Journal of Electrical Engineering, Vol. 75, No. 5, 2024, pp. 363-371

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PAPERS

Performance evaluation of symbol detection algorithms in massive MIMO communication systems

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Massive MIMO (mMIMO) is the essential technique for attaining the exponential increase in the data rate needed for future communication systems. These advancements have been significantly supported by progress in VLSI technology, which enables the integration of an enormous number of antennas and the complex signal processing essential for massive MIMO systems on a single chip. This work conducts a comprehensive analysis of the intricacy of seven matrix decomposition techniques for symbol detection in future mMIMO communication systems: ADMM-based infinity norm (ADMIN), Neumann series (NS), Newton iteration (NI), Jacobi iteration (Ja), improved Gauss-Seidel (IGS), conjugate gradient (CG), and QR decomposition (QR), against linear and near-optimal minimum mean square error (MMSE). QR, GS, Ja, and CG belong to linear algebraic methods based on matrix decomposition. MMSE and ADMIN are nonlinear optimization methods; NS and NI are iterative methods. The analysis examines into the complexity of these detection algorithms, considering the symbol error rate, the convergence rate, the initial solution vector, and the correlation factor. Performance evaluations are conducted on 8 and 16-user mMIMO systems with 64 and 128 base station antennas, modulation schemes (32-QAM and 64-QAM), iteration counts (p = 1, 2, 3, 4), correlation factors (α = 0.2, 0.4, and 0.6), and initial solution vectors (zero vector, D^{-1} and $W_2^{-1}\,y_{MF}$). The result clearly shows that if the initial solution and number of iterations are chosen properly, the IGS-based linear detector achieves near-optimal performance with a lower computational complexity of $O(K^2)$ compared to the nonlinear MMSE-based detector with a computational complexity of $O(K^3)$.

Keywords: massive MIMO, matrix decomposition, MMSE, ADMIN, QR, NSA, NI, JA, IGS, CG

1 Introduction

Next-generation wireless innovation, known as massive MIMO (mMIMO), enables the transmission of numerous data streams inside a single frequency band [1]. It improves spectrum efficiency, data rates, coverage, interference control, energy efficiency, diversity gain, spatial resolution, and scalability [2-3]. But it poses challenges to the receiver's ability to discern messages amid noise and interference [4]. The maximum-likelihood (ML)-based symbol detection method is optimal but becomes increasingly complex as the number of antennas increases [5]. Therefore, near-perfect mMIMO detectors are used, which maintain a balance between algorithm complexities and hardware requirements [6]. The presence of numerous antennas and the utilization of high-order modulation techniques present challenges in the detection of mMIMO data at the receiver end [7-8]. Conventional non-linear data detectors, such as the sphere decoder (SD) and tabu search (TS), work well for small-scale MIMO but are too complicated for mMIMO detection [9]. As an alternative, linear data detection techniques such as minimum mean-square error (MMSE)-based equalization or zero-forcing (ZF) are taken into consideration [10-11]. These algorithms provide a balance between complexity and performance, but they require high computational complexity because of matrix inversion operations [12-13]. The computational complexity of $O(K^3)$, for K number of users, is given by exact inversion-based methods, such as QR-Gram Schmidt, Gauss-Jordan, or Cholesky decomposition, which further necessitates a significant amount of processing power [14]. The mMIMO detector is a crucial component in wireless technologies like the Internet of Things (IoT), Vehicle to Everything (V2X), Wireless Sensor Network (WSN), and Machine to Machine (M2M), provided that they offer minimal complexity, latency, power, and space [15].

1.1 Notations used in this paper

This study uses lower and capital-case boldface characters to indicate vectors and matrices, respectively. The notation $\boldsymbol{A}_{ij}$ is used to designate the i^{th} row and the

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https://doi.org/10.2478/jee-2024-0044, Print (till 2015) ISSN 1335-3632, On-line ISSN 1339-309X

j^{th} column of matrix $\boldsymbol{A}$. Similarly, the k^{th} element of vector $\boldsymbol{a}$ is expressed as $\boldsymbol{a}_k$. Additionally, the conjugate transpose and inverse are represented by the operations $(.)^H$ and $(.)^{-1}$, respectively. The sign $|.|$ denotes the absolute value operator. $E\{.\}$ is a mathematical construct used to calculate expectations. The abbreviation I_m is used to represent the MxM identity matrix. Further, p^{th} iteration of the detection method is represented by $(.)^p$.

