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COMMUNICATIONS

UTILIZATION OF 1st ERLANG FORMULA IN ASYNCHRONOUS NETWORKS

Erik Chromý — Ivan Baroňák *

This paper emphasizes a possibility for the utilization the 1st Erlang formula in asynchronous networks. This statement is possible on the basis of research of statistical features of traffic source input flow. We consider ATM network type where in the case of VP important parameters are as follows: loss - B, link utilization - ρ and bandwidth - c. The evidence for utilization the 1st Erlang formula in asynchronous networks is achieved through the model of video source traffic. To design the video source traffic discreet Markov chain has been used.

K e y w o r d s: ATM, 1st Erlang formula, Markov model, video source model

1 INTRODUCTION

It is obvious that with help of the 1st Erlang formula, where such parameters as loss - B, link utilization - ρ and bandwidth - c are used, we are able to describe the traffic in synchronous networks. On the other hand, to describe the traffic in asynchronous networks of ATM network type the Markov chains [1, 2, 9, 11] are often used. Their disadvantage is that they are very complicated and at the same time demand complicated estimations. Papers [3, 4] deal with the idea of utilization ot the 1st Erlang formula. As in the case of ATM networks, the optical fibre is used as a transmission medium, it is possible to consider ATM networks to be networks with low losses. The networks with low losses are characterized by the 1st Erlang formula which expresses the connection loss probability in virtual path VP [5, 6]. The question if the utilization of the 1st Erlang formula is possible in asynchronous networks. In this paper we give some evidence while utilizing this formula in asynchronous networks

$$\frac{\frac{\rho^c}{c!}}{\sum_{k=0}^{c} \frac{\rho^k}{k!}} = B, \tag{1}$$

where ρ represents the percentage utilization of a link, c - is the bandwidth and B - is the link loss.

2 SEARCH FOR B, ρ, C RELATIONS

The aim is to estimate the relations between B, ρ, c parameters on the basis of (1) formula in such a manner that always one parameter is kept constant and the dependence between remaining two parameters is observed.

2.1 Link utilization and bandwidth at constant loss

Dependence of the link utilization ρ on bandwidth c at a given loss B is shown Fig. 1 and Tab. 1.

From the figure it can be seen that with the increasing bandwidth it is possible to increase the link utilization while that loss can be preserved. The narrower the loss requirement (*ie* parameter B gets smaller value), the less link utilization is possible.

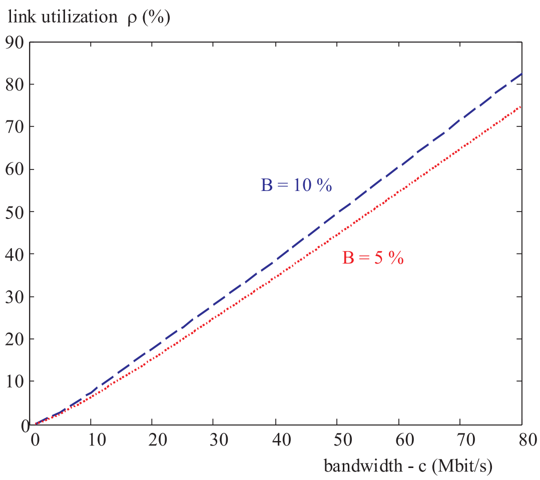

Fig. 1. Dependence of link utilization on bandwidth at given loss

Table 1. Data for Fig. 1

$B = 10\%$		$B = 5\%$	
c (Mbit/s)	ρ (%)	c (Mbit/s)	ρ (%)
1	0.11	1	0.05
2	0.59	2	0.38
5	2.88	5	2.21
10	7.51	10	6.21
20	17.61	20	15.24
25	22.83	25	19.98
40	38.78	40	34.59
50	49.56	50	44.53
60	60.40	60	54.56
80	82.20	80	74.81

* Slovak University of Technology, Faculty of Electrical Engineering and Information Technology, Department of Telecommunications, Ilkovičova 3, 812 19 Bratislava, Slovakia; chromy@ktl.elf.stuba.sk; baronak@ktl.elf.stuba.sk

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2.2 Link loss and link utilization at constant bandwidth

Dependence of the link loss B on the link utilization ρ at a constant bandwidth c is shown in Fig. 2 and Tab. 2.

Estimations were made for different constant bandwidths c = 4 Mbit/s, 10 Mbit/s and 50 Mbit/s. From Fig. 2 it is clear that with the increasing link utilization for a given bandwidth the loss increases. Further it is seen that at the same link utilization ρ and at different values c the loss is different. With the increasing bandwidth at the same link, utilization loss decreases.

