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DECENTRALIZED CONTROL DESIGN USING LMI MODEL REDUCTION

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Decentralized control design approach based on partial aggregation is proposed. The control is designed on subsystem level, the dynamics of the remaining part of the system is considered through its reduced order model. Model reduction scheme based on generalized balanced realization is developed to consider the dominant part of the remaining system dynamics.

K e y w o r d s: decentralized control, model reduction, balanced realization, stabilization, linear matrix inequalities (LMI)

1 INTRODUCTION

Decentralized control belongs to the most frequent control structures for (large scale) systems. Over the past twenty years, the analysis and design methods for decentralized control scheme have been intensively studied, overview of the existing methods and problems can be found in [1]. These approaches consider a system as an interconnection of subsystems. For each subsystem, a local controller is designed to ensure the subsystem stability with a (partial) measure of its performance — the local objective. Furthermore, the local controllers must ensure the stability and the performance of the overall system — the global objective. The connective stability should be ensured as well, that is, the on-off participation of subsystems does not destabilize the overall system. The local controllers form the decentralized controller; these controllers can be designed *eg* using "classical" approaches as LQ theory [2].

Two basic approaches can be adopted in decentralized control design procedure: sequential and independent local controller design. The latter consists in designing independently the local controller and then checking the properties of the overall closed-loop system. Thus, it is an iterative, "trial and error" approach. The former — sequential approach consists in designing the local controllers one by one, taking into consideration the already designed controlled part of the system. This approach is especially appropriate for cases, when the subsystem should be connected to an already existing large-scale system or the subsystems can be joined or disconnected in a prescribed way.

In this paper we adopt the sequential design approach, taking into consideration the global objective, thus avoiding the "trial and error" process. A "large scale" system is modelled as the interconnection of subsystems. For each subsystem a local controller should be designed such that each closed loop is stable and the remaining part of the system is considered as well to keep the overall system stability and certain level of performance. A possible approach to solve this design problem was introduced in [3], where the local controller was designed taking into consideration the reduced model of the remaining part of the system. The proposed control law aimed at minimizing interactions. The modal aggregation model reduction scheme was used. We propose another approach: to consider the remaining part of the system, representation based on the reduced model coming from balanced realization is used. This model reduction approach can be realized also using LMI formulation with possibility to define the special objective function.

The paper is organized as follows. The problem formulation is given in Section 2. In Section 3, the model reduction procedure based on balanced realization and the controllability and observability gramians and generalized gramians both for stable and unstable systems are introduced. The resulting model reduction schemes are based on LMI and LME (linear matrix equations) solution for gramians and generalized gramians respectively. Section 4 provides the main result — decentralized control design scheme. The approach is illustrated on the example in Section 5, where both LMI and LME are used.

2 DECENTRALIZED CONTROL: PROBLEM FORMULATION

A "large scale" system can be modelled as the interconnection of n subsystems denoted as S_i. Each subsystem is assumed as linear time invariant (LTI) and causal. Let us consider the control design procedure for one of subsystems, denote it S_1. The remaining part of the system is denoted as S_2.

$$
\begin{aligned}
S_1: \quad & \dot{\mathbf{x}}_1(t) = \mathbf{A}_{11}\mathbf{x}_1(t) + \mathbf{A}_{12}\mathbf{x}_2(t) + \mathbf{B}_1\boldsymbol{u}_1(t) \\
& \boldsymbol{y}_1(t) = \mathbf{C}_1\mathbf{x}_1(t) \\
S_2: \quad & \dot{\mathbf{x}}_2(t) = \mathbf{A}_{21}\mathbf{x}_1(t) + \mathbf{A}_{22}\mathbf{x}_2(t)
\end{aligned}
\tag{1}
$$

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where $\mathbf{x}_1(t) \in R^{n_1}$, $\mathbf{x}_2(t) \in R^{n_2}$ are state vectors of the subsystem to be controlled and the remaining part of the system (supposed to be stable or stabilized already) respectively; $\boldsymbol{u}_1(t) \in R^{m_1}$, $\boldsymbol{y}_1(t) \in R^{p_1}$ are control and output vectors of the subsystem S_1 respectively; matrices $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, $\mathbf{C}_1$ are constant matrices with the respective dimensions. We assume that S_1 described by the triplet $(\mathbf{A}_{11}, \mathbf{B}_1, \mathbf{C}_1)$ is controllable (and observable if output feedback is considered in control design).