2 System framework

Let us examine a mMIMO system with K users and N base station (BS) antennas, where N is much greater than K (e.g., $N = 64$ *and* $K = 8$), as shown in Fig. 1. Many users transmit information to the base station using spatial multiplexing at the same time. Every user sends a symbol s_i selected from a modulation constellation set. Furthermore, it is presumed that the temporal synchronization of communication applies to all users. The transmit symbol vector $\boldsymbol{s}$ is formed by combining all the information symbols, denoted as $[s_1, s_2, s_3, \dots, s_K\ ]^K$. The vector received at the base station after demodulation and sampling can be represented as

$$\boldsymbol{y} = \boldsymbol{Hs} + \boldsymbol{n}, \tag{1}$$

In Eqn. (1), $\boldsymbol{H}$ and $\boldsymbol{n}$ represent the channel state information (CSI) and additive white Gaussian noise (AWGN), respectively. Received vector is shown by $\boldsymbol{y}$. The Rayleigh flat fading channel between users and the base station is taken into consideration. The channel matrix is distributed according to a complex normal distribution with a mean of zero and a variance of one. Furthermore, each component of the AWGN vector $\boldsymbol{n}$ is characterized by being independent and identically distributed as a complex normal random variable.

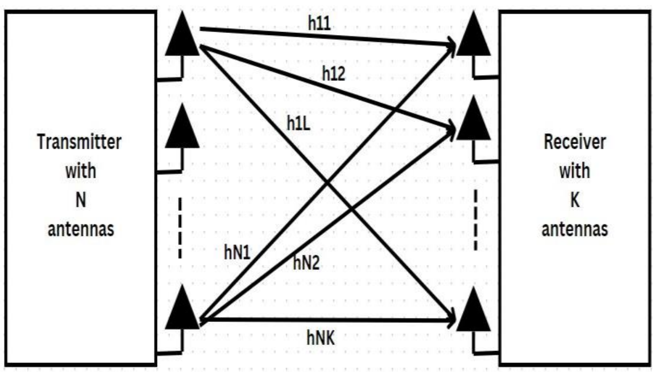

Fig. 1. Massive MIMO communication

3 Various symbol detection methods and their complexity calculation

3.1 *Minimum mean square error (MMSE) based detection*

The linear detection techniques, such as the MMSE algorithm, produce near-optimal bit error rate (BER) performance in mMIMO systems due to the impact of the channel hardening phenomenon [4, 16]. The channel hardening phenomenon occurs when the number of base station antennas is increased and the mMIMO system becomes immune to channel fluctuations. The MMSE detection involves the formulation of a linear transformation matrix, $\boldsymbol{s}_{\text{MMSE}}$ which aims to minimize the mean squared error between the linear transformation of the received vector, $\boldsymbol{y}$, and the sent symbol vector, $\boldsymbol{s}$ [17]. Equation (2) provides the expression for MMSE-based detection.

$$\boldsymbol{s}_{\text{MMSE+1}} = (\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I_U})^{-1}\boldsymbol{H}^H\boldsymbol{y}. \tag{2}$$

Here, $\boldsymbol{H}$ is the channel matrix, $\boldsymbol{y}$ is the received signal vector, σ^2 is noise variance, $\boldsymbol{H}^H\boldsymbol{H}$ is a Gramian matrix. $(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I_U})^{-1}$ is a pseudo-inverse of $\boldsymbol{H}$ along with noise variance, σ^2 and $\boldsymbol{H}^H\boldsymbol{y}$ is the matched filter. The MMSE method handles noise and interference statistically. Noise-resistant MMSE, although more computationally demanding than the ZF detector, performs better in difficult channel circumstances [2, 7].

The complexity analysis of MMSE can be outlined as follows:

Matrix multiplications $\boldsymbol{H}^H\boldsymbol{H}$: $O(K^2N)$
Matrix Vector multiplication $\boldsymbol{H}^H\boldsymbol{y}$: $O(KN)$
Matrix Addition $\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I_U}$: $O(K^2)$
Matrix Inversion $(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I_U})^{-1}$: $O(K^3)$
Final Matrix Multiplication:
$(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I_U})^{-1}\boldsymbol{H}^H\boldsymbol{y}$: $O(K^2N)$
Total complexity

$$= O(2K^2N + KN + K^2 + K^3) \longrightarrow O(K^3)$$

In MMSE, the matrix inversion step primarily dictates the overall complexity, particularly concerning the number of receivers and transmitters.

3.2 *Jacobi based detection*

A simple iterative method called the Jacobi method can be applied to solve a system which is diagonally dominant. In this method, the detected signal is determined as shown in Eqn. (3).

$$\boldsymbol{s}_{JA+1} = \boldsymbol{D}^{-1}\left(\boldsymbol{s}_{MF} + (\boldsymbol{D} - \boldsymbol{W})\boldsymbol{s}_{JA}\right) \tag{3}$$

Here, the channel matrix is decomposed as $\boldsymbol{H} = \boldsymbol{D} + (\boldsymbol{H} - \boldsymbol{D})$. $\boldsymbol{D}$ is the diagonal matrix and $\boldsymbol{s}_{MF}$ is the matcher filter vector. The computational expense of a Jacobi-based detector is less than that of the NS-based detector [4, 15, 18]. A detector utilizing the Jacobi approach has been developed in order to ensure that the initial iteration is devoid of multiplication, thereby reducing the overall complexity.

