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OUTPUT STABLE MODEL PREDICTIVE CONTROL DESIGN WITH INPUT CONSTRAINTS

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The paper addresses the problem to design a quadratic stable output/state feedback model predictive control for linear systems with input constraints. In the proposed design technique the model predictive control is designed for N_2 step ahead prediction using the Lyapunov function approach with the cost function guaranteeing input constraints. Output gain matrix calculation is realized off line and through dynamic behavior with respect to quadratic stability of closed-loop system only modification of output gain matrix is realized to guarantee input constraints. Two examples are given to demonstrate the effectiveness of the proposed methods.

K e y w o r d s: model predictive control, quadratic stability, Lyapunov function, input constraints

1 INTRODUCTION

Predictive control algorithms compute the control variable minimizing a cost function considering expected future errors in a given prediction horizon. In each sampling time the series of the control signals is calculated, but according to receding horizon strategy only the first value is applied as a manipulated variable, and in the next sampling point the procedure is repeated. Predictive algorithms provide good performance especially in case of a big dead time and if the future reference trajectory is known. Nowadays in industrial control, besides PID control, predictive control gains more and more applications. The main criticism related to predictive control is that because of the finite prediction horizon the algorithm in its original formulation does not guarantee stability. Different extensions as e.g. including punishing of the end point deviation of the state variables in the cost function, or considering bigger weighting factors for punishing the output deviation at the last points of the prediction horizon would ensure stable performance. There are some approaches to guarantee the stability of closed-loop systems. The first type of approaches has originated by Rawlings and Muske [14] where the central idea is that if the minimization problem is feasible, the cost function can be interpreted as a monotonically decreasing Ljapunov function and asymptotic stability is therefore guaranteed. The main idea of the next approach, Clarke and Mohtadi [4], Clarke and Scattolini [5], Dermicioglu and Clarke [6], is to impose state terminal constraints to force the predicted output to exactly follow the reference during a sufficiently large horizon. Stability results for constrained MPC have been obtained by Rossiter and Kouvaritakis [17] who found that for any reference $w(t)$ which assumes a constant value after a number of sampling periods, if the constrained MPC is feasible for sufficiently large values of the horizon, the closed-loop will be stable. The reader can consult other approaches in Veselý and Bars [18], Rossiter [16], Camacho and Bordons [3], Maciejowski [12], Haber *et al* [9], Mayne *et al* [13]. Here closed-loop system stability guarantees are given creating a Lyapunov function for the design of a state or output feedback controller with input constraints.

The paper is organized as follows. The next section gives a problem formulation and preliminary about a predictive output/state model. In Section 3, the approach of output feedback predictive controller design using linear matrix inequality is presented. In Section 4, the input constraints are formulated to LMI feasible solution. Two examples in Section 5 illustrate the effectiveness of the proposed method. Finally, some conclusions are given.

2 PRELIMINARIES AND PROBLEM FORMULATION

A time invariant linear discrete-time system is given by

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t)\,, \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n, u(t) \in R^m, y(t) \in R^l$ are state, control and output variables of the system, respectively; A, B, C are known matrices of corresponding dimensions.

The state of the model for instant $t + N_2$ can be computed recursively (see [3]) applying (1), the result is

$$x(t+N_2) = A^{N_2}x(t) + \sum_{i=0}^{N_2-1} A^{N_2-i-1}Bu(t+i) \tag{2}$$

and the corresponding output is

$$y(t+N_2) = Cx(t+N_2)\,. \tag{3}$$

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Consider a set of N_2 steps ahead predictions, one obtains

$$\begin{aligned} z(t+1) &= A_f z(t) + B_f v(t)\,, \\ y_f(t) &= C_f z(t) \end{aligned} \tag{4}$$

where

$$\begin{aligned}
z(t)^\top &= [\,x(t)^\top \quad x(t+1)^\top \quad \dots \quad x(t+N_2-1)^\top\,]\,,\\
v(t)^\top &= [\,u(t)^\top \quad u(t+1)^\top \quad \dots \quad u(t+N_2-1)^\top\,]\,,\\
y_f(t)^\top &= [\,y(t)^\top \quad y(t+1)^\top \quad \dots \quad y(t+N_2-1)^\top\,]\,,\\
A_f &= \begin{bmatrix} A & 0 & \dots & 0 \\ A^2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ A^{N_2} & 0 & \dots & 0 \end{bmatrix} \in R^{(nN_2)\times(nN_2)},\\
B_f &= \begin{bmatrix} B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ \dots & \dots & \dots & \dots \\ A^{N_2-1}B & \dots & \dots & B \end{bmatrix} \in R^{(nN_2)\times(mN_2)},\\
C_f &= \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & C \end{bmatrix} \in R^{(lN_2)\times(nN_2)}.
\end{aligned}$$

If the state $x(t)$ for state feedback is not accessible, Kalman filter is required.