The control aim is to stabilize the overall system and achieve the performance minimizing the quadratic objective function

$$J = \int\limits_{t=t_0}^{\infty} \big(\mathbf{x}_1^\top(t)\mathbf{Q}_1\mathbf{x}_1(t) + \boldsymbol{u}_1^\top(t)\mathbf{R}_1\boldsymbol{u}_1(t)\big)\mathrm{d}t \tag{2}$$

where $\mathbf{Q}_1$ and $\mathbf{R}_1$ are positive definite matrices. Therefore LQ control design for subsystem S_1 should be realized.

Let $n_1 \ll n_2$ and we want to consider the relevant dynamics of S_2 in subsystem S_1 control design. Thus, there is a need to simplify the consideration of S_2 taking its reduced model (of the dimension $\sim n_1$) instead of the full one (of the dimension $= n_2$). Various methods for dynamic system model reduction have been developed. In [3] the "classical" linear aggregation model reduction scheme is considered. Lately, the model reduction method based on the so called balanced realization has attracted considerable interest [4], [5]; this approach is described and applied in this paper.

3 MODEL REDUCTION BASED ON BALANCED REALIZATION

The term balanced realization denotes the dynamic system time domain mathematical model, where the state variables are defined in such a way that the respective controllability and observability Gramians are balanced, *ie* they are equal, given by diagonal matrix, [4]. Recall that controllability Gramian for linear continuous time invariant given by the triplet $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is defined as

$$\mathbf{G}_C = \int\limits_0^{\infty} e^{\mathbf{A}t}\mathbf{B}\mathbf{B}^\top e^{\mathbf{A}^\top t}\mathrm{d}t \tag{3}$$

and is related to minimum control energy. The observability Gramian is defined as

$$\mathbf{G}_O = \int\limits_0^{\infty} e^{\mathbf{A}^\top t}\mathbf{C}^\top\mathbf{C}e^{\mathbf{A}t}\mathrm{d}t \tag{4}$$

and is related to minimum estimation error. It is well known that $\mathbf{G}_C$ and $\mathbf{G}_O$ are also solutions to the following Lyapunov equations:

$$\mathbf{A}\mathbf{G}_C + \mathbf{G}_C\mathbf{A}^\top + \mathbf{B}\mathbf{B}^\top = 0\,, \tag{5}$$

$$\mathbf{A}^\top\mathbf{G}_O + \mathbf{G}_O\mathbf{A} + \mathbf{C}^\top\mathbf{C} = 0\,. \tag{6}$$

For balanced realization

$$\mathbf{G}_C = \mathbf{G}_O = \Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \dots, \sigma_n)\,. \tag{7}$$

The main idea behind balanced realization is to achieve "the accordance" between the orientation of "the best controllable" directions and "the best observable" directions. Recently, see [4], so called generalized controllability and observability Gramians $\widetilde{\mathbf{G}}_C$ and $\widetilde{\mathbf{G}}_O$ has been introduced as solutions to LMI:

$$\mathbf{A}\widetilde{\mathbf{G}}_C + \widetilde{\mathbf{G}}_C\mathbf{A}^\top + \mathbf{B}\mathbf{B}^\top < 0\,, \tag{8}$$

$$\mathbf{A}^\top\widetilde{\mathbf{G}}_O + \widetilde{\mathbf{G}}_O\mathbf{A} + \mathbf{C}^\top\mathbf{C} < 0\,. \tag{9}$$

Generalized gramians $\widetilde{\mathbf{G}}_C$ and $\widetilde{\mathbf{G}}_O$ are not unique, in fact, there exist many solutions of LMI (8) and (9), if these inequalities are feasible.

Similarly to (7), for the generalized balanced realization:

$$\widetilde{\mathbf{G}}_C = \widetilde{\mathbf{G}}_O = \Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \dots, \sigma_n)\,. \tag{10}$$

Having balanced realization, the model reduction can be accomplished by simple "truncation" of "the worst controllable (observable)" states. Let $\mathbf{G} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & 0 \end{bmatrix}$ be a minimal realization and $\mathbf{A}$ be Hurwitz, then $\mathbf{G}_r = \begin{bmatrix} \mathbf{A}_r & \mathbf{B}_r \\ \mathbf{C}_r & 0 \end{bmatrix}$ is a reduced model, respective to gramians (7) or generalized gramians (10), taking the first r states of $(\mathbf{A}, \mathbf{B}, \mathbf{C})$.