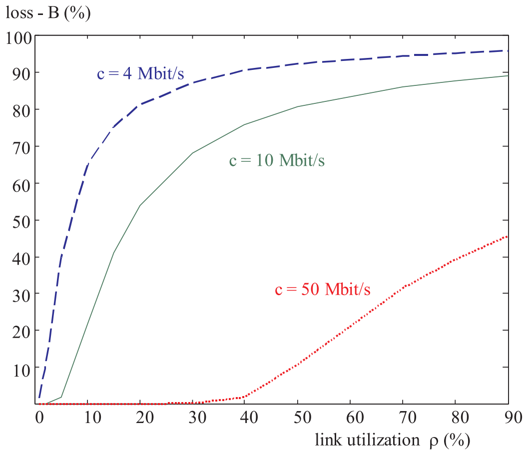

Fig. 2. Loss dependence on the link utilization at constant bandwidth

Table 2. Data for Fig. 2

c (Mbit/s)=	4	10	50
ρ (%)		B (%)	
1	1.53	0	0
5	39.83	1.83	0
10	64.66	21.45	0
20	81.09	53.79	0
30	187.13	68.13	0.02
40	90.26	75.76	1.86
50	92.16	80.47	10.47
70	94.36	85.94	31.38
90	95.60	89.02	45.70

2.3 Link loss and bandwidth at constant link utilization

Dependence of the link loss B on the bandwidth c at a constant link utilization ρ is shown in Fig. 3 and Tab. 3.

Tab. 3 and Fig. 3 show that at the constant link utilization ρ and increasing bandwidth loss B decreases. Further it is visible that at the same bandwidth and different utilizations the loss is different. With the increasing link utilization, loss increases as well.

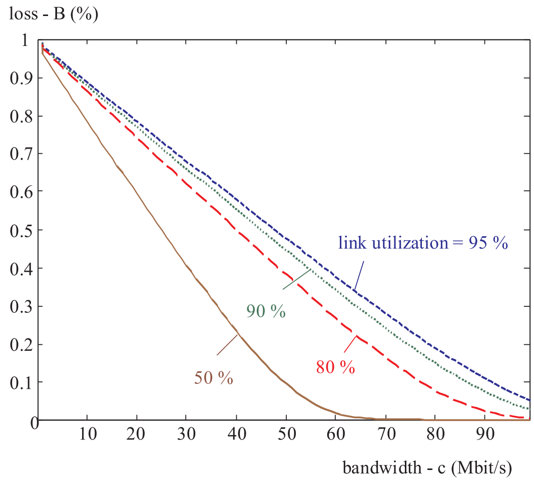

Fig. 3. Loss dependence on the bandwidth at constant link utilization

Table 3. Data for Fig. 3

ρ (%) =	95	90	80	50
c (Mbit/s)		B (%)		
1	98.96	98.90	98.77	98.04
5	94.79	94.51	93.83	90.21
10	89.59	89.02	87.67	80.47
20	79.22	78.08	75.40	61.21
30	68.89	67.20	63.21	42.48
40	58.62	56.39	51.14	24.98
50	48.44	45.70	39.31	10.48
70	28.69	25.15	17.29	00.14
90	11.24	07.96	2.62	0

3 EVIDENCE OF 1st ERLANG FORMULA'S APPLICATION IN ASYNCHRONOUS NETWORKS WITH THE HELP OF VIDEO SOURCE MODEL

The aim is to prove the possibility of the 1st Erlang formula utilization in asynchronous networks on the basis of simulations with the help of designed mathematical model. Parameters were used from a real video source during the creation of the model for video traffic source, which is shown in [7] for video Ice Age DVD.

On the basis of the method shown in [8] the parameters necessary for the video source model creation are estimated as follows:

- Transition matrix $P = [p_{ij}]$
- Vector size of the video source model state $s = [s_1, \dots, s_M]$

Here M expresses the number of the states in Markov chain and $s_1, \dots, s_M$ the video source model states are representing the rate of the cell generation.

