Journal of ELECTRICAL ENGINEERING, VOL. 59, NO. 6, 2008, 322–327

SLIDING MODE APPLICATION IN POSITOIN CONTROL OF AN INDUCTION MACHINE

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In this paper, we propose a sliding mode technique to control the position of the field-oriented induction machine. Our aim is to make the position control robust to parameter variations. The variation of motor parameters during operation degrades the performances of the controllers. The use of the nonlinear sliding mode method provides best performances for motor operation and the robustness of the control law despite the external/internal perturbations. Simulation results are given to highlight the performances of the proposed control technique under load disturbances and parameter uncertainties.

K e y w o r d s: induction machine, field-oriented control, robustness, sliding mode control, position and speed control

1 INDRODUCTION

The induction machine is one the most widely used actuators for industrial applications owing to its reliability, ruggedness and relatively low cost. Furthermore, its use of position and speed tracking control is expected to increase in the near future. Based on the forecast, many researchers have proposed the use of modern control techniques for precise control of the induction machine. [6–9]

The control of the induction machine (IM) must take into account machine specificities: high order of the model, non-linear functioning as well as coupling between the different variables of control. Furthermore, the machine parameters depend generally on the operating point and vary either with the temperature (resistance) or with the magnetic state of the induction machine. These parametric variations modify the performances of the control system when we use a regulator or a control law with fixed parameters. The new industrial applications necessitate position and speed variators having high dynamic performances, good precision in permanent regime, and a high capacity of overload over the whole range of position and speed, and robustness to different perturbations. Thus, the recourse to robust control algorithms is desirable in stabilization and in tracking trajectories. The variable structure control (VSC) possesses this robustness using the sliding mode control that can offer many good properties such as good performance against unmodelled dynamics, insensitivity to parameter variation, external disturbance rejection and fast dynamics [1–5]. These advantages of sliding mode control can be employed in the position and speed control of an alternative current servo system.

This paper is organized as follows. The oriented model of an induction motor is introduced in section 2. In section 3, the observers of the rotor flux are presented. Then, the sliding mode theory and the design of the sliding mode controllers of rotor flux and motor position are presented in section 4. In section 5, the proposed control of IM using the sliding mode is delineated and some simulation results are presented. Finally, we give some concluding remarks on the proposed control of IM using the sliding mode.

2 INDUCTION MOTOR ORIENTED MODEL

The induction machine model can be described by the following state equations in the synchronous reference frame whose axis d is aligned with the rotor flux vector, ($\Phi_{rd} = \Phi_r$ and $\Phi_{rq} = 0$) [10–11]:

$$\dot{I}_{sd} = -\gamma I_{sd} + \omega_s I_{sq} + \frac{K}{T_r}\Phi_{rd} + \frac{1}{\sigma L_s}U_{sd}\,, \qquad (1)$$

$$\dot{I}_{sq} = -\omega_s I_{sd} - \gamma I_{sq} - P\Omega K\Phi_{rd} + \frac{1}{\sigma L_s}U_{sq}\,, \qquad (2)$$

$$\dot{\Phi}_{rd} = \frac{M_{sr}}{T_r}M_{sr}I_{sd} - \frac{1}{T_r}\Phi_{rd}\,, \qquad (3)$$

$$\dot{\Phi}_{rq} = \frac{M_{sr}}{T_r}I_{sq} - (\omega_s - P\Omega)\Phi_{rd}\,, \qquad (4)$$

$$\dot{\Omega} = \frac{PM_{sr}}{JL_r}\big(\Phi_{rd}I_{sq}\big) - \frac{T_l}{J} - \frac{f}{J}\Omega\,. \qquad (5)$$

Here

$$T_r = \frac{L_r}{R}, \qquad \sigma = 1 - \frac{M_{sr}^2}{L_s L_r},$$

$$K = \frac{M_{sr}}{\sigma L_s L_r}, \qquad \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M_{sr}^2}{\sigma L_s L_r^2}.$$

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ISSN 1335-3632

Here, Φ_{rd}, Φ_{r1} are the rotor flux components, U_{sd}, U_{sq} are the stator voltage components, I_{sd}, I_{sq} are the stator current components, σ is the leakage factor and p is the number of pole pairs. R_s and R_r are stator and rotor resistances, L_s and L_r denote stator and rotor inductances, whereas M_{sr} is the mutual inductance. T_e is the electromagnetic torque, T_l is the load torque, J is the moment of inertia of the IM, Ω is the mechanical speed, ω_s is stator pulsation, f is the damping coefficient, T_r is the rotoric time-constant.