The complexity of Jacobi can be broken down as below:

Matrix-Vector Multiplications:

1) $\boldsymbol{D}^{-\mathbf{1}}\boldsymbol{s}_{MF}$: $O(K)$

2) $(\boldsymbol{D}-\boldsymbol{W})\boldsymbol{s}_{JA}$: $O(K^2)$

Vector Additions: $\boldsymbol{s}_{MF} + (\boldsymbol{D}-\boldsymbol{W})\boldsymbol{s}_{JA}$: $O(K)$

Component-wise Division:

$\boldsymbol{D}^{-1}\left(\boldsymbol{s}_{MF} + (\boldsymbol{D}-\boldsymbol{W})\boldsymbol{s}_{JA}\right)$: $O(K)$

Total complexity: $O(3K + K^2) \longrightarrow O(K^2)$

3.3 Newton iteration based detection

The Newton-Raphson approach, sometimes known as the Newton iteration (NI), is an iterative method for approximating a matrix's inverse [15, 18]. Equation (4) represents the detector based on the NI.

$$\boldsymbol{s}_{NI+1} = \boldsymbol{D}^{-1}(\sigma^2\boldsymbol{I}_U - \boldsymbol{s}_{NI}\,\boldsymbol{D}^{-1})\boldsymbol{s}_{MF} \tag{4}$$

Similar to the NS approach, it solely necessitates a straightforward calculation to expedite the detection process. Despite the matrix multiplication required in each iteration, the NI outperforms the NS method in terms of convergence speed [4, 5].

The complexity of NI can be broken down as below:

Matrix Inversion: $\boldsymbol{D}^{-1}$: $O(K)$

Matrix Multiplications:

1) $\boldsymbol{D}^{-1}(\sigma^2\boldsymbol{I}_U)$: $O(K^2)$

2) $\boldsymbol{D}^{-1}(\boldsymbol{s}_{NI})$: $O(K)$

3) $\boldsymbol{D}^{-1}(\sigma^2\boldsymbol{I}_U - \boldsymbol{s}_{NI}\,D^{-1})$: $O(K^2)$

Vector-Matrix Multiplication:

$\boldsymbol{D}^{-1}(\sigma^2\boldsymbol{I}_U - \boldsymbol{s}_{NI}\,\boldsymbol{D}^{-1})\boldsymbol{s}_{MF}$: $O(K^2)$

Total complexity = $O(2K + 3K^2) \longrightarrow O(K^2)$

The complexity is influenced by the operations of matrix inversions, scalar multiplications, and matrix multiplications.

3.4 ADMM based infinity norm (ADMIN) based detection

Typically, non-linear detectors, initially configured with a linear detector value, attempt to update the results in subsequent iterations. The ADMIN algorithm employs the alternate direction method of multipliers (ADMM) to address a detection problem involving box constraints [19]. The ADMM (Alternating Direction Method of Multipliers) technique can be employed to solve a convex issue by decomposing it into smaller sub problems and solving them iteratively [18]. β represents a scaled version of noise variance σ^2. Two more iterations of the ADMIN algorithm are used to calculate the values of the $\boldsymbol{z}\ and\ \boldsymbol{\lambda}$ vectors. Equation (5) bears a resemblance to the MMSE equation when $\boldsymbol{z} = \boldsymbol{0}$ and $\boldsymbol{\lambda} = \boldsymbol{0}$.

Estimated signal vector $\boldsymbol{x}_{\mathrm{ADMIN}}$ is given by

$$\boldsymbol{s}_{\mathrm{ADMIN}+1} = (\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I}_U)^{-1}\left(\boldsymbol{H}^H\boldsymbol{y} + \beta(\mathbf{z} - \boldsymbol{\lambda})\right) \tag{5}$$

The number of operations and time complexity calculation of the ADMM is similar to MMSE, which is already been explained.

3.5 NSA based detection

The Neumann series is a more widely used technique for approximating matrix inversion. Here, the channel matrix can be broken down into two parts: the off-diagonal matrix $\boldsymbol{E}$ and the diagonal matrix $\boldsymbol{D}$ [4, 15]. Estimated signal vector $\boldsymbol{s}_{\mathrm{NSA}}$ is given as

$$\boldsymbol{s}_{NSA+1} = (\boldsymbol{D}^{-1}\boldsymbol{E})^p\boldsymbol{D}^{-1}\,\boldsymbol{s}_{MF} \tag{6}$$

The complexity of NSA can be broken down as below.

Matrix Inversion: $\boldsymbol{D}^{-1}$: $O(K)$

Matrix Multiplication: $\boldsymbol{D}^{-\mathbf{1}}\boldsymbol{E}$: $O(K^2)$

Vector-Matrix Multiplication:

$(\boldsymbol{D}^{-1}\boldsymbol{E})^p\boldsymbol{D}^{-1}\,\boldsymbol{s}_{MF}$: $O(K^3)$

Total complexity

$= O((p-1)(K^3 + K^2 + K) \longrightarrow O((p-1)K^3)$

The number of iterations p plays a significant role in the complexity calculation of NSA.