Let the cost function to be minimized be in the form [10]

$$J = \sum_{t=t_0}^{\infty} J(t) \tag{5}$$

where

$$J(t) = \min_{v(t)} \big\{ [y_f(t+1) - w_f(t+1)]^\top Q [y_f(t+1) - w_f(t+1)] + v(t)^\top R v(t) \big\}$$

and

$$w_f(t+1)^\top = [\,w(t+1)^\top \quad \dots \quad w(t+N_2)^\top\,]$$

gives the predicted setpoints. Q, R are positive definite matrices.

The problem studied in this part of the paper is to design a model predictive controller with output (state) feedback in the form

$$v(t) = F(y_f(t) - w_f(t)) \tag{6}$$

where F is the output feedback gain matrix which guarantees stability of the closed-loop system and minimum value of the cost function $J(t)$ subject to (4).

3 MODEL PREDICTIVE CONTROLLER DESIGN

Let us consider a set of $y_f(t+j)$, $j \in \langle 1, N_2\rangle$ ahead output predictions affecting the cost function (5) and the vector of $v(t+j)$, $j \in \langle 0, N_u\rangle$ future control in the control horizon. A new system and input matrices are formed by the corresponding submatrices of $A_f, B_f \in R^{(nN_2)\times(mN_u)}$ respectively. For more detail see [3]. We will proceed without changing the denotation.

$$v(t) = F(y_f(t) - w_f(t)) = F(C_f z(t) - w_f(t)) \tag{7}$$

and the closed-loop system is given by

$$\begin{aligned} z(t+1) &= (A_f + B_f F C_f) z(t) - B_f F w_f(t) \\ &= A_c z(t) - B_f F w_f(t)\,. \end{aligned} \tag{8}$$

Because the vector $w_f(t)$ is independent of vector $z(t)$ and if vector $w_f(t)$ belongs to the class of L_2, the stability of the closed-loop system (8) is determined by matrix A_c. The origin of the state vector $z(t)$ has to be recalculated to a new steady state given by the set point vector $w_f(t)$. Due to Lyapunov function approach and because of the stability of the closed-loop system determined by the matrix A_c we assume that $w(t) = w(t+1) = \dots = 0$. The closed-loop system will be stable if and only if the first difference of Lyapunov function $V(t) = z(t)^\top P z(t)$, $P = P^\top > 0$ on the solution of (8) will be negative definite (semidefinite) that is

$$\Delta V(t) = V(t+1) - V(t) = z(t)^\top (A_c^\top P A_c - P) z(t) \le 0\,, \quad t = 1, 2, \dots \tag{9}$$

Substituting (7) and (8) to cost function (5) one obtains

$$J(t) = \min_F z(t)^\top (A_c^\top C_f^\top Q C_f A_c + C_f^\top F^\top R F C_f) z(t) \ge 0 \tag{10}$$

From (10) one can see that $J(t)$ for $t = 1, 2, \dots$ is a positive definite (semidefinite) function. Closed-loop system (8) will be stable and cost function $J(t)$ reaches minimal value iff the following inequality holds (Krokavec and Filasova, 2008)

$$B_e(t) = z^\top(t)[A_c^\top P A_c - P + A_c^\top C_f^\top Q C_f A_c + C_f^\top F^\top R F C_f] z(t) \le 0 \tag{11}$$

for $t = 1, 2, \dots$ *ie* the matrix expression in (11) will be negative definite (semidefinite). From (11) one can recast the following bilinear matrix inequality (BMI)

$$\begin{bmatrix} -P & A_c^\top & A_c^\top C_f^\top Q & C_f^\top F^\top R \\ * & -P^{-1} & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -R \end{bmatrix} \le 0 \tag{12}$$

