & 0 \\ 0 & \bar{x}_2 \end{bmatrix} \right\} \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$$

The control algorithm consists of two parts: linear and non-linear. Assume that the Lyapunov function for the global system is given as follows: $\mathbf{V} = \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{x}$ and its time derivative is:

$$\dot{\mathbf{V}} = \mathbf{x}^T\left[\left(\mathbf{A}_{\mathbf{L}}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathbf{L}}\right) + \left(\mathbf{A}_{\mathbf{N}}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathbf{N}}\right)\right]\mathbf{x} < 0$$

Assume that linear part of the system matrix

$$\mathbf{A}_{\mathbf{L}} = \mathbf{A} + \mathbf{B}\begin{bmatrix} d_1K_1 & 0 \\ 0 & d_2K_2 \end{bmatrix}$$

is asymptotically stable, then it holds

$$\mathbf{x}^T\left(\mathbf{A}_{\mathbf{L}}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathbf{L}}\right)\mathbf{x} = -\mathbf{x}^T\mathbf{Q}\mathbf{x}, \mathbf{Q} > 0$$

For the time derivative of the Lyapunov function for complex system one obtains

$$\dot{\mathbf{V}} = \mathbf{x}^T\mathbf{Q}\mathbf{x} - \mathbf{x}^T[(\mathbf{BNK})^T\mathbf{P} + \mathbf{P}(\mathbf{BNK})] \leq$$
$$-\lambda_{min}(\mathbf{Q}) + 2\|\mathbf{BNK}\|\|\mathbf{P}\|\|\mathbf{x}\|^2$$

where $\mathbf{N} = \mathrm{diag}\{\overline{x}_1, \overline{x}_2\}$, $\quad \mathbf{K} = \mathrm{diag}\{K_1, K_2\}$

Note that if $\mathbf{x}$ approaches zero, then $\overline{x}_1$ and $\overline{x}_2$ go to zero.

Lemma 2. There exists some $\mathbf{x} \in (0, \delta)$ for which $\dot{\mathbf{V}} < 0$, meaning that V is the Lyapunov function of the complex non-linear system. Assume that $\delta = \|\mathbf{BNK}\|$, then for δ one obtains:

$$\delta = \frac{\alpha_{min}(\mathbf{Q})}{2\|\mathbf{P}\|}$$

For the system given below, the stability results of calculation are as follows:

Linear part of the complex system.

$$\dot{\mathbf{x}} = \begin{bmatrix} a_{11} + b_1d_1K_1 & a_{12} \\ a_{21} & a_{22} + b_2d_2K_2 \end{bmatrix}$$

where

$$a_{11} = -1, a_{12} = 0.6, a_{21} = 0.55, a_{22} = -0.8$$
$$d_1 = d_2 = 1, K_1 = K_2 = -20$$

Matrix

$$\mathbf{Q} = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}$$

The stability boundary of the complex system is given by the value of $\alpha = 20.3160$. This means the maximal eigenvalues of the linear part of the system may move to the right by the value of α, and stability is still guaranteed. For the above stability boundary, the obtained $\delta = 33.1064$, which means the complex system is asymptotically stable for the state $\|\mathbf{x}\| \in \langle 0{,}33.1064\rangle$. Subsystems for decentralized controller design are:

$$\dot{x}_1 = -2x_1 + b_1u_1, \qquad \dot{x}_2 = -1.8x_2 + b_2u_2,$$

and control algorithms are

$$u_1 = (d_1 - \bar{x}_1)K_1x_1, \quad u_2 = (d_2 - \bar{x}_2)K_2x_2$$

In this case, we want to design two decentralized P controllers, which will guarantee the stability of complex systems and the soft constraints of two output variables y_1, y_2. This results in constant deviations between the desired values w_1, w_2 and the system outputs. The designed P-decentralized controllers are $K_1 = K_2 = -20$. The simulation was carried out with $b_1 = b_2 = 1$. The nonlinear characteristics of $\overline{x}_1$ and $\overline{x}_2$ as functions of x_1 and x_2 are shown in Fig. 1. Two outputs with soft constraints and their desired values are shown in Fig. 2. The simulation results indicate that the complex system is asymptotically stable. When the outputs are greater than 0.5 (this value can be changed), the open-loop gain $D_\mathrm{i} = (d_\mathrm{i} - \overline{x}_\mathrm{i})$ decreases, and in this way, the outputs cannot grow. The output constraints are soft.

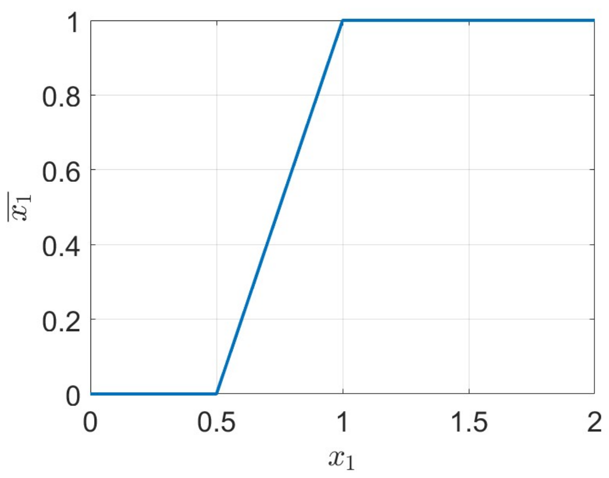

Fig. 1. Nonlinear part system for example 2

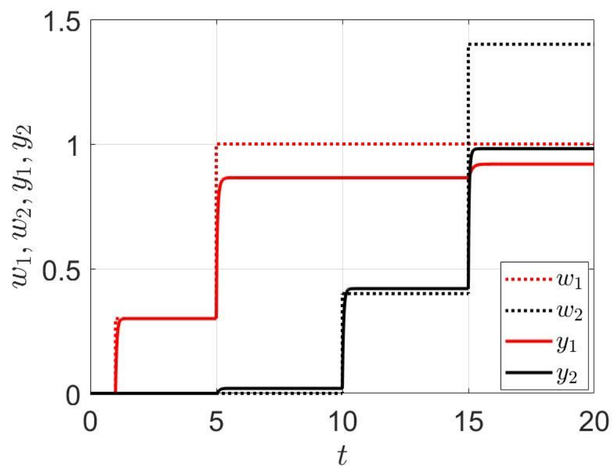

Fig. 2. Two outputs with soft constraints and their desired values for example 2

4 Conclusion

The proposed decentralized control method for nonlinear systems has many benefits. It introduces a new approach that improves stability and performance. By breaking down complex systems into controllable linear subsystems, it simplifies stability analysis and controller design. This method uses Lyapunov-based techniques to ensure the whole system remains stable, even when subsystems interact. The use of Proportional-Integral (PI) controllers fine-tunes subsystem dynamics, greatly enhancing quality and expanding the stability region. This allows the system to handle larger disturbances while maintaining optimal performance. Its scalability and adaptability make it suitable for various nonlinear systems, enabling independent subsystem tuning and easier implementation. Case studies show its practical use, with designed controllers effectively enhancing stability, proving its feasibility for real-world applications in robotics and industrial automation. Overall, this innovative approach offers a robust, effective, and flexible solution for advanced control systems.

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