3.6 QR decomposition-based detector

In the context of the QR algorithm, it is pertinent to note that $\boldsymbol{Q}$ is a unitary matrix, $\boldsymbol{R}$ represents an upper triangular matrix, and $\boldsymbol{s}_{MF} = \boldsymbol{H}^H\boldsymbol{y}$ is matched filter vector. A QR decomposition-based detector is represented by Eqn. (7).

$$\boldsymbol{s}_{QR+1} = \boldsymbol{Q}\boldsymbol{R}^{-1}\,\boldsymbol{s}_{MF} \tag{7}$$

Here, the norm of a vector is computed as $r_{i,i} = \|\boldsymbol{q}_i\|^2$.

Off diagonal elements of the upper triangular matrix $\boldsymbol{R}$ are computed as

$$r_{i,j} = \boldsymbol{q}_i^H\,\boldsymbol{q}_j\,. \tag{8}$$

Further, $\boldsymbol{q}_i$ and $\boldsymbol{q}_j$ are vectors computed based on $r_{i,i}$ and $r_{i,j}$ as given below.

$$\boldsymbol{q}_{i+1} = {}^{\boldsymbol{q}_i}/_{r_{i,i}} \tag{9}$$

$$\boldsymbol{q}_{j+1} = \boldsymbol{q}_j - r_{i,j}\boldsymbol{q}_i \tag{10}$$

Complexity calculation of QR:

Orthogonal matrix: $O(K^3)$
Matrix Inversion: $\boldsymbol{R}^{-1}$: $O(K^2)$
Matrix-Vector Multiplication: $\boldsymbol{QR}^{-1}\, \boldsymbol{s}_{MF}$: $O(K^2)$
Total complexity $= O(K^3 + 2K^2) \longrightarrow O(K^3)$

The dominant factor is the matrix-vector multiplication.

3.7 Improved Gauss-Seidel

The improved Gauss-Seidel detection method solves the linear system. The diagonal component $\boldsymbol{D}$, the strictly lower triangular component $\boldsymbol{L}$, and the strictly upper triangular component $\boldsymbol{R}$, respectively, are the three components that make up the Gramian matrix. $\boldsymbol{s}_{MF} = \boldsymbol{H}^H \boldsymbol{y}$ is a matched filter vector. The IGS method's mathematical representation is as given in Eqn. (11).

$$\boldsymbol{s}_{IGS+1} = (\boldsymbol{D} + \boldsymbol{L})^{-1} (\boldsymbol{s}_{MF} - \boldsymbol{R}\, \boldsymbol{s}_{IGS}) \qquad (11)$$

Complexity calculation:
Matrix Inversion: $(\boldsymbol{D} + \boldsymbol{L})^{-1}$: $O(K)$
Matrix-Vector Multiplication: $\boldsymbol{R}\, \boldsymbol{s}_{IGS}$: $O(K^2)$
Vector Addition: $\boldsymbol{s}_{MF} - \boldsymbol{R}\, \boldsymbol{s}_{IGS}$: $O(K)$
Matrix-Vector Multiplication:
$(\boldsymbol{D} + \boldsymbol{L})^{-1} (\boldsymbol{s}_{MF} - \boldsymbol{R}\, \boldsymbol{s}_{IGS})$: $O(K^2)$
Total complexity $= O(2K + 2K^2) \longrightarrow O(K^2)$

Matrix inversion and multiplication is the domination part of complexity in IGS.

3.8 Conjugate gradient based detector

The CG method is another more approximate method used in mMIMO detection which can be articulated as Eqn. (12).

$$\boldsymbol{s}_{CG+1}^{(p)} = \boldsymbol{s}_{CG}^{(p)} + \alpha_{CG}^{(p)}\, \boldsymbol{q}_{CG}^{(p)} \qquad (12)$$

Here, $\boldsymbol{q}_{CG}$ refers to the conjugate direction in relation to the Gramian matrix and α_{CG} is a scalar parameter a scalar parameter typically referred to as the step size [18]. In the Conjugate Gradient (CG) method, the determination of the conjugate direction relies on the Gramian matrix. The CG technique typically involves iterative updates that calculate the conjugate direction using the previous residual and the previous conjugate direction.

The complexity calculation of CG is as below.

Matrix-vector multiplication: $O(K^2)$
Element-wise vector addition and scalar multiplication
$\boldsymbol{s}^{(p)} + \alpha^{(p)} \boldsymbol{q}^{(p)}$: $O(K)$
Total Complexity
$= O((p+1)(K^2 + K)) \longrightarrow O((p+1)K^2)$

The number of iterations is the dominant factor in CG detection.

To summarize, Tab. (1) provides a comparative chart dealing with the computational complexities of various detection algorithms.