3 FLUX ESTIMATOR

For the direct field-oriented control of induction machine, accurate knowledge of the magnitude and position of the rotor flux vector is necessary. In a normal cage motor, as rotor current are not measurable, the rotor flux should be estimated. Various types of estimators and observers have been proposed in the literature. The flux estimator used in this work is based on the integration of the stator voltage equations in the stationary frame. The flux estimator can be obtained by the following equations [14]:

$$\dot{\hat{\Phi}}_{r\alpha} = \frac{L_r}{M_{sr}}\big(U_{s\alpha} - R_s I_{s\alpha} - \sigma L_s \dot{I}_{s\alpha}\big)\,, \tag{6}$$

$$\dot{\hat{\Phi}}_{r\beta} = \frac{L_r}{M_{sr}}\big(U_{s\beta} - R_s I_{s\beta} - \sigma L_s \dot{I}_{s\beta}\big)\,, \tag{7}$$

θ_s is the angle between the rotoric vector flux Φ_r and the axis of the (α,β) frame

$$\theta_s = \arctan\frac{\hat{\Phi}_{r\beta}}{\hat{\Phi}_{r\alpha}}\,. \tag{8}$$

Where $\hat{\Phi}_{r\alpha}$, $\hat{\Phi}_{r\beta}$ are the estimated rotor flux components, $I_{s\alpha}$, $I_{s\beta}$ are the measured stator current components.

4 SLIDING MODE CONTROL DESIGN

Sliding mode control is developed from variable structure control. It is a form of linear control providing robust means of controlling the nonlinear plants with disturbances and parameters uncertainties.

The sliding mode is a technique to adjust the feedback by previously defining a surface so that the system which is controlled will be forced to that surface, then the behaviour system slides to the desired equilibrium point.

This control consists of two phases:
The first phase is choosing a sliding manifold having a desired dynamics, usually linear and of a lower order.
The second phase is designing a control law, which will drive the state variable to the sliding manifold and will keep them there.

The design of the control system will be demonstrated for a following non-linear system [1–5]:

$$\dot{x} = f()X,t) + B(X,t)u(x,t)\,, \tag{9}$$

Here $X \in \Re^n$ is the state vector, $U \in \Re^m$ is the control vector, $f(X,t) \in \Re^n$, $B(X,t) \in \Re^{n\times m}$.

From system (9), it possible to define a set S of the state trajectories x such as:

$$S = \{X(t) \mid s(X,t) = 0\}\,, \tag{10}$$

where

$$s(X,t) = [s_1(X,t), s_2(X,t), \dots, s_m(X,t)]^\top \tag{11}$$

and $[\,\cdot\,]^\top$ denotes the transposed vector, S is called the sliding surface. To bring the state variable to the sliding surfaces, the following two conditions have to be satisfied:

$$s(X,t) = 0\,, \quad \dot{s}(X,t) = 0\,. \tag{12}$$

The control law satisfying the precedent conditions is presented in the following form:

$$U = U^{eq} + U^n. \tag{13}$$

Here U is the control vector, U^{eq} is the equivalent control vector, U^n is the switching part of the control (the correction factor). U^{eq} can be obtained by considering the condition for the sliding regime, $s(X,t) = 0$. The equivalent control keeps the state variable on the sliding surface, once they reach it. U^n is needed to assure the convergence of the system states to sliding surfaces in finite time.

In order to alleviate the undesirable chattering phenomenon, J.J. Slotine proposed an approach to reduce it, by introducing a boundary layer of width ϕ on either side of the switching surface [3]. Then, U^n is defined by

$$U^n = \mathrm{sat}\big(s(X)/\phi\big)\,. \tag{14}$$

Here $\mathrm{sat}\big(s(X)/\phi\big)$ is the proposed saturation function, ϕ is the boundary layer width, K is the controller gain designed from the Lyapunov stability

$$V = \frac{1}{2}s^2\,, \tag{15}$$

$$\dot{V} = s\dot{s}\,, \tag{16}$$

where η is a strictly positive constant.