Furthermore, it was shown that the approximation error of the reduced model obtained by taking the first r states of balanced realization and truncating the $r+1, \dots, n$ states is upper bounded by

$$er = 2(\sigma_{r+1} + \dots + \sigma_n)\,. \tag{11}$$

For controllable and observable system $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ there always exists a transformation matrix $\mathbf{T}$ such that $\big(\mathbf{T}\mathbf{A}\mathbf{T}^{-1}, \mathbf{T}\mathbf{B}, \mathbf{C}\mathbf{T}^{-1}\big)$ is balanced. Matrix $\mathbf{T}$ can be computed as:

$$\mathbf{T}^{-1} = \mathbf{G}_C^{\frac{1}{2}}\mathbf{U}\Theta^{-\frac{1}{2}}, \tag{12}$$

where matrices $\mathbf{U}$ and Θ are obtained from singular value decomposition:

$$\mathbf{G}_C^{\frac{1}{2}}\mathbf{G}_O\mathbf{G}_C^{\frac{1}{2}} = \mathbf{U}\Theta^2\mathbf{U}^\top\,. \tag{13}$$

The general procedure to reduce system model using balanced realization is summarized in the following steps:

- transform the system into balanced state space realization using transformation matrix (12), controllability and observability gramians are computed from (5), (6), or, alternatively, from (8) and (9);
- reduce the balanced realization model to dimension r by "removing" the last $(n-r)$ states.

3.1 Gramians for unstable systems

In this part we show definitions of controllability and observability Gramians, which better characterize the input-output properties of the unstable system and the respective model reduction scheme for unstable system [5].

Consider

$$\mathbf{G}(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{0} \end{array}\right], \tag{14}$$

$$\begin{aligned} \mathbf{P}_C &= \frac{1}{2\pi}\int_{-\infty}^{\infty} (j\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}\mathbf{B}^\top\big(-j\omega\mathbf{I}-\mathbf{A}^\top\big)^{-1}\mathrm{d}\omega\,, \\ \mathbf{Q}_O &= \frac{1}{2\pi}\int_{-\infty}^{\infty} (-j\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{C}^\top\mathbf{C}\big(j\omega\mathbf{I}-\mathbf{A}^\top\big)^{-1}\mathrm{d}\omega \end{aligned} \tag{15}$$

where $\mathbf{P}_C$ and $\mathbf{Q}_O$ are the controllability and observability Gramians. We assume, that matrix $\mathbf{A}$ has no eigenvalues on the imaginary axis. Consider then the transformation matrix $\mathbf{T}$ which transforms the matrix $\mathbf{A}$ into block diagonal matrix with blocks $\mathbf{A}_1$ and $\mathbf{A}_2$.

$$\left[\begin{array}{c|c} \mathbf{TAT}^{-1} & \mathbf{TB} \\ \hline \mathbf{CT}^{-1} & \mathbf{0} \end{array}\right] = \left[\begin{array}{cc|c} \mathbf{A}_1 & \mathbf{0} & \mathbf{B}_1 \\ \mathbf{0} & \mathbf{A}_2 & \mathbf{B}_2 \\ \hline \mathbf{C}_1 & \mathbf{C}_2 & \mathbf{0} \end{array}\right] \tag{16}$$

where matrix $\mathbf{A}_1$ is stable(contains stable poles) and $\mathbf{A}_2$ is anti-stable (contains unstable poles). Let $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{Q}_1$, $\mathbf{Q}_2$ are solutions of the following LMI:

$$\begin{aligned} \mathbf{A}_1\mathbf{P}_1 + \mathbf{P}_1\mathbf{A}_1^\top + \mathbf{B}_1\mathbf{B}_1^\top &= \mathbf{0} \\ \mathbf{Q}_1\mathbf{A}_1 + \mathbf{A}_1^\top\mathbf{Q}_1 + \mathbf{C}_1^\top\mathbf{C}_1 &= \mathbf{0} \\ (-\mathbf{A}_2)\mathbf{P}_2 + \mathbf{P}_2(-\mathbf{A}_2)^\top + \mathbf{B}_2\mathbf{B}_2^\top &= \mathbf{0} \\ \mathbf{Q}_2(-\mathbf{A}_2) + (-\mathbf{A}_2)^\top\mathbf{Q}_2 + \mathbf{C}_2^\top\mathbf{C}_2 &= \mathbf{0} \end{aligned} \tag{17}$$