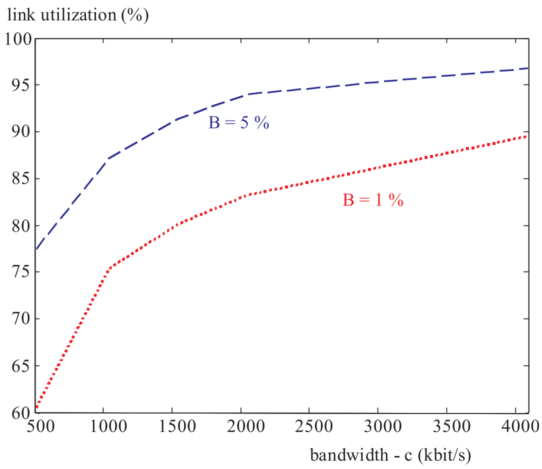

Fig. 4. Dependence of link utilization on bandwidth at given loss

Table 4. Data for Fig. 4

B (%) =	1	5
c (kbit/s)	ρ (%)	
512	60.53	77.49
1024	75.40	87.21
1544	80.20	91.40
2048	83.19	93.94
4096	89.53	96.78

The result is a video source model for $M = 7$.

$$P = \begin{bmatrix} 0.7888 & 0.1472 & 0.0439 & 0.0171 & 0.0028 & 0.0003 & 0 \\ 0.4178 & 0.4595 & 0.0976 & 0.0214 & 0.0031 & 0.0005 & 0 \\ 0.7256 & 0.2451 & 0.0185 & 0.0091 & 0.0017 & 0 & 0 \\ 0.7877 & 0.1872 & 0.0219 & 0.0011 & 0.0021 & 0 & 0 \\ 0.7933 & 0.1733 & 0.0233 & 0.0100 & 0 & 0 & 0 \\ 0.7500 & 0.2500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

- $s_1 = 26.873\ kbit/s$, $s_2 = 75.711\ kB/s$, $s_3 = 135.687\ kbit/s$
- $s_4 = 189.924\ kbit/s$, $s_5 = 241.409\ kbit/s$, $s_6 = 302.333\ kbit/s$
- $s_7 = 389.520\ kbit/s$

For the purposes of simulation the necessary simulation model consisting of ATM node without buffer store and with video traffic source [10, 12] on input of an ATM node has been created. The input video traffic sources have different requirements from the point of view of bandwidth, loss and link utilization. There is an output link on the other end of ATM node with defined parameters: bandwidth, loss requirement and link utilization. All these parameters were adjusted according to the requirements stemming from requirements of particular situations.

3.1 Dependence of link utilization on link bandwidth at a constant loss

Dependence of link utilization ρ on the bandwidth c (512 kbit/s, 1024 kbit/s, 1544 kbit/s, 2048 kbit/s a 4096 kbit/s) at two constant losses B (1 % a 5 %) has been simulated. Results are recorded in Tab. 4 and Fig. 4. They clearly show that with the increasing bandwidth it is possible to increase the link utilization by that way the loss is preserved. The narrower the loss requirement, the less the link utilization is possible.

3.2 Dependence of link loss on link utilization at a constant bandwidth

Dependence of loss B on link utilization ρ at few constant bandwidths c = 512 kbit/s, 1024 kbit/s and 2048 kbit/s has been simulated and results are recorded in Tab. 5.

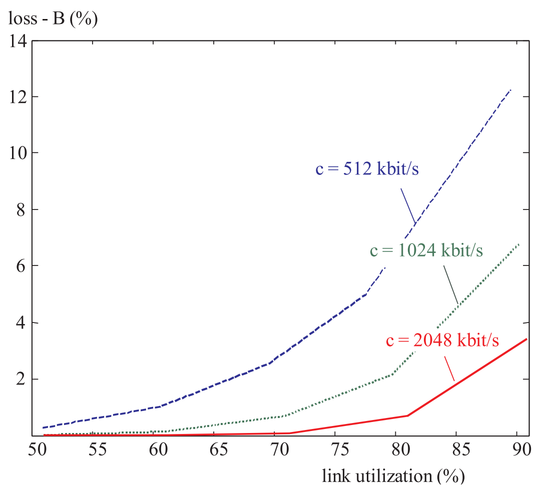

Fig. 5. Loss dependence on link utilization at constant bandwidth

Table 5. Data for Fig. 5

$c = 512$ kbit/s		$c = 1.024$ Mbit/s		$c = 2.048$ Mbit/s	
ρ (%)	B (%)	ρ (%)	B (%)	ρ (%)	B (%)
50.8	0.29	50.9	0.01	0.0003	98.04
60.5	1.03	61.0	0.14	0.0050	90.21
69.5	2.57	70.8	0.69	0.0900	80.47
77.5	4.95	79.7	2.15	0.6800	61.21
89.5	12.24	90.2	6.74	3.4300	42.48

It is obvious that with the increasing link utilization for a given bandwidth the loss increases as well. Further it is visible that with the same link utilization ρ and at different values c the loss is different as well. With the increasing bandwidth the loss decreases.

3.3 Dependence of link loss on link bandwidth at a constant link utilization

Dependence of loss B in bandwidth $c =$ (512 kbit/s, 1024 kbit/s, 1544 kbit/s, 2048 kbit/s a 4096 kbit/s) at a constant link utilization has been simulated. Results are recorded in Tab. 6 and Fig. 6.