Table 1. Computational complexity comparison

Algorithms	Computational complexity
MMSE	$O(2K^2N + KN + K^2 + K^3) = O(K^3)$
ADMIN	$O(2K^2N + KN + K^3 + K^2 + K) = O(K^3)$
NSA	$O\big((p-1)(K^3 + K + K)\big) = O(pK^3)$
IGS	$O(2K + 2K^2) = O(K^2)$
QR	$O(K^3 + 2K^2) = O(K^3)$
JA	$O(3K + K^2) = O(K^2)$
NI	$O(2K + 3K^2) = O(K^2)$
CG	$O((p+1)(K^2 + K)) = O(pK^2)$

This analysis helps to understand how the algorithm's performance scales with input size. The notation "O" represents the upper bound of computational complexity, with K representing the number of users in the mMIMO system and N is number of the base station antennas.

The comparison reveals that both MMSE and ADMIN share the same computational complexity $O(K^3)$, indicating that as the input size increases, both algorithms exhibit cubic growth in their running times. This suggests that the MMSE and ADMIN algorithms share similarities in their underlying mathematical operations, leading to similar computational demands. QR also exhibits the complexities of cubic growth $O(K^3)$. NSA's and CG's performance is more computationally intensive due to the significant role played by the number of iterations, in addition to the cubic and quadrature terms of the user count respectively. JA and IGS have the lowest complexity of $O(K^2)$. In addition to complexity, the algorithm is selected based on various other characteristics of the communication system, such as the size of the input data, available computational resources, CSI information, number of iterations, etc.

4 Simulation results and discussions

Figures 2 to 10 present numerical data illustrating the SER performance across varying SNR levels for MMSE, NS, NI, JA, GS, CG, QR, and ADMIN for mMIMO communication systems. These simulations were conducted using MATLAB R2021a with the parameters shown in Tab. (2).

Table 2. Parameter specifications used in simulations using MATLAB 2021a

Parameters	Value
Channel characteristics	Massive MIMO with AWGN
Fading model	Rayleigh fading
Modulation index M	32 and 64 QAM
Number of transmitting (BS) antennas N	64 and 128
Number of receiving (user) antennas K	8 and 16
Number of symbols transmitted per user	10^5
Tolerance for convergence ε	10^{-6}
SNR range	5 to 15 (dB)
Iteration count p	1 to 4
User-to-BS ratio for N = 64, K = 8	0.125
User-to-BS ratio for N = 64, K = 16	0.25
User-to-BS ratio for N = 128, K = 8	0.0625
Correlation factor α	0.4, 0.6, 0.8

4.1 Performance analysis with regard to the symbol error rate

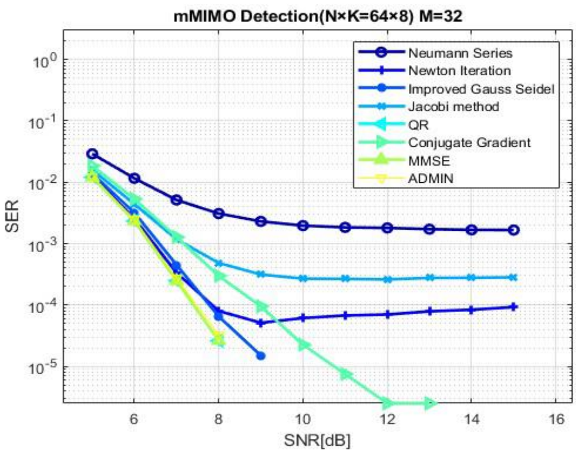

Fig. 2. SER performance for $N \times K = 64 \times 8$, $p = 3$, $M = 32$ and user-to-BS ratio of 0.125

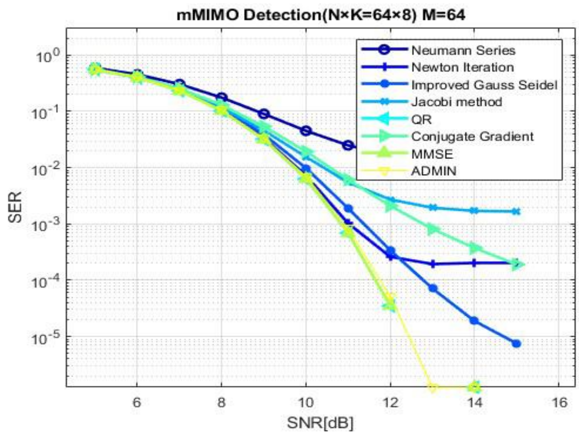

Fig. 3. SER performance for $N \times K = 64 \times 8$, $p = 3$, $M = 64$ and user-to-BS ratio of 0.125