Then for $\mathbf{P}_C$ and $\mathbf{Q}_O$ we get

$$\mathbf{P}_C = \mathbf{T}^{-1}\begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_2 \end{bmatrix}\big(\mathbf{T}^{-1}\big)^\top,\ \mathbf{Q}_O = \mathbf{T}^\top\begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix}\mathbf{T}. \tag{18}$$

3.2 Generalized gramians for unstable systems

Generalized gramians are defined analogically as in (8), (9). We come out from (17) but using inequalities.

$$\begin{aligned} \mathbf{A}_1\mathbf{P}_1 + \mathbf{P}_1\mathbf{A}_1^\top + \mathbf{B}_1\mathbf{B}_1^\top &\le \mathbf{0} \\ \mathbf{Q}_1\mathbf{A}_1 + \mathbf{A}_1^\top\mathbf{Q}_1 + \mathbf{C}_1^\top\mathbf{C}_1 &\le \mathbf{0} \\ (-\mathbf{A}_2)\mathbf{P}_2 + \mathbf{P}_2(-\mathbf{A}_2)^\top + \mathbf{B}_2\mathbf{B}_2^\top &\le \mathbf{0} \\ \mathbf{Q}_2(-\mathbf{A}_2) + (-\mathbf{A}_2)^\top\mathbf{Q}_2 + \mathbf{C}_2^\top\mathbf{C}_2 &\le \mathbf{0} \end{aligned} \tag{19}$$

Final generalized gramians are

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_v \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_v \end{bmatrix}. \tag{20}$$

Using the singular value decomposition we receive transformation matrices $\mathbf{T}_{S1}$ and $\mathbf{T}_{S2}$:

$$\begin{aligned} \mathbf{P}_1^{\frac12}\mathbf{Q}_1\mathbf{P}_1^{\frac12} &= \mathbf{U}_1\Sigma_1^2\mathbf{U}_1^*, \quad \mathbf{T}_{S_1} = \Sigma_1^{\frac12}\mathbf{U}_1^{\frac12}\mathbf{P}_1^{-\frac12}, \\ \mathbf{P}_2^{\frac12}\mathbf{Q}_2\mathbf{P}_2^{\frac12} &= \mathbf{U}_2\Sigma_2^2\mathbf{U}_2^*, \quad \mathbf{T}_{S_2} = \Sigma_2^{\frac12}\mathbf{U}_2^{\frac12}\mathbf{P}_2^{-\frac12}. \end{aligned} \tag{21}$$

The final transformation matrix $\mathbf{T}_S$ is:

$$\mathbf{T}_S = \begin{bmatrix} \mathbf{T}_{S_1} & 0 \\ 0 & \mathbf{T}_{S_2} \end{bmatrix}. \tag{22}$$

In the following, the above described general model reduction procedure is applied to obtain the model reduction.

4 DECENTRALIZED CONTROL DESIGN

The main result — the new decentralized control design procedure is proposed in this section. The main contribution is the application of model reduction approach introduced in previous section for the subsystem control design.

According to the control design methodology outlined in Section 2, the crucial point is in model reduction applied to S_2. We adopt the following approach. Consider matrices describing S_2 and its interaction with S_1 as:

$\mathbf{A}_{22} \to \mathbf{A}$ system matrix,
$\mathbf{A}_{12} \to \mathbf{C}$ output matrix (output from S_2 to S_1),
$\mathbf{A}_{21} \to \mathbf{B}$ input matrix (input from S_1 to S_2).