where * denote the transpose of the corresponding symmetric part of the matrix. Using linearization approach with respect to P^{-1} (see [7]) the following inequality holds

$$-P^{-1} \leq Y_k^{-1}(P - Y_k)Y_k^{-1} - Y_k^{-1} = lin(-P^{-1}). \quad (13)$$

Due to (13), (12) becomes LMI in the form

$$\begin{bmatrix} -P & A_c^\top & A_c^\top C_f^\top Q & C_f^\top F^\top R \\ * & lin(-P^{-1}) & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -R \end{bmatrix} \leq 0 \quad (14)$$

where $Y_k, k = 1, 2, \dots$ in the iteration process $Y_k = P$. Because the linearization approach has been used for (12), BMI conditions (12) "if and only if" reduces to "if" for (14). If the LMI (14) is feasible with respect to matrices P and F, the closed-loop system will be quadratically stable with minimal value of the cost function. Note that using (7) with matrix F one could calculate the input (control) vector $v(t)$ for $N_u - 1$ predicted control horizon from N_2 predicted output vector $y_f(t)$. As a receding horizon strategy is used, only the first element of the control vector $u(t)$ is sent to the plant and all the computation is repeated at the next sampling time. Let us rewrite (4) as follows

$$z(t+1) = A_f z(t) + (B_1 + B_2)v(t) \quad (15)$$

where $B_f = B_1 + B_2$, $B_1 = [B_{f1} \quad 0_1]$, $B_2 = [0_2 \quad B_{f2}]$ and

$$B_{f1} \in R^{(nN_2)\times m}, \quad 0_1 \in R^{(nN_2)\times(N_u-1)m}$$
$$B_{f2} \in R^{(nN_2)\times(N_u-1)m}, \quad 0_2 \in R^{(nN_2)\times m}$$

and 0_i, $i = 1, 2$ are zero matrices with corresponding dimensions.

From (15) we get

$$(B_1+B_2)v(t) = B_{f1}u(t)+O_1 v(t+1)+0_2 u(t)+B_{f2}v(t+1). \quad (16)$$

Equation (16) implies: to guarantee the stability of the closed-loop system for control variable $u(t)$ instead of matrix B_f in (14) one has to use the matrix of B_1.

We can conclude that if the following two LMIs

$$\begin{aligned} \{in\, eq\,(14) \quad with \quad B_f\} \leq 0, \\ \{eq\,(14) \quad with \quad B_1\} \leq 0 \end{aligned} \quad (17)$$

are feasible with respect to $P = P^\top => 0$ and matrix F, the closed-loop system with control variables $v(t)$ and $u(t)$ will be quadratically stable and the cost function will have a minimal value. Note that only the first m rows of matrix F are used for real plant control with control input $u(t)$. The above results can be summarized in the following theorem.

THEOREM 1. *We are given a discrete linear-time invariant system* (1). *Assume that control algorithm for model predictive control is given by* (6), *where* $w(t + j) = 0$ *or constant,* $j = 1, 2, \dots, N_u - 1$. *(For the case of* $w(t + j) =$*cons the origin of state space of system* (1) *has to be recalculated to achieve the steady state given by constant set points). If the two LMIs* (17) *are feasible with respect to* $P = P^\top > 0$ *and a matrix* F, *then the cost function* (6) *has minimal value and quadratic stability of closed-loop system is guaranteed for two cases*

- *output feedback with control algorithm given by* (6) *and*
- *output feedback for control variable* $u(t) = Fy_f(t)$ *when only first* m *rows of matrix* F *are used.*

4 MPC DESIGN FOR INPUT CONSTRAINTS

To design model predictive control (Adamy and Flemming [1], Camacho and Bordons [3]) with any constraints on input, state and output variables at each sampling time, starting from the current state, an open-loop optimal control problem is solved over the defined finite horizon. The first element of the optimal control sequence is applied to the plant. At the next time step, the computation is repeated. Thus, the implementation of the MPC strategy requires a QP solver for the on-line optimization which still requires significant on-line computational effort, which limits MPC applicability. In this paper we propose the off-line calculation of two control gain matrices and using analogy to SVSS approach, Adamy and Flemming, 2004 we significantly reduce the computational effort for MPC suboptimal control with input constraints.