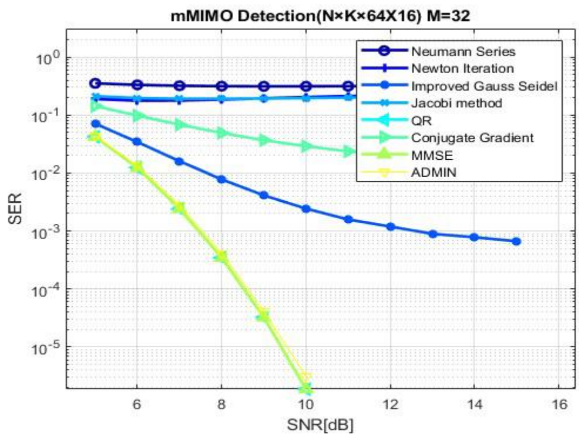

Fig. 4. SER performance for $N \times K = 64 \times 16$, $p = 3$, $M = 32$ and user-to-BS ratio of 0.25

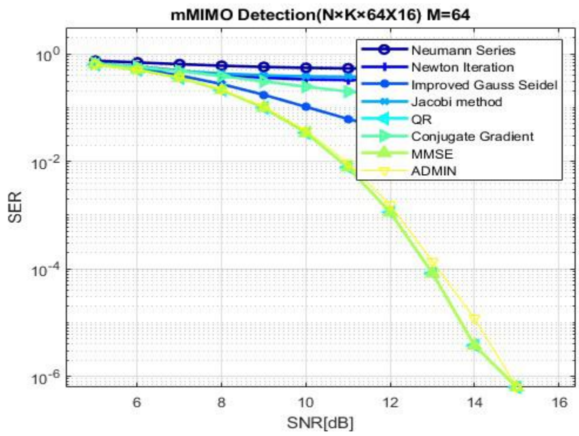

Fig. 5. SER performance for $N \times K = 64 \times 16, p = 3$, $M = 64$ and user-to-BS ratio of 0.25

This section involves a thorough evaluation of several detection methods for mMIMO detection systems in terms of the symbol error rate (SER) performance against the signal-to-noise ratio (SNR) expressed in (dB).

This section will look at how the ratio of user-to-BS antennas and the modulation index affect the detection methods' performance. Comparing the results in Figs. 2, 3, 4, and 5, it is clear that the consistency of the performance of detection techniques heavily depends on the ratio of user-to-BS antennas. High ratios yield promising results, but their usefulness decreases as the ratio decreases. This impact can clearly be seen where 64 base station antennas are fixed and users are changed from 16 to 8, giving a user-to-BS ratio of 0.25 and 0.125, respectively. The performance of NS, NI, CG, and JA significantly deteriorates as the ratio increases from 0.125 to 0.25. A similar effect is seen with 128 BS antennas and a user-to-BS ratio of 0.0625 too. IGS is moderately okay, whereas the performance of ADMIN and QR are close to the MMSE. In fact, for user-to-BS ratio of 0.25, all methods except MMSE, ADMIN, and QR perform poorly.

The results shown in Figs. 2, 3, and 8 demonstrate that increasing the number of base stations from 64 to 128, keeping the number of users fixed, significantly improves the performance of NS, NI, CG, and JA, but also achieves desirable symbol error at lower SNR levels due to the channel hardening phenomenon. But, using a very large number of antennas is not always a good choice, as the hardware resources needed are also huge. Therefore, the tradeoff between user-to-BS ratio and modulation index must be worked out to avoid the use of extra resources for the hardware implementation of detector algorithms.

The modulation index also has a direct impact on SER performance. Here it can be seen that a higher modulation index achieves higher data rates at the cost of more SNR compared to a lower modulation index at the expense of a heightened susceptibility to noise. Notably, 64-QAM demands a higher SNR for the same error performance compared to 32-QAM. So, detector with 32-QAM works well in channels with a lot of noise.

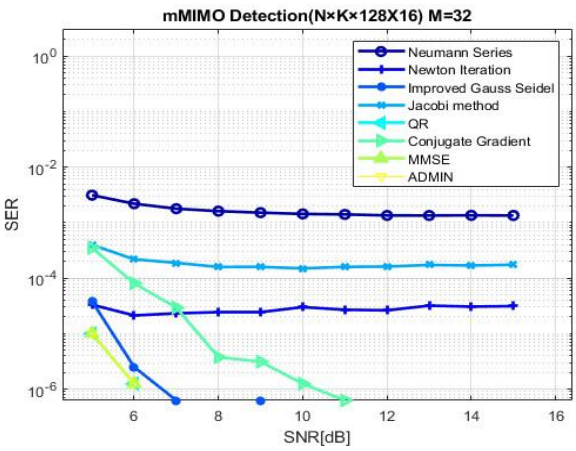

Fig. 6. SER performance for $N \times K = 128 \times 16$, $p = 3$, $M = 32$ and user-to-BS ratio of 0.125