$$\begin{aligned} S_1 &: \dot{\boldsymbol{x}}_1(t) = \mathbf{A}_{11}\boldsymbol{x}_1(t) + \mathbf{A}_{12}\boldsymbol{x}_2(t) + \mathbf{B}_1\boldsymbol{u}_1(t)\,, \\ S_2 &: \dot{\boldsymbol{x}}_2(t) = \mathbf{A}_{21}\boldsymbol{x}_1(t) + \mathbf{A}_{22}\boldsymbol{x}_2(t)\,. \end{aligned} \tag{23}$$

Denote as $\mathbf{T}$ transformation matrix for system S_2 described by the triplet $(\mathbf{A}_{22}, \mathbf{A}_{21}, \mathbf{A}_{12})$, *ie* $\big(\mathbf{TA}_{22}\mathbf{T}^{-1}, \mathbf{TA}_{21}, \mathbf{A}_{12}\mathbf{T}^{-1}\big)$ is balanced realization for $(\mathbf{A}_{22}, \mathbf{A}_{21}, \mathbf{A}_{12})$. Then the respective model reduction transformation is:

$$\boldsymbol{x}_{2r} = \mathbf{L}\boldsymbol{x}_2 = [\,\mathbf{I}\quad \mathbf{0}\,]\,\mathbf{T}\,. \tag{24}$$

$\mathbf{I}$ is $n_1 \times n_1$ identity matrix, respective to model reduction of S_2 to dimension n_1; $\mathbf{0}$ is zero matrix of the respective dimension.

Denote the balanced-reduced model for $(\mathbf{A}_{22}, \mathbf{A}_{21}, \mathbf{A}_{12})$ as $(\mathbf{A}_{22r}, \mathbf{A}_{21r}, \mathbf{A}_{12r})$, then the resulting reduced model of (23) is:

$$\begin{aligned} S_{1r} &: \ \dot{\boldsymbol{x}}_1(t) = \mathbf{A}_{11}\boldsymbol{x}_1(t) + \mathbf{A}_{12r}\boldsymbol{x}_{2r}(t) + \mathbf{B}_1\boldsymbol{u}_1(t)\,, \\ S_{2r} &: \dot{\boldsymbol{x}}_{2r}(t) = \mathbf{A}_{21r}\boldsymbol{x}_1(t) + \mathbf{A}_{22r}\boldsymbol{x}_{2r}(t)\,, \\ &\quad \boldsymbol{y}_1(t) = \mathbf{C}_1\boldsymbol{x}_1(t)\,. \end{aligned} \tag{25}$$

Having the reduced order model (25), recall that its dimension is $\sim 2n_1$ instead of the overall system dimension

$n_1 + n_2$ $(n_1 \ll n_2)$, now the local control law for subsystem S_{1r} can be designed as follows.

Consider $\boldsymbol{v}_1(t) = [\boldsymbol{x}_1(t) \quad \boldsymbol{x}_{2r}(t)]^\top$ as a new state vector for the system:

$$\begin{aligned} \dot{\boldsymbol{v}}_1(t) &= \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12r} \\ \mathbf{A}_{21r} & \mathbf{A}_{22r} \end{bmatrix} \boldsymbol{v}_1(t) + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{0} \end{bmatrix} \boldsymbol{u}_1(t)\,, \\ \boldsymbol{y}_1(t) &= [\,\mathbf{C}_1 \quad \mathbf{0}\,]\, \boldsymbol{v}_1(t)\,. \end{aligned} \tag{26}$$

The appropriate control law for (26) can be designed applying common approaches to state feedback or output feedback design. We adopt the output feedback control law based on classical LQ design, considering objective function (2) extended for the state vector $\boldsymbol{v}_1(t)$:

$$J_r = \int_{t=t_0}^{\infty} \big(\boldsymbol{x}_1^\top(t)\mathbf{Q}_1\boldsymbol{x}_1(t) + \boldsymbol{v}_1^\top \mathbf{Q}_{1v}\boldsymbol{v}_1(t) + \boldsymbol{u}_1^\top(t)\mathbf{R}_1\boldsymbol{u}_1(t)\big)\mathrm{d}t\,. \tag{27}$$

The additional matrix $\mathbf{Q}_{1v}$ should be positive definite, its choice enables to minimize this interaction.