Consider the system (4) where the control $v(t)$ is constrained to evolve in the following set

$$\Gamma = \{v \in R^{mN_2} : |v_i(t)| \leq U_i\,,\ i = 1, \dots mN_2\}. \quad (18)$$

The aim of this part of paper is to design the stabilizing control output feedback law for system (4) in the form

$$v(t) = FC_f z(t) \quad (19)$$

which guarantees that for the initial state $z_0 \in \Omega(P) = \{z(t) : z(t)^\top P z(t) \leq \theta\}$ control $v(t)$ belongs to the set (18) for all $t \geq 0$, where $P = P^\top > 0$ is positive definite matrix, θ is positive real parameter which determines the size of $\Omega(P)$. Furthermore, $\Omega(P)$ should be such that all $z(t) \in \Omega(P)$ provide $v(t)$ satisfying the relation (18), restricting the values of the control parameters. Moreover, the following ellipsoidal Ljapunov function level set

$$\Omega(P) = \{z(t) \in R^{nN_2} : z(t)^\top P z(t) \leq \theta\} \quad (20)$$

can be proven to be a robust positively invariant region with respect to motion of the closed-loop system in the sense of the following definition, Rohal-Ilkiv [15], Ayd and *et al* [2].

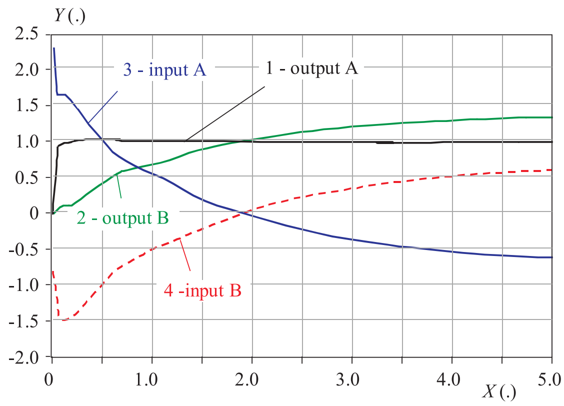

Fig. 1. Dynamic behavior of unconstrained controlled system

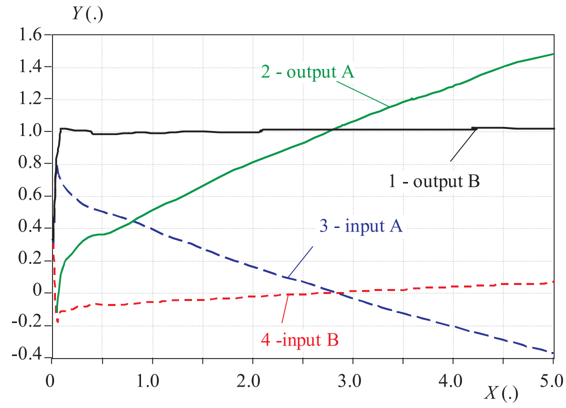

Fig. 2. Dynamic behavior of constrained controlled system

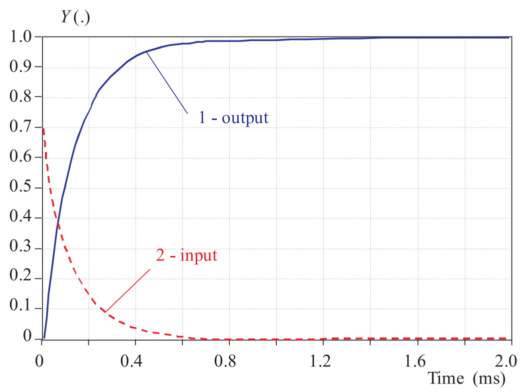

Fig. 3. Dynamic behavior of controlled system for unconstrained case

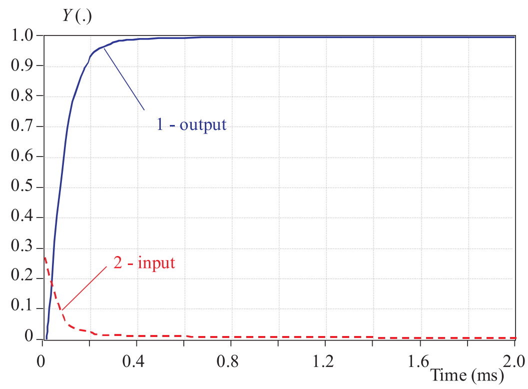

Fig. 4. Dynamic behavior of controlled system for constrained case

DEFINITION. A subset $S \in R^{(nN_2)}$ is said to be positively invariant with respect to motion of system (4) with control algorithm (7) if for every initial state $z(0)$ inside S the trajectory $z(t)$ remains in S for all $t \geq 0$.