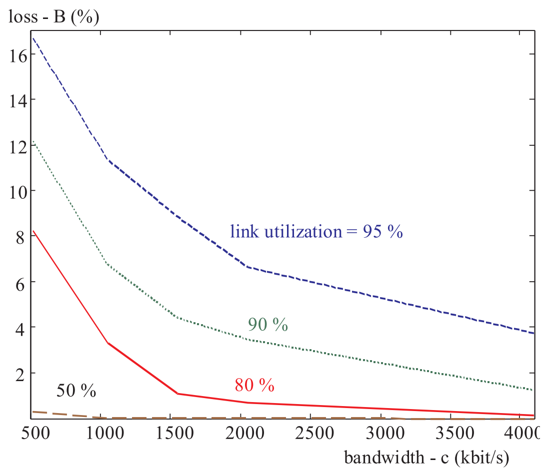

Fig. 6. Loss dependence on the bandwidth at constant link utilization

Table 6. Data for Fig. 6

ρ (%) =	95	90	80	50
c (kbit/s)		B (%)		
512	16.7	12.18	8.25	0.283
1024	11.35	6.72	3.31	0.01000
1544	8.85	4.42	1.07	0.00050
2048	6.61	3.44	0.68	0.00007
4096	3.74	1.23	0.08	0

Tab. 6 and Fig. 6 verify that at constant link utilization and increasing bandwidth the loss decreases. Further it is visible that with the same bandwidth and different link utilizations the loss is different. With the increasing link utilization the loss increases as well.

4 RESULT ANALYSIS

The following tendencies were shown with the help of 1st Erlang formula:

- with the increasing bandwidth it is possible to increase link utilization in the way that the given transfer path loss is preserved,
- at a constant bandwidth and increasing link utilization the transfer path loss increases as well,
- at constant link utilization and increasing bandwidth the transfer path loss decreases.

On the basis of simulations with the help of designed mathematical model of video source model (built-up with the help of Markov chains) listed changes gained by 1st Erlang formula have been verified. This is proved by tables and diagrams.

In the case of dependencies between the transfer path loss and the link utilization at a constant bandwidth just a small difference occurred. It is visible from the comparison of Fig. 2 and Fig. 5. In the case of estimations with the help of 1st Erlang formula sudden loss growth at smaller bandwidths occurred, whereas in the case of mathematical video source model, a progressive growth of transfer path loss with progressive growth of link utilization occurred.

Consequently, simulation results justified a possibility of using the 1st Erlang formula also in asynchronous networks, where such parameters as loss, bandwidth and link utilization are taken into consideration.

5 CONCLUSION

Nowadays we witness development in telecommunication field. Converged technologies and convergence processes are extremely attractive topics. Convergence represents an evolution path of a switch from actual telecommunications to the future modern multimedia telecommunication infrastructure for land-lines as well as for the mobile networks. Next generation networks (NGN) are results of convergence.

There exist several important fields within the frame of NGN matter, which draw a lot of attention of professionals from the telecommunication field. We speak about improvement of QoS parameters, management systems, multimedia, routing matter in NGN networks, dimensioning of networks, optimization of system components of NGN platform and many other.

It is becoming more evident that the matter of network dimensioning in asynchronous network can be researched with a help of the 1st Erlang formula. On the basis of simulations using a video source model the prospective of the 1st Erlang formula utilization in asynchronous networks has been demonstrated. All results gained with the help of the 1st Erlang formula and video source model are shown from diagrams and approve this by evidence.

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Erik Chromý (Ing) was born in Veľký Krtíš, Slovakia, in 1981. He received the engineering degree in telecommunications in 2005 from the Faculty of Electrical Engineering and Information Technology of Slovak University of Technology (FEI STU) Bratislava. In 2007 he submitted PhD work from on observation of statistical properties of the input flow of traffic sources on virtual paths dimensioning, his scientific research is focused on optimizing of the processes in convergent networks.

Ivan Baroňák (Prof, Ing, PhD) was born in Žilina, Slovakia, on July 1955. He received the electronic engineering degree from the Slovak Technical University Bratislava in 1980. Since 1981 he has been a lecturer at Department of Telecommunications STU Bratislava. In 1992 he submitted PhD work from the field of Terminal telephone equipment. In 1995 he became an Associate Professor for the subject applied information. Nowadays he works as a full Professor in the Department of Telecommunications of FEI STU in Bratislava. Scientifically, professionally and pedagogically he focuses on problems of digital switching systems, ATM, telecommunication management (TMN), next generation networks, problems of optimal modelling of private telecommunication networks and services.