The respective output feedback control law is:

$$\boldsymbol{u}_1(t) = \mathbf{K}_y \boldsymbol{y}_1(t) = \mathbf{K}_y\,[\,\mathbf{C}_1 \quad \mathbf{0}\,]\,\boldsymbol{v}_1(t)\,. \tag{28}$$

The ideal case would be if the output feedback law was equal to the LQ state feedback one:

$$\mathbf{K}_y = [\,\mathbf{C}_1 \quad \mathbf{0}\,]\,\boldsymbol{v}_1(t) = \mathbf{K}_x \boldsymbol{v}_1(t) \Leftrightarrow \mathbf{K}_y\,[\,\mathbf{C}_1 \quad \mathbf{0}\,] = \mathbf{K}_x\,. \tag{29}$$

Since the last equality cannot be fulfilled in general, we make use of the approximate (heuristic) solution which often provides reasonable results:

$$\mathbf{K}_y = \mathbf{K}_x pinv\big([\,\mathbf{C}_1 \quad \mathbf{0}\,]\big)\,. \tag{30}$$

Stability must be checked afterwards, since the control law (30) does not automatically guarantee the stability. The example of the outlined design approach is provided in the next section.

5 EXAMPLE

Consider the linear time invariant dynamical system, described as

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{x}}_1(t) \\ \dot{\boldsymbol{x}}_2(t) \\ \dot{\boldsymbol{x}}_3(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \\ \boldsymbol{x}_3(t) \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{B}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1(t) \\ \boldsymbol{u}_2(t) \\ \boldsymbol{u}_3(t) \end{bmatrix} \\ \boldsymbol{y}_1(t) = \boldsymbol{x}_1(t)\,,\ &\boldsymbol{y}_2(t) = \boldsymbol{x}_2(t)\,,\ \boldsymbol{y}_3(t) = \boldsymbol{x}_3(t)\,. \end{aligned} \tag{31}$$

where

$$\begin{aligned} &\mathbf{A}_{11} = \begin{bmatrix} -0.9 & 0.3 & 0.7 \\ 0.1 & -1.7 & -0.1 \\ 0.6 & 0 & -0.4 \end{bmatrix},\ \mathbf{A}_{12} = \begin{bmatrix} 0.12 & 0.2 \\ 0 & 0 \\ 0.19 & 0.05 \end{bmatrix}, \\ &\mathbf{A}_{13} = \begin{bmatrix} 0.31 & 0 \\ 0.22 & 0.2 \\ -0.05 & 0 \end{bmatrix},\ \mathbf{A}_{21} = \begin{bmatrix} 0.6 & 1.0 & 0 \\ -0.7 & 0.28 & -0.3 \end{bmatrix}, \\ &\mathbf{A}_{22} = \begin{bmatrix} -0.36 & 0.4 \\ 0.22 & -0.1 \end{bmatrix},\ \mathbf{A}_{23} = \begin{bmatrix} 0.11 & -0.01 \\ 0.2 & -3.6 \end{bmatrix}, \\ &\mathbf{A}_{31} = \begin{bmatrix} 0.3 & 0 & 0.6 \\ 0.82 & -0.15 & -0.025 \end{bmatrix},\ \mathbf{A}_{32} = \begin{bmatrix} -0.01 & 0.42 \\ 0 & 0.1 \end{bmatrix}, \\ &\mathbf{A}_{33} = \begin{bmatrix} -1.2 & 0 \\ 0.6 & -3.6 \end{bmatrix},\ \mathbf{B}_1 = \begin{bmatrix} 1.2 & 0 \\ 0.1 & 0 \\ 0 & 0.7 \end{bmatrix}, \\ &\mathbf{B}_2 = \begin{bmatrix} 1.2 & 0 \\ 0 & 0.3 \end{bmatrix},\ \mathbf{B}_3 = \begin{bmatrix} 0.4 \\ 0.85 \end{bmatrix}. \end{aligned} \tag{32}$$

Matrices $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3$, $\mathbf{Q}_{1r}$, $\mathbf{Q}_{2r}$, $\mathbf{Q}_{3r}$, $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R}_3$ are identities.