Consider that vector F_i denotes the i-th row of matrix F and define

$$L(F) = \{z(t) \in R^{(nN_2)} \colon |F_i C_f z(t)| \leq U_i, \}\,, \qquad i = 1, 2, \dots, mN_2\,.$$

The above set can be written as

$$L(F) = \{z(t) \in R^{(nN_2)} \colon |D_i F C_f z(t)| \leq U_i, \}\,, \qquad i = 1, 2, \dots, mN_2 \tag{21}$$

where $D_i \in R^{1 \times mN_2} = \{dij\}$, $d_{ij} = 1$, $i = j$; $d_{ij} = 0$, $i \neq j$. The results are summarized in the following theorem.

THEOREM 2. *The inclusion $\Omega(P) \subseteq L(F)$ is for output feedback control equivalent to*

$$\begin{bmatrix} P & C_f^\top F^\top D_i^\top \\ D_i F C_f & \lambda_i \end{bmatrix} \geq 0\,,\ i = 1, 2, \dots, mN_2 \tag{22}$$

where $\lambda_i \in \left\langle 0, \frac{U_i^2}{\theta} \right\rangle$.

P r o o f . To prove the inclusion $\Omega(P) \subseteq L(F)$ is equivalent to (22) we use $S-$ procedure in following way. Rewrite (20) and (21) in the following form

$$p(z) = z^\top(t) P Z(t) - \theta \leq 0\,,$$

$$g_i(z) = z^\top(t) C_f^\top F^\top D_i^\top D_i F C_f z(t) - U_i^2 \leq 0\,.$$

According to $S-$ procedure there exists a positive scalar λ_i such that

$$g_i(z) - \lambda_i p(z) \leq 0$$

or equivalently

$$z(t)^\top (C_f^\top F^\top D_i^\top D_i F C_f - \lambda_i P) z(t) - U_i^2 + \lambda_i \theta \leq 0\,. \tag{23}$$

Taking any scalar $\varrho \neq 0$ and multiplying (23) from left and right side we obtain

$$\begin{bmatrix} C_f^\top F^\top D_i^\top D_i F C_f - \lambda_i P & 0 \\ 0 & -U_i^2 + \lambda_i \theta \end{bmatrix} \leq 0\,, \qquad i = 1, 2, \dots, mN_2\,. \tag{24}$$

The above diagonal matrix is equivalent to two inequalities. Using Schur complement formula for the first one one obtains the inequality (22) which proves the theorem.

In order to check the value of θ_i for i-th input one solves the optimization problem $z(t)^\top Pz(t) \to \max$ subject to constraints (18), whose solution yields

$$\theta_i = \frac{U_i^2}{D_i F C_f P^{-1} C_f^\top F^\top D_i^\top} \,.$$

In the design procedure it should be verified that when parameter θ decreases the obtained robust positively invariant regions $\Omega(P)$ are nested to region obtained for $\theta+\epsilon$, $\epsilon>0$.

Assume that one calculates two output feedback gain matrices F_1 for unconstrained case and F_2 for constrained one. Normally, closed-loop system with the gain matrix F_2 gives the dynamic behavior slower than one obtain for F_1. Consider the resulting output feedback gain matrix F in the form

$$F = \gamma F_1 + (1-\gamma)F_2, \gamma \in (0,1)\,. \tag{25}$$

For gain matrices F_i, $i=1,2$ one obtains two closed-loop system in the form (8), $A_{ci} = A_f + B_f F_i C_f$, $i=1,2$. Consider the edge between within A_{c1} and A_{c2}, that is

$$A_c = \alpha A_{c1} + (1-\alpha)A_{c2}, \alpha \in \langle 0,1\rangle\,. \tag{26}$$

The following lemma gives the stability conditions for matrix A_c (26).