In this work, $s(X,t)$ is the sliding mode vector proposed by J. J. Slotine [3].

$$s(X) = \Big(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\Big)^{n-1} e\,, \tag{17}$$

where $X = \Big[X, \dot{X}, \dots, X^{n-1}\Big]^\top$ is the state vector, $X^d = \Big[X^d, \dot{X}^d, X^d\Big]^\top$ is the desired state vector, $e = X^d - X = \Big[e, \dot{e}, \dots, e^{n-1}\Big]$ is the error vector, and λ is a vector of slopes of the S.

Commonly, in IM control using sliding mode theory, the surfaces are chosen as functions of the error between the reference input signals and the measured signals [2, 3]. After this step, the objective is to determine a control law which drives the state trajectories along the surface.

4.1 position control

To control the position of the induction machine, three surfaces are chosen. Variables of control are the rotation position and the flux Φ_r. The flux will be maintained at its nominal value to have a maximal torque.

We take $n = 1$, the position control manifold equations can be obtained as:

$$S(\theta) = \lambda_\theta\big(\theta_{ref} - \theta\big) + \frac{\mathrm{d}}{\mathrm{d}t}\big(\theta_{ref} - \theta\big)\,, \tag{18}$$

$$\dot S(\theta) = \lambda_\theta\big(\dot\theta_{ref} - \dot\theta\big) + \frac{\mathrm{d}}{\mathrm{d}t}\big(\dot\theta_{ref} - \dot\theta\big)\,, \tag{19}$$

with

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta = \dot\theta = \Omega\,, \tag{20}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Omega = \frac{PL_m}{JL_r}\Phi_{rd}^{rf} I_{sq} - \frac{f}{J}\Omega - \frac{T_l}{J}\,. \tag{21}$$

Substituting the expression of $\dot\Omega$ defined by equation (21) in equation (19), we obtain:

$$\dot S(\theta) = \lambda_\theta \dot\theta_{ref} + \ddot\theta_{ref} + \Big(\frac{f}{J} - \lambda_\theta\Big)\Omega + \frac{T_l}{J} - \frac{PM_{sr}}{JL_r}\Phi_{rd}^{ref} I_{sq}\,. \tag{22}$$

Substituting the expression of I_{sq} by $I_{sq}^{eq} + I_{sq}^{n}$ the command clearly appears in the equation before.

$$\dot S(\theta) = \lambda_\theta \dot\theta_{ref} + \ddot\theta_{ref} + \Big(\frac{f}{J} - \lambda_\theta\Big)\Omega + \frac{T_l}{J} - \frac{PM_{sr}}{JL_r}\Phi_{rd}^{ref}(I_{sq}^{eq} + I_{sq}^{n})\,. \tag{23}$$

During the sliding mode and in permanent regime, we have

$$S(\Omega) = 0\,, \quad \dot S(\Omega) = 0\,, \quad I_{sq}^{n} = 0\,, \tag{24}$$

where the equivalent control is:

$$I_{sq}^{eq} = \frac{JL_r}{PM_{sr}\Phi_{rd}^{ref}}\Big(\lambda_\theta\dot\theta_{ref} + \ddot\theta_{ref} + \Big(\frac{f}{J} - \lambda_\theta\Big) + \frac{T_l}{J}\Big). \tag{25}$$

During the convergence mode, the condition must be verified. We obtain:

$$\dot S(\Omega) = -\frac{PM_{sr}\Phi_{rd}^{ref}}{JL_r} I_{sq}^{n}\,. \tag{26}$$

Therefore, the correction factor is given by

$$I_{sq}^{n} = K_{isq}\,\mathrm{sign}\big(S(\Omega)\big). \tag{27}$$

To verify the system stability condition, parameter K_{isq} must be positive.

In order to limit all possible overshoots of the current I_{qs}, we add a limiter of current defined by

$$I_{sq}^{\lim} = I_{sq}^{\max}\,\mathrm{sat}(I_{sq})\,. \tag{28}$$

4.2 Stator current control

In order to limit all possible overshoots of the current I_{qs}, we add a limiter of current defined by

$$I_{sq}^{\lim} = I_{sq}^{\max}\,\mathrm{sat}(I_{sq})\,. \tag{29}$$

The current control manifold is

$$S(I_{sq}) = I_{sq}^{\lim} - I_{sq}\,, \tag{30}$$

$$\dot S(I_{sq}) = \dot I_{sq}^{\lim} - \dot I_{sq}\,, \tag{31}$$

Substituting the expression for $\dot I_{sq}$ in equation 2 into equation (31), we obtain:

$$\dot S(I_{sq}) = \dot I_{sq}^{\lim} - \Big(-\omega_s Isd - \gamma I_{sq} - P\Omega K\Phi_{rd} + \frac{1}{\sigma L_s}U_{sq}\Big). \tag{32}$$