5.1 Model reduction using balanced realization

Firstly, the feedback matrix for the first isolated subsystem is calculated

$$\mathbf{K}_1 = \begin{bmatrix} 0.6526 & 0.0956 & 0.4596 \\ 0.2663 & 0.0211 & 0.7100 \end{bmatrix}. \tag{33}$$

Secondly, the first two subsystems are considered together as in (26), the first, already controlled system being represented by its reduced model of dimension 2. The resulting control gain obtained using (30) is

$$\mathbf{K}_2 = \begin{bmatrix} 0.8271 & 0.4286 \\ 0.1072 & 0.6322 \end{bmatrix}. \tag{34}$$

Finally, the control law for the third subsystem is designed applying the similar procedure: the first two already controlled subsystems are considered together and represented by the reduced model of 2 states.

$$\mathbf{K}_3 = [\,0.2328 \quad 0.1268\,]\,. \tag{35}$$

The resulting closed loop overall system is stable.

5.2 Model reduction using generalized balanced realization

Similarly as in 5.1 but using LMI (8), (9) we get the following feedback matrices.

The first feedback matrix for the first subsystem is the same as in 5.1:

$$\mathbf{K}_1 = \begin{bmatrix} 0.6526 & 0.0956 & 0.4596 \\ 0.2663 & 0.0211 & 0.7100 \end{bmatrix}, \tag{36}$$

$$\mathbf{K}_2 = \begin{bmatrix} 0.8252 & 0.4449 \\ 0.1112 & 0.6725 \end{bmatrix}, \tag{37}$$

$$\mathbf{K}_3 = [\,0.2298 \quad 0.1264\,]\,. \tag{38}$$

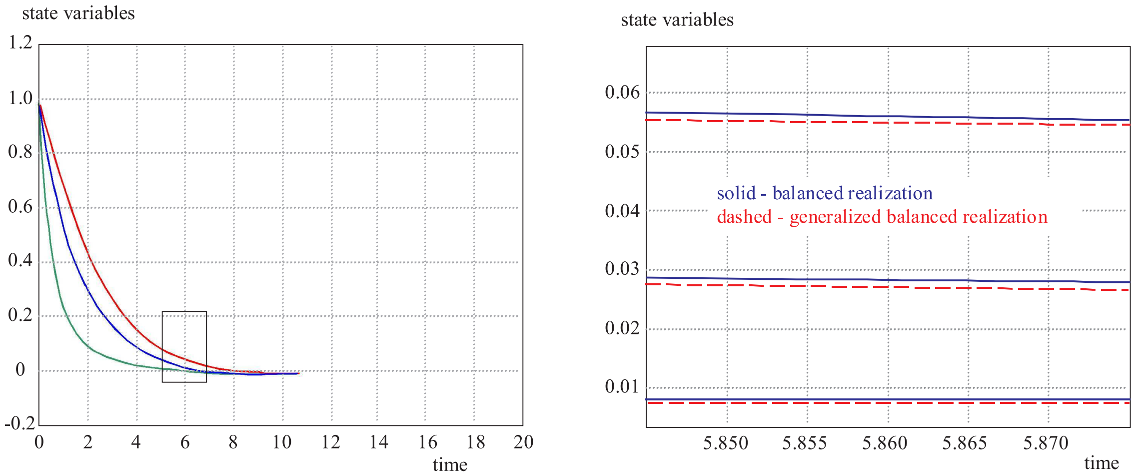

Fig. 1. State variable responses to non-zero initial conditions for subsystem1: comparison of LME(5.1) and LMI(5.2) solution