LEMMA 1. *Consider the stable closed-loop matrices A_{ci}, $i=1,2$.*

- *If there exists a positive definite matrix P_q such that*

$$A'_{ci}P_q + P_q A_{ci} \le 0\,,\ i=1,2 \tag{27}$$

  then matrix A_c is quadratically stable.
- *If there exist two positive definite matrices P_1, P_2 such that they satisfy the stability conditions given by Grman et al [8], the closed-loop system A_c is parameter dependent quadratically stable (PDQS).*

Remarks.

- If the closed-loop matrices $A_{ci}, i=1,2$ satisfy (27) the scalar γ in (25) may be change with any rate without violating the closed-loop stability.
- If the closed-loop matrices A_{ci}, $i=1,2$ are only PDQS, the scalar γ in (25) has to be constant but may be unknown.
- The proposed control algorithm (25) is similar to Soft Variable-Structure Control (SVSC) [1] but in our case, when $v_i < U_i$ the feedback gain matrix F (25) gives rather more quick dynamic behavior to the closed-loop system then when v_i approaches U_i. The algorithm for calculation of γ (25) may be as follows

$$\gamma = \min_i \frac{U_i - |v_i|}{U_i}$$

  If accidentally some $v_i > U_i$, $\gamma = 0$.

5 EXAMPLES

The first example has been borrowed from [3, p. 147]. The model corresponds to the longitudinal motion of a Boeing 747 airplane. The multivariable process is controlled using a predictive controller based on the output model of the aircraft. Two of the usual command outputs that must be controlled are airspeed that is, velocity with respect to air, and climb rate. Continuous model has been converted to discrete time one with sampling time of 0.1 s, the model turns to (1) where

$$A = \begin{bmatrix} .9996 & .0383 & .0131 & -.0322 \\ -.0056 & .9647 & .7446 & .0001 \\ .002 & -.0097 & .9543 & 0 \\ .0001 & -.0005 & .0978 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} .0001 & .1002 \\ -.0615 & .0183 \\ -.1133 & .0586 \\ -.0057 & .0029 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 7.74 \end{bmatrix}.$$

Note that matrix A is unstable. For the weighted matrices $Q = 10I$, $R = I$, $N_u = N_2 = 6$ the following results are obtained

- Unconstrained case
  $Closed-loopmaxeig = 0.8371 \pm 0.2873i$.
- Constrained case with $U_i = 1, \theta = 1$,
  $Closed-loopmaxeig = 0.7546 \pm 0.3010i$.

Closed-loop step responses of the above two cases are given in Fig. 1 and Fig. 2. The closed-loop system (26) is quadratically stable.

The second example serves as a benchmark. The model of double integrator turns to (1) where

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C = [0 \quad 1]\,.$$

For the case of $N_2 = 6$, $N_u = 6$, $Q = 5I$, $R = I$ for unconstrained case the gain matrix to control variable $v(t) = F1y_f(t)$ is as follows

$F1 =$

$$\begin{bmatrix} -0.0206 & 0.0097 & -0.0081 & -0.0177 & -0.0241 & -0.0293 \\ -0.6432 & 0.0057 & -0.0052 & -0.0111 & -0.0151 & -0.0183 \\ 0.4136 & -0.0038 & 0.0033 & 0.0072 & 0.0097 & 0.0118 \\ 0.1700 & -0.0016 & 0.0014 & 0.0029 & 0.0040 & 0.0049 \\ 0.0582 & -0.0005 & 0.0005 & 0.0010 & 0.0014 & 0.0017 \\ 0.0310 & -0.0002 & 0.0003 & 0.0005 & 0.0007 & 0.0009 \end{bmatrix}$$

and for constrained case with $U_i = 0.3$, $i = 1,2,\dots,6$, $\theta = 1$

$F2 =$

$$\begin{bmatrix} 0.0441 & 0.0600 & -0.0111 & -0.0476 & -.0707 & -.0881 \\ -0.2271 & 0.0182 & -0.0014 & -0.0112 & -0.0171 & -0.0215 \\ 0.2389 & -0.0163 & 0.0023 & 0.0119 & 0.0178 & 0.0222 \\ 0.0947 & -0.0065 & 0.0009 & 0.0047 & 0.0071 & 0.0088 \\ 0.0407 & -0.0028 & 0.0004 & 0.0020 & 0.0030 & 0.0038 \\ 0.0266 & -0.0018 & 0.0003 & 0.0013 & 0.0020 & 0.0025 \end{bmatrix}$$

Closed-loop step responses for unconstrained and constrained cases are given in Fig. 3 and Fig. 4. The closed-loop system (26) is quadratically stable.