The control voltage is

$$U_{sq}^{ref} = U_{sq}^{eq} + U_{sq}^{n}\,, \tag{33}$$

$$U_{sq}^{eq} = \sigma L_s\big(\dot I_{sq}^{\lim} + \omega_s I_{sd} + \gamma I_{sq} + P\Omega K\Phi_{rd}\big)\,, \tag{34}$$

$$U_{sq}^{n} = Ku_{sq}\,\mathrm{sat}\big(S(I_{sq})\big)\,. \tag{35}$$

To verify the system stability condition, parameter K_{usq} must be positive.

4.3. Rotor flux control

In the proposed control, we take $n = 2$ to appear control U_{sd}, the manifold equation can be obtained by:

$$s(\Phi_r) = \lambda_\Phi\big(\Phi_r^{ref} - \Phi_r\big) + \big(\dot\Phi_r^{ref} - \dot\Phi_r\big), \tag{36}$$

$$\dot s(\Phi_r) = \lambda_\Phi\big(\dot\Phi_r^{ref} - \dot\Phi_r\big) + \big(\ddot\Phi_r^{ref} - \ddot\Phi_r\big)\,. \tag{37}$$

The control voltage

$$U_{sd} = U_{sd}^{eq} + U_{sd}^{n}\,, \tag{38}$$

$$U_{sd}^{eq} = \sigma L_s\Big(\ddot\Phi_r^{ref} + \lambda_\Phi\dot\Phi_r^{ref} + \Big(\frac{1}{T_r} - \lambda_\Phi\dot\Phi_r\Big)\frac{T_r}{M_{sr}} - \sigma L_s\Big(-\gamma_{sd} + \omega_s I_{sq} + \frac{K}{T_r}\Phi_{rd}\Big), \tag{39}$$

$$U_{sd}^{n} = Ku_{sd}\,\mathrm{sat}\big(s(\Omega)\big)\,. \tag{40}$$

To verify the system stability condition, parameter Ku_{sd} must be positive.

The selection of coefficients Ki_{sq}, Ku_{sd}, Ku_{sq} and λ_Φ must be made so as to satisfy the following requirements:

- Existence condition of the sliding mode, which requires that the state trajectories are directed toward the sliding manifold,
- Hitting condition, which requires that the system trajectories encounter the manifold sliding irrespective of their starting point in the state space (insure the rapidity of the convergence),
- Stability of the system trajectories on the sliding manifold,
- Not saturate the control to allow the application of the control discontinuous.