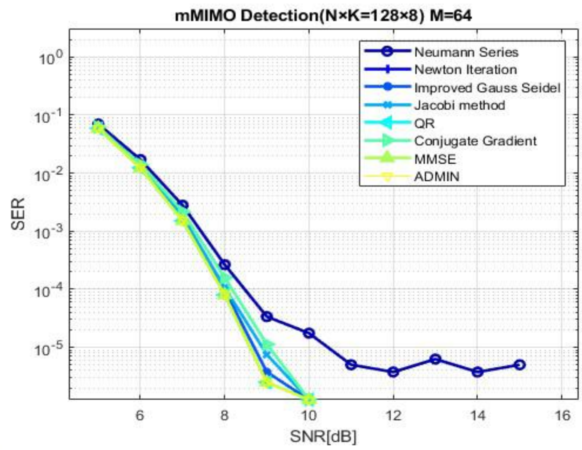

Fig. 8. SER performance for $N \times K = 128 \times 8$, $p = 3$, $M = 64$ and user-to-BS ratio of 0.0625

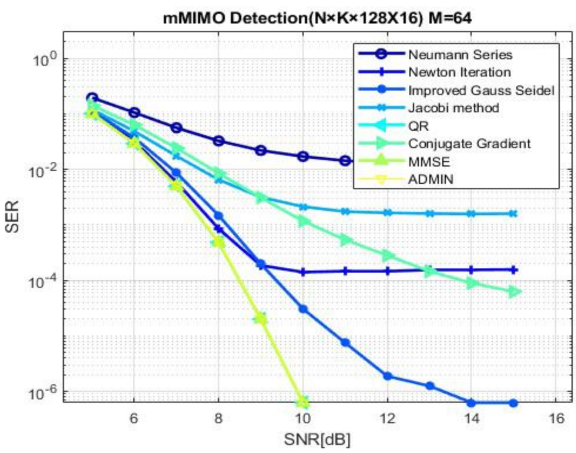

Fig. 7. SER performance $N \times K = 128 \times 16$, $p = 3$, $M = 64$ and user-to-BS ratio of 0.125

To summarize, non-linear detectors like ADMIN and QR-based perform best near MMSE but have the highest complexity of $O(K^3)$. NS, NI, CG, and Jacobi detectors fail to converge. IGS is performing moderately close to MMSE, especially when the user-to-BS ratio is at its minimum. In all scenarios, nonlinear algorithms with cubic complexity perform well, particularly when the ratio between base station (BS) antennas and number of users is limited. These kinds of scenarios are common in 5G and beyond networks. Consequently, nonlinear detectors based on perfect inversion are anticipated to remain attractive in the future as reliable options due to their ability to meet industry requirements for durability and consistent performance across diverse circum-stances. ADMIN and QR's superior performance faces hardware complexity challenges, suggesting the use of less complex, moderately performing algorithms like IGS for hardware implementation in 5G and 6G communication networks and emerging wireless technologies like IoT, WSN, V2X, and M2M. The next section will delve into the effect of the initial solution vector on the IGS algorithm.

4.2 Effect of initial solution and iterations on the convergence analysis of IGS

The convergence of detection algorithms in mMIMO communication systems depends on a variety of factors, including initial conditions, channel matrix conditioning, modulation scheme, computational requirements, and numerical stability. This section explains the effect of the initial solution and number of iterations on the convergence of the IGS algorithm, which is less complex than ADMIN and QR but performs moderately throughout. In Fig. 9, the SER performance of IGS for the initial solution of $\boldsymbol{s}^{(0)} = \boldsymbol{D}^{-1}$ and $\boldsymbol{s}^{(0)} = \boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$ can be seen for the number of iterations from 1 to 4. As compared to zero vector as the initial solution, the performance of IGS is better for $\boldsymbol{s}^{(0)} = \boldsymbol{D}^{-1}$, which further improves for $\boldsymbol{s}^{(0)} = \boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$. As shown in Fig. 9, IGS is performing closer to MMSE for $K = 4$, and $\boldsymbol{s}^{(0)} = \boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$.

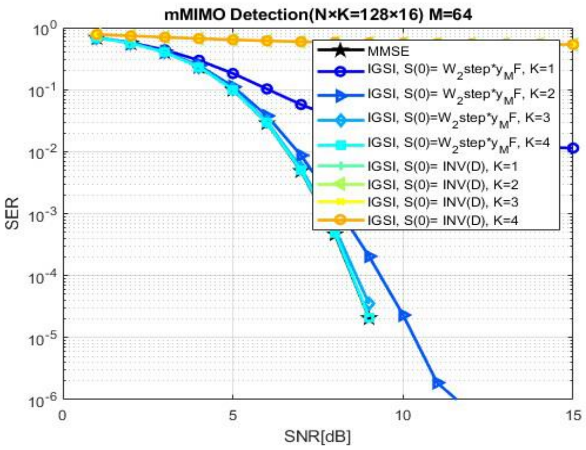

Fig. 9. SER performance for $N \times K = 128 \times 16$, $M = 64$, $p = 1, 2, 3, 4$, $\boldsymbol{s}^{(0)} = \boldsymbol{D}^{-1}$ and $\boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$