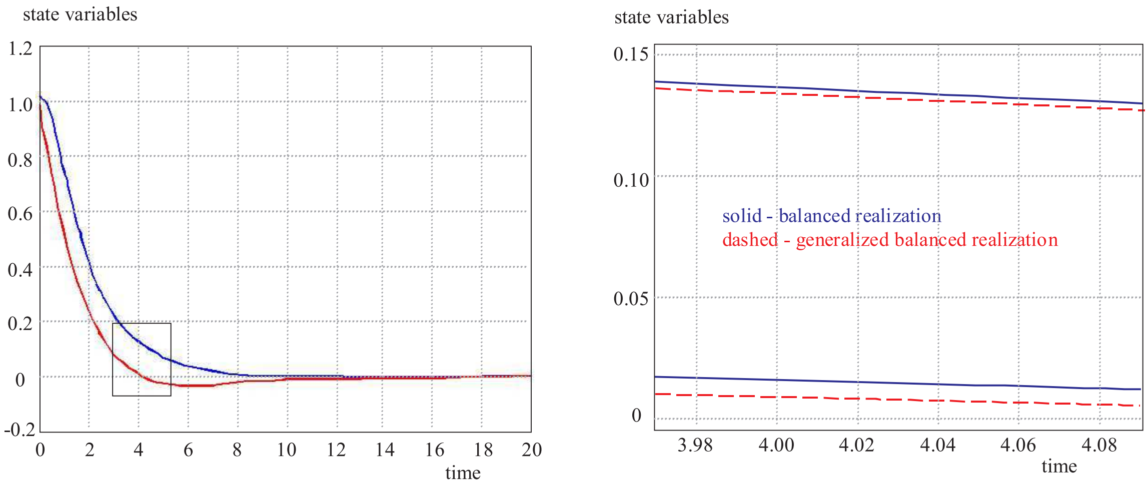

Fig. 2. State variable responses to non-zero initial conditions for subsystem2: comparison of LME(5.1) and LMI(5.2) solution

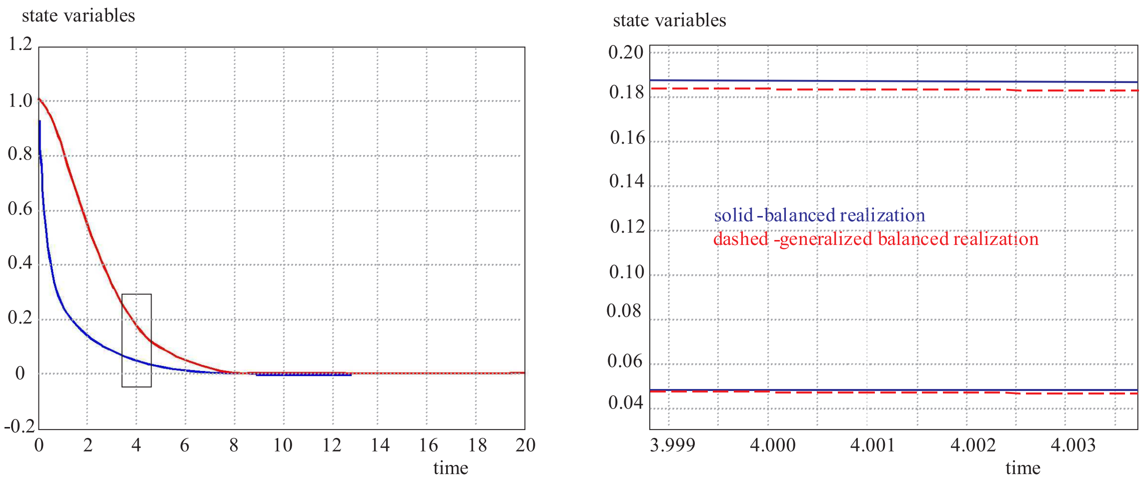

Fig. 3. State variable responses to non-zero initial conditions for subsystem3: comparison of LME(5.1) and LMI(5.2) solution

The resulting closed loop overall system is stable too.

In Figs 1–3 the simulation results of the closed loop system with control matrix gains from 5.1 and 5.2 are shown.

In the program realization a free software product SEDUMI in software environment MATLAB has been used for the solution of LMI together with the SEDUMI interface. In Figs. 1,2,3 the state variable responses to nonzero initial condition are shown for each subsystem of system (32). There are little differences between simulations for solutions with LME (5.1) and LMI (5.2), which means, that the generalized balanced realizations is also applicable; in this case the results are nearly the same since there were no additional requirements to obtain different solution from feasible region of (8), (9).

6 CONCLUSION

The decentralized control design scheme has been proposed, which considers the interactions of the remaining part of the overall system when the subsystem control law is designed. The approach based on reduced model representation of this remaining part of the system, that reduces the problem size and complexity of control design is applied. The proposed model reduction scheme comes from balanced realization or generalized balanced realization. The latter variant (using the respective LMI solution) is still under research to check a possibility to include the additional criterion into model reduction scheme. The generalized balanced realization scheme provides the set of feasible solutions (on the contrary to "classical" balanced realization with unique solution). Therefore there is possibility to choose the appropriate criterion "to tune" the solution according to additional requirements. The problem of determining appropriate criterion (additional requirements) is still open.

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