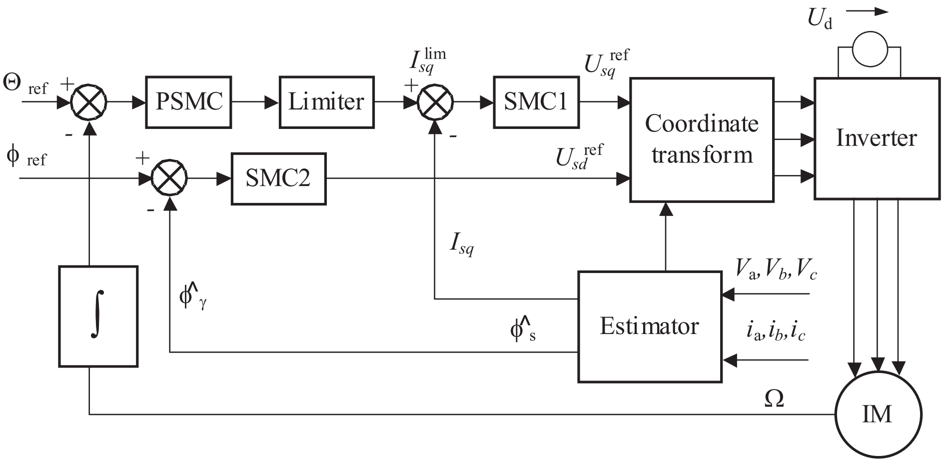

Fig. 1. Block diagram of the proposed control scheme of the IM

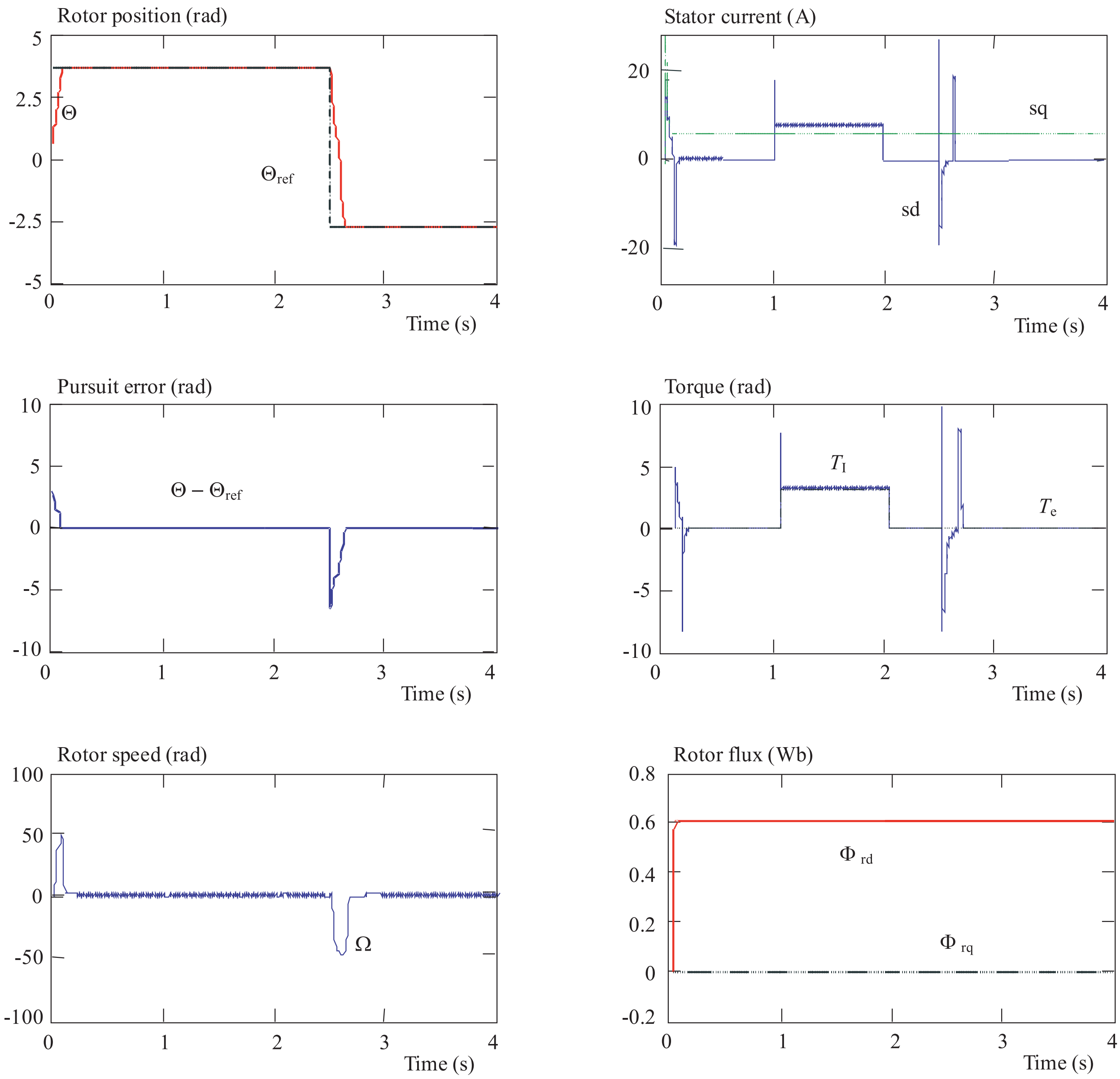

Fig. 2. System responses in nominal case

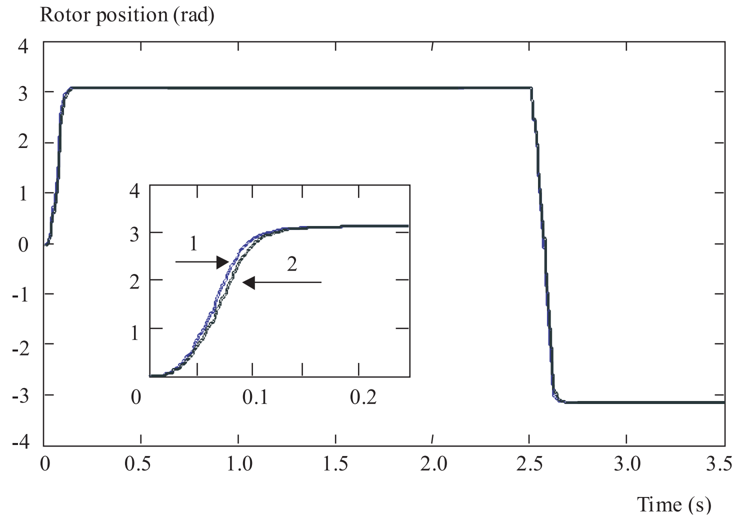

Fig. 3. Simulation results with the variation of parameters. 1: Nominal case, 2: ($1.5\,R_s$, $1.5\,R_r$, $1.2\,L_s$, $1.2\,L_r$, $1.2\,M_{sr}$, $2J$)

5 SIMULATION RESULTS

The proposed robust control scheme of the machine drive system is shown in Fig. 1. Blocks PSMC, SMC2, SMC3 represent the proposed sliding mode controllers, rotor position, stator current, flux, respectively. The block limiter limits the current within the limit values. Block 'Coordinate transform' makes the conversion between the synchronously rotating and stationary reference frame. 'Inverter' shows that the motor is voltage fed. 'Estimator' represents respectively the estimated stator current I_{sq} and the rotor flux Phi_r. Block 'IM' represents the induction motor. The IM used in this work is a 1.5 kW, $U = 220$ V, 50 Hz, $= 2$, $I_n = 6.1$ A, $\Phi_n = 0.595$ Wb.

IM parameters: $R_s = 1.47\,\Omega$, $R_r = 0.79\,\Omega$, $L_r = M_{sr} = 0.094$ H, $L_s = 0.105$ H. The system has the following mechanical parameters: $J = 0.00256\,\mathrm{Nm/rad/s^2}$, $f = 0.0029\,\mathrm{Nm/rad/s}$.

The global system is simulated using MATLAB/SIMULINK software.

5.1 Results and comments

The proposed control has been tested to illustrate its performances. We simulated a loadless starting up mode with the reference speed $\pm\pi$ and an application of the load torque ($T_l = \pm 10$ Nm) at time 1 s to 2 s and at time 3 s to 4 s, the reference flux is 0.595 Wb (Fig. 2).

In order to test the robustness of the proposed control, we have studied the position performances control with current limitation. The introduced variations in the tests simulate such work conditions as magnetic circuit overheating and saturation. The considered cases are: Inertia variation, stator and rotor resistance variations, stator and rotor inductance and mutual variations.