4.3 Effect of correlation factor on the convergence analysis of IGS

The correlation between antenna elements in mMIMO communication systems influences detection performance due to spatial closeness and spatial multiplexing capacity, limiting potential data rate and capacity increases. Strong antenna correlation can cause channel estimation inaccuracies, impacting symbol detection algorithms. Tightly spaced antennas streamline hardware implementation but require meticulous attention for signal detection. The effect of the correlation factor on IGS performance can be seen in Fig. 10. Here, IGS performance for iteration $p = 3$ and 4, $\boldsymbol{s}^{(0)} = \boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$ and $\alpha = 0.2,\ 0.4$ and 0.8 can be seen. As the correlation factor increases, IGS performance deteriorates, which implies that IGS will not be a better choice in highly correlated environments.

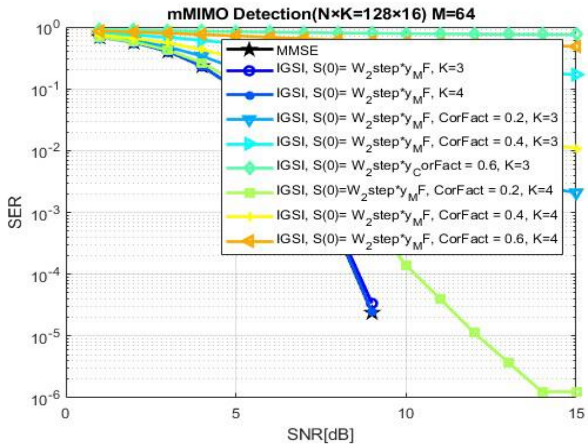

Fig. 10. SER performance for $N \times K = 128 \times 16$, $M = 64$, $p = 3, 4$, $\alpha = 0.2, 0.4, 0.6$, and $\boldsymbol{s}^{(0)} = \boldsymbol{D}^{-1}$ and $\boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$

5 Conclusion

In this study, a comprehensive analysis of the computation complexity of seven symbol detection algorithms (ADMIN, NS, NI, JA, IGS, CG, and QR) is conducted against a linear near-optimal MMSE based detector in terms of symbol error rate, convergence rate, initial solution vector, and correlation factor. SER performance is evaluated in an 8 and 16 user mMIMO system with 64 and 128 base station antennas, considering different user-to-base station ratios (0.0625, 0.125, and 0.25), modulation scheme (M = 32 and 64 QAM), iterations (p = 1, 2, 3, and 4), correlation factor (α = 0.2, 0.4, and 0.6), and initial solution vector ($\boldsymbol{s}^{(0)}$= zero vector, $\boldsymbol{D}^{-1}$ and $\boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$).

From SER vs. BER performance, it is observed that nonlinear methods like ADMIN and QR achieve performance close to MMSE, with a time complexity of $O(K^3)$. Conversely, NS, NI, CG, and JA exhibit instability. Additionally, 64-QAM requires a higher SNR than 32-QAM for equivalent error performance. 32-QAM exhibits better error performance at lower SNRs, demonstrating its robustness in noisy channels. However, it's important to note that higher-order QAM schemes are advantageous for achieving higher data rates despite being more vulnerable to noise.

Furthermore, when the user-to-base station ratio is increased from 0.125 to 0.25, the performance of NS, NI, CG, and JA significantly deteriorates. IGS performance is moderately satisfactory for all considered user-to-BS ratios with a lower time complexity of $O(K^2)$. Non-linear ADMIN and QR detectors are close to MMSE with a time complexity of $O(K^3)$.

Upon evaluating detector performance against different initial solutions, it is found that IGS performs better with the initial solution $\boldsymbol{s}^{(0)} = \boldsymbol{D}^{-1}$, as compared to the zero vector, which further improves with $\boldsymbol{s}^{(0)} = \boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$. The performance of $\boldsymbol{s}^{(0)} = \boldsymbol{W}_2^{-1}\boldsymbol{y}_{MF}$ for $p = 3, 4$ iterations closely matches MMSE with a lower time complexity of $O(K^2)$. Lastly, when assessing the impact of correlation, nonlinear detectors like ADMIN and QR outperform linear detectors like IGS.

The results clearly indicate that, with a properly selected initial solution and an appropriate number of iterations, the IGS-based linear detector can achieve performance that is nearly optimal. More importantly, IGS convergence is achieved with a significantly lower computational complexity of $O(K^2)$ as compared to other detection methods. This highlights the IGS-based detector's suitability for balancing performance and computational demands for the hardware realization of mMIMO future-generation detectors. The in-depth look at different detection methods in this study, including SER, user-to-base station ratios, modulation scheme, iterations, correlation factor, and initial solution vector, will also help researchers choose the best detection methods for next-generation communication. Future studies could extend these techniques to various fading models.

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Received 29 May 2024