6 CONCLUSION

The paper addresses the problem of the design of a state or output feedback model predictive controller with input constraints for N_2 step prediction. The obtained control algorithm guarantees the closed-loop system quadratic stability and optimal value of performance function using Lyapunov function approach. Finally two examples are given which show the effectiveness of the proposed methods.

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References

[1] ADAMY, J.—FLEMMING, A.: Soft Variable-Structure Controls: a Survey, Automatica **40** (2004), 1821–1844.

[2] AYD, H.—MESQUINE, F.—AITRAMI, M.: Robust Control for Uncertain Linear Systems with State and Control Constraints, In: Proc. of the 17th World Congress IFAC, Seoul, Korea, 2008, 1153–1158.

[3] CAMACHO, E. F.—BORDONS, C.: Model Predictive Control, Springer-Verlag, London, 2004.

[4] CLARKE, D. W.—MOHTADI, C.: Properties of Generalized Predictive Control, Automatica **25** No. 6 (1989), 859–875.

[5] CLARKE, D. W.—SCATTOLINI: Constrained Receding-Horizon Predictive Control, Proceedings IEE **138** No. 4 (1991), 347–354.

[6] DERMICIOGLU, H.—CLARKE, D. W.: Generalized Predictive Control with End-Point Weighting, IEE Proc **140,** Part D No. 4 (1993), 275–282.

[7] de OLIVEIRA, M. C.—CAMINO, J. F.—SKELTON, R. E.: A Convexifying Algorithm for the Design of Structured Linear Controllers, In: Proc.39th IEEE Conference on Decision and Control, Sydney, 2000, 2781–2786.

[8] GRMAN, L.—ROSINOVÁ, D.—VESELÝ, V.—KOZÁKOVÁ, A.: Robust Stability Conditions for Polytopic Systems, Int. Journal of Systems Science **36** No. 15 (2005), 961–973.

[9] HABER, R.—SCHMITZ, U.—BARS, R.: Stabilizing Predictive Control with Infinite Control Error Weighting, In Conf. Process Control, Kouty nad Desnou, Czech Republik, 2002, paper R221a, 1–11.

[10] KOTHARE, M. V.—BALAKRISHNAN, V.—MORARI, M.: Robust Constrained Model Predictive Control Using Linear Matrix Inequalities, Automatica **32** No. 10 (1996), 1361–1379.

[11] KROKAVEC, D.—FILASOVÁ, A.: Discrete Systems, Elfa, 2008. (in Slovak)

[12] MACIEJOWSKI, J. M.: Predictive Control with Constraints, Prentice Hall, 2002.

[13] MAYNE, D. Q.—RAWLINGS, J. B.—RAO, C. V.—SCOKAERT, P. O. M.: Constrained Model Predictive Control: Stability and Optimality, Automatica **36** (2000), 789–814.

[14] RAWLINGS, J.—MUSKE, K.: The Stability of Constrained Receding Horizon Control, IEEE Trans. on Automatic Control **38** (1993), 1512–1516.

[15] ROHAL-ILKIV, B.: A Note on Calculation of Polytopic Invariant and Feasible Sets for Linear Continuous-Time Systems, Annual Rewiew in Control **28** (2004), 59–64.

[16] ROSSITER, J. A.: Model Based Predictive Control: A Practical Approach, CRC Press, Control Series, 2003.

[17] ROSSITER, J. A.—KOUVARITAKIS, B.: Advances in Model Based Predictive Control, Chapter Advances in Generalized and Constrained Predictive Control, Oxford University Press, 1994.

[18] VESELÝ, V.—BARS, R.: Stable Output Feedback Model Predictive Control Design: LMI Approach, Archives of Control Sciences **18** No. 3 (2008), 385–394.

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