Figure 2 presents the system responses in the nominal case. Figure 3 shows the robustness tests in relation to variations of parameters.

From the system responses given in Fig. 2., the position tracks the desired position with an instantaneously perturbation reject. The decoupling between the flux and torque is assured in the permanent regime. The flux tracks the desired flux. It shows also the limited start torque. The I_{sd}, corresponding to the component, remains constant because the flux is maintained constant and the current I_{sq}, corresponding to the torque component varies with the torque.

Figure 3 shows that the parameter variation does not deteriorate performances of the proposed control. The position tracks the desired position and it is insensitive to parameter variations of the machine, without overshoot and without static error permanent regime, the perturbation reject is instantaneous.

The results show that high precision tracking can be achieved using the sliding mode controller in spite of large parameter variations.

6 CONCLUSION

A sliding mode control method has been proposed and used for the position control of an induction machine using the field oriented control. A simple algorithm to estimate the rotor flux is presented. Simulation results show good performances obtained with the proposed control. The proposed controller presents high robustness in the presence of internal and external perturbations. With a good choice of the control parameters, the chattering effects are reduced, and the torque fluctuations are decreased. The position control operates with sufficient stability and has strong robustness to parameter variations. Furthermore, this regulation presents a simple robust control algorithm that has the advantage to be easily implemented in a calculator.

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Received 22 November